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Input-Dependent Uncorrelated Weighting for Monte Carlo Denoising

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Connecting STORIES

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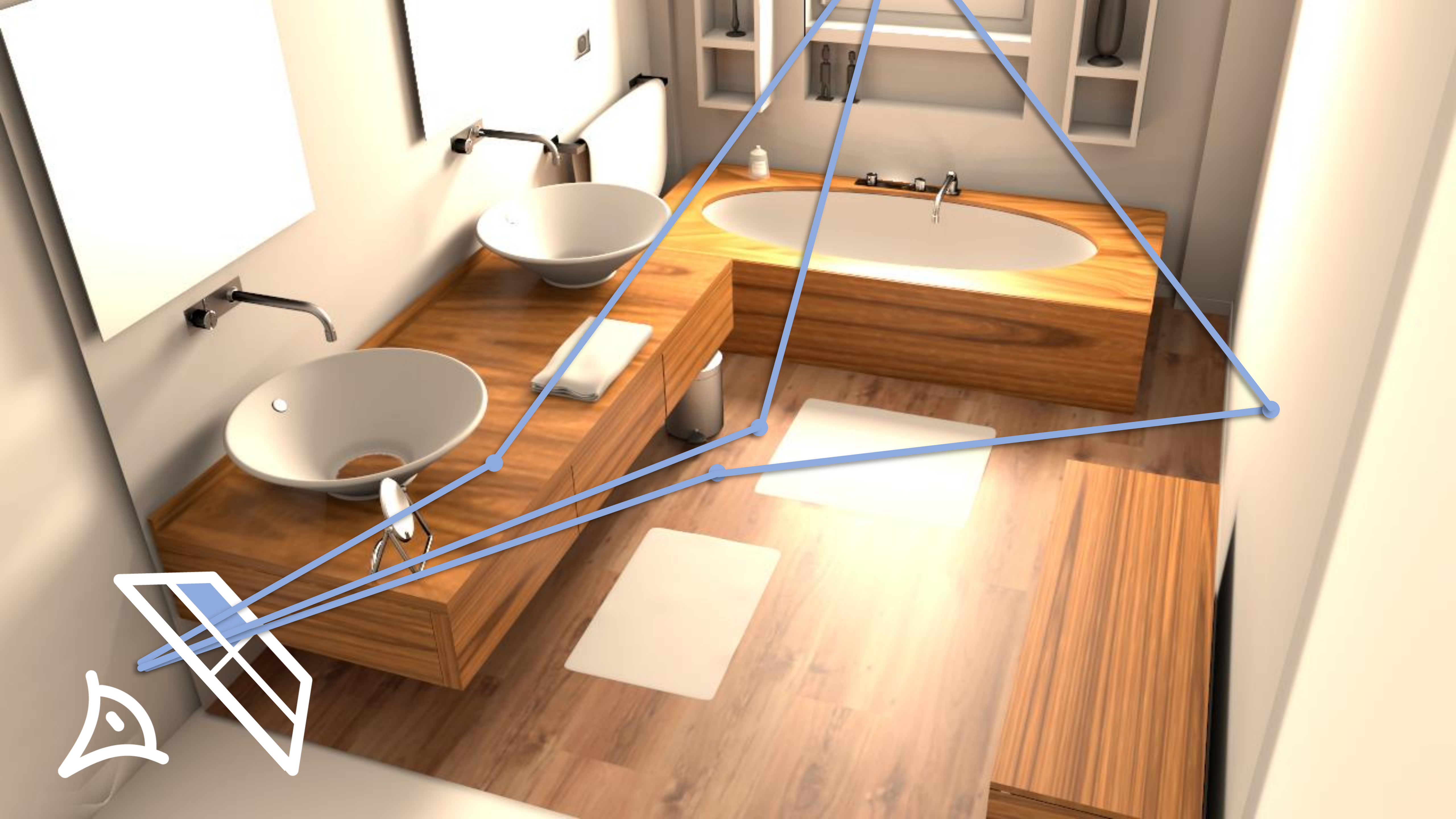
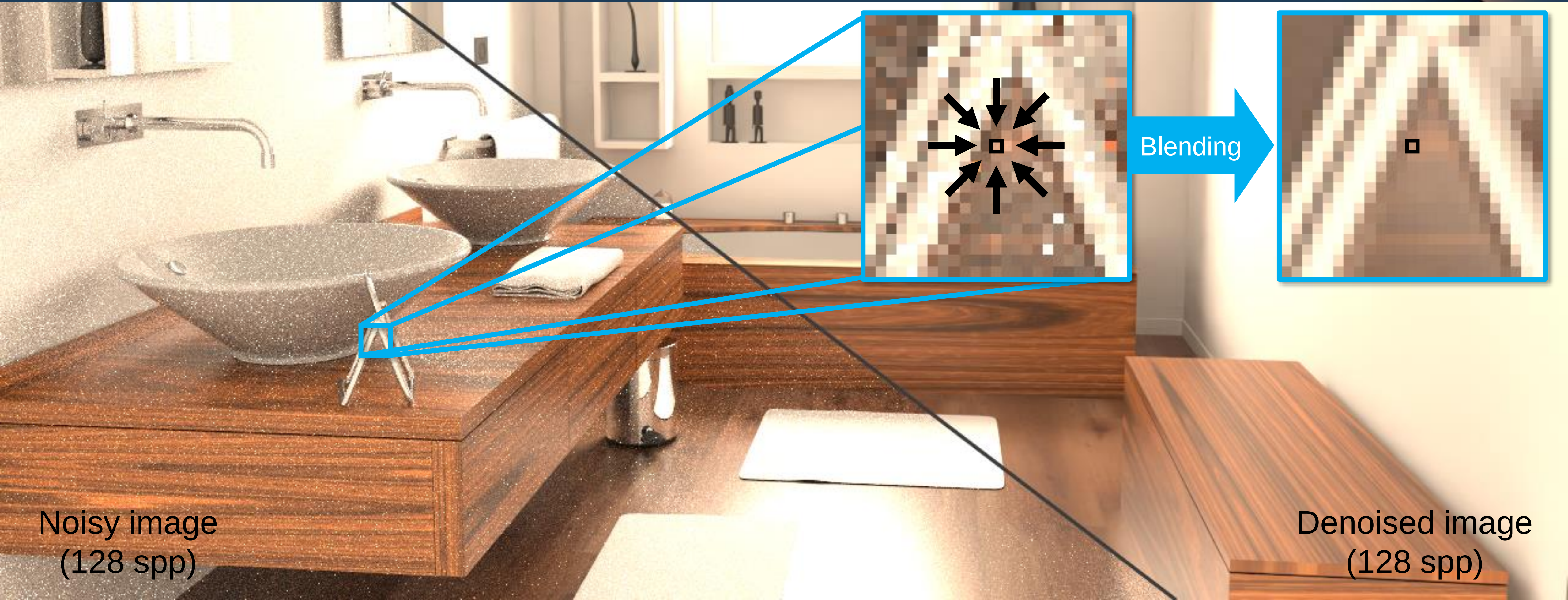


Path tracing
(128 spp)
Path tracing
(65,536 spp)

Monte Carlo Denoising

- Effective approach to reduce noise in Monte Carlo rendering



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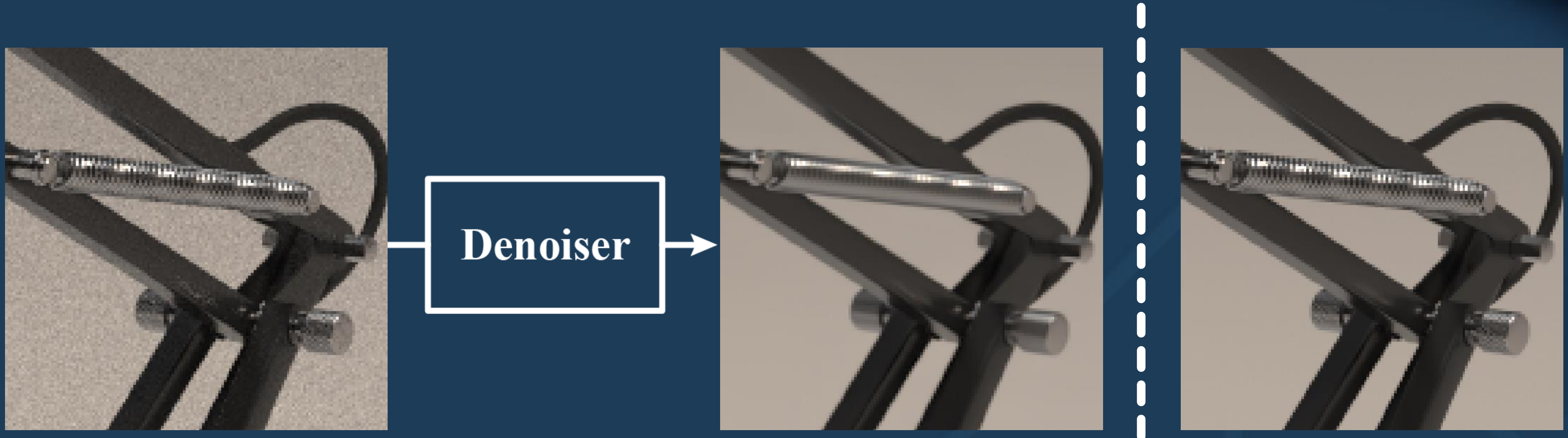
Monte Carlo Denoising

- Showing effective variance reduction but severe denoising bias



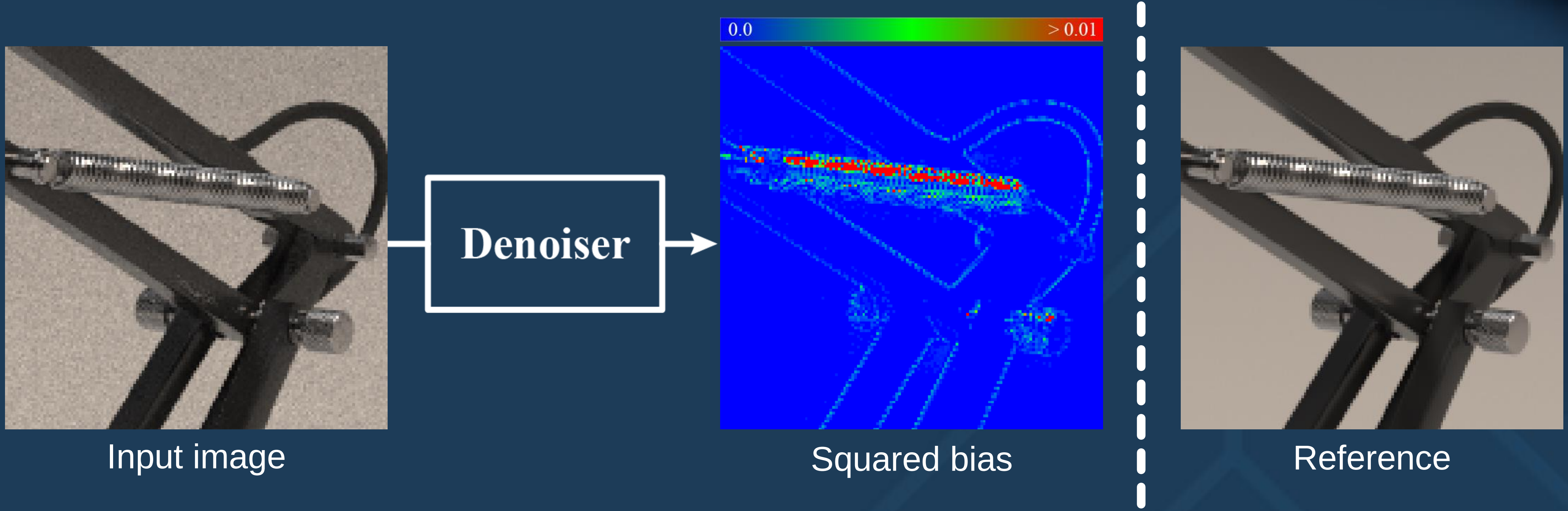
Input image

Denoised image

Reference

Monte Carlo Denoising

- Showing effective variance reduction but severe denoising bias



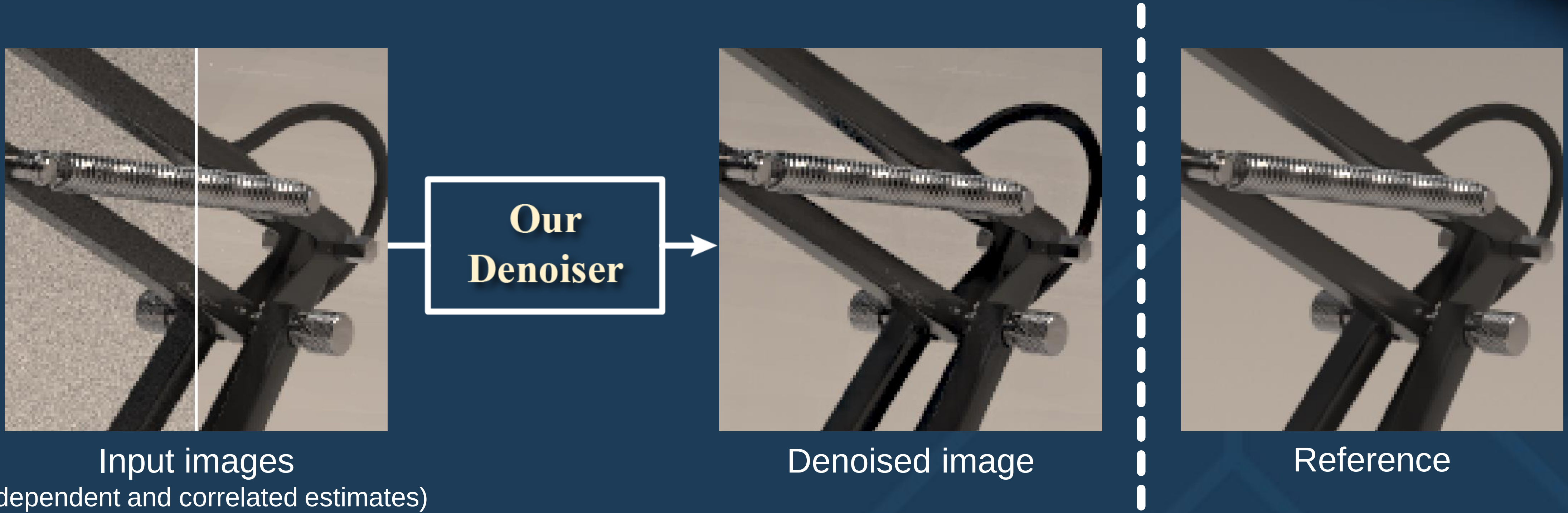
Input image

Squared bias

Reference

Our Goal

- Designing a denoising framework enabling to reduce some noise while limiting bias



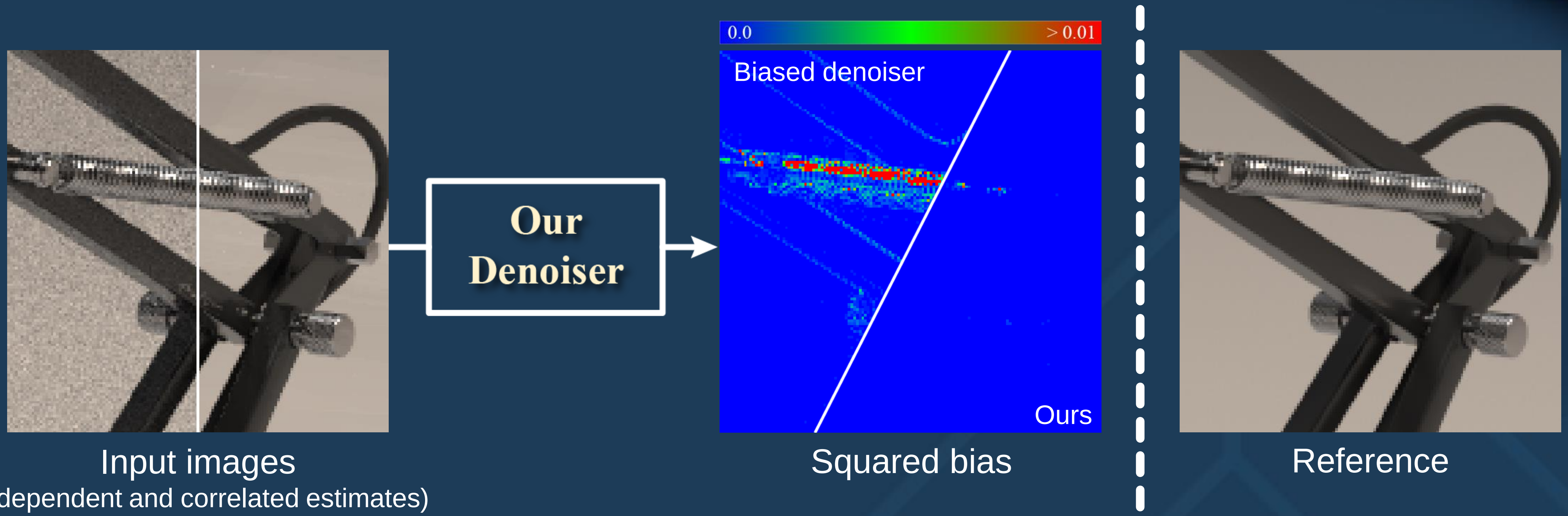
Input images
(independent and correlated estimates)

Denoised image

Reference

Our Goal

- Designing a denoising framework enabling to reduce some noise while limiting bias



Input images
(independent and correlated estimates)

Squared bias

Reference

Related Work

- Denoising for Monte Carlo ray tracing [Zwicker 2015]
  - Classical approaches
    ([Li 2012], [Sen 2012], [Rousselle 2012, 2013], [Moon 2014, 2016], [Bitterli 2016])
  - Learning-based approaches
    ([Kalantari 2015], [Bako 2017], [Vogels 2018], [Gharbi 2019], [Meng 2020], [Işık 2021], [Yu 2021])

- Reconstruction for gradient-domain rendering [Hua 2019]
  - Classical approaches
    ([Kettunen 2015], [Rousselle 2016], [Manzi 2016])
  - Learning-based approaches
    ([Guo 2019], [Kettunen 2019])

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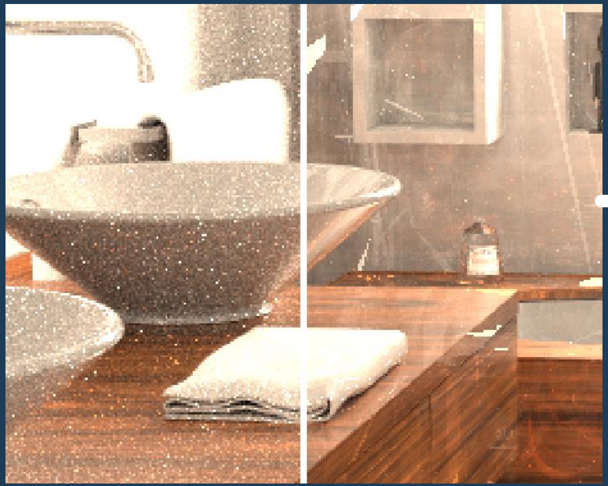
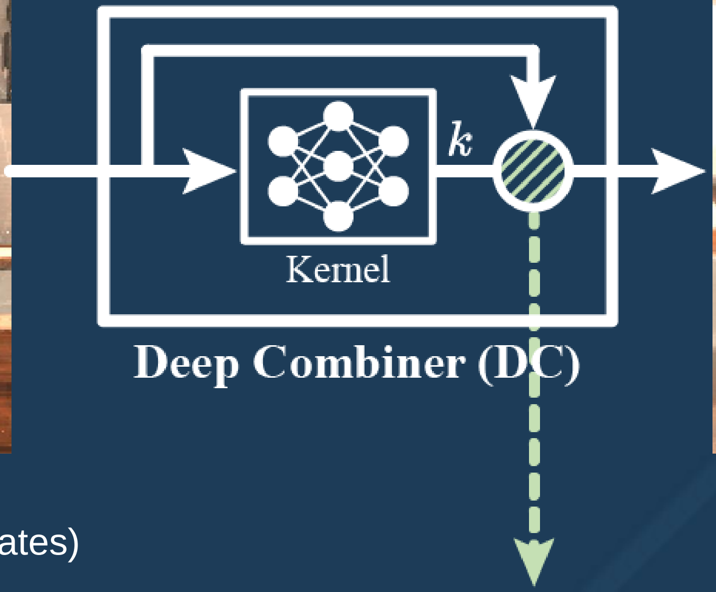
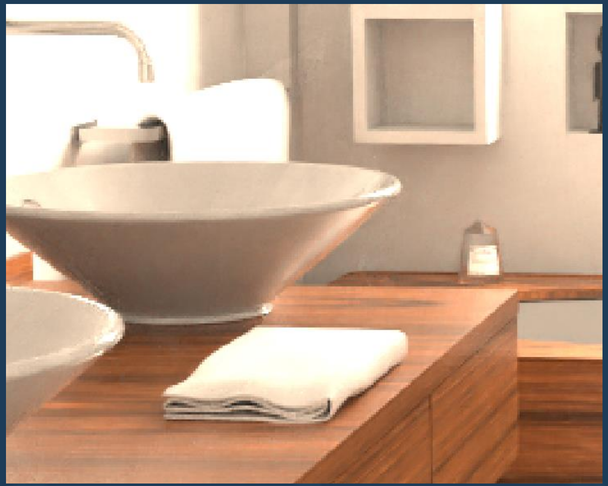
Background and Motivation

- Combining technique between independent and correlated pixel estimates [Back 2020]



Input images
(independent and correlated pixel estimates)





Combined image

μ_c, μ_i : ground truth at pixels c and i
y_c, y_i : independent pixel estimate at pixels c and i
z_c, z_i : correlated pixel estimate at pixels c and i
k_i : normalized weight for pixel i
Ω_c : set of neighboring pixels nearby pixel c

$$\hat{\mu}_c = k_c y_c + \sum_{i \in \Omega_c} k_i y_i + \sum_{i \in \Omega_c} k_i (z_c - z_i)$$

Combination function

Background and Motivation

- Bias analysis of combination function
  - Denoising using two unbiased independent and correlated estimates, y and z

Unbiased denoising if a kernel is uncorrelated with its inputs y and z

$$E[\hat{\mu}_c] = \boxed{E[k_c]E[y_c]} + \sum_{i\in\Omega_c} \boxed{E[k_i]E[y_i]} + \sum_{i\in\Omega_c} \boxed{E[k_i]E[z_c - z_i]}$$

$$= E[k_c]\mu_c + \sum_{i\in\Omega_c} E[k_i]\mu_i + \sum_{i\in\Omega_c} E[k_i](\mu_c - \mu_i) = \mu_c$$

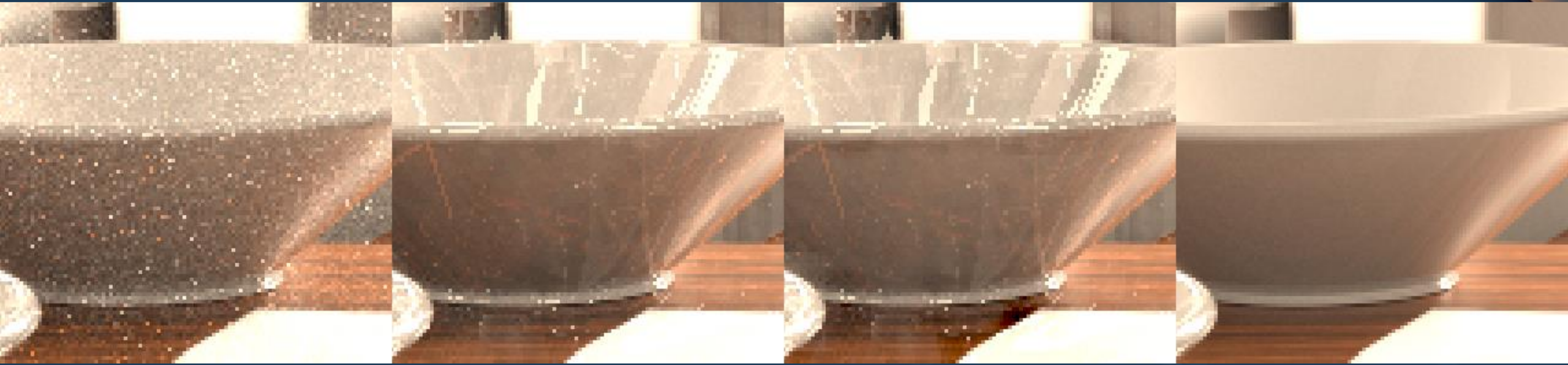
μ_c, μ_i : ground truth at pixels c and i
y_c, y_i : independent pixel estimate at pixels c and i
z_c, z_i : correlated pixel estimate at pixels c and i
k_i : normalized weight for pixel i
Ω_c : set of neighboring pixels nearby pixel c

$$\hat{\mu}_c = k_c y_c + \sum_{i\in\Omega_c} k_i y_i + \sum_{i\in\Omega_c} k_i(z_c - z_i)$$

Combination function

Background and Motivation

- A straightforward way to achieve the condition is to use an input-independent kernel.
  - Unbiased but not effective in reducing variance



$RelL_2$ 0.043677	$RelL_2$ 0.034509	$RelL_2$ 0.020037	64K spp
Input image (PT†, 256 spp)	Input image (CRN‡, 256 spp)	Denoised image (w/ uniform kernel)	Reference

† PT: path tracing with independent sampling, ‡ CRN: path tracing with correlated sampling using common random numbers (CRN)

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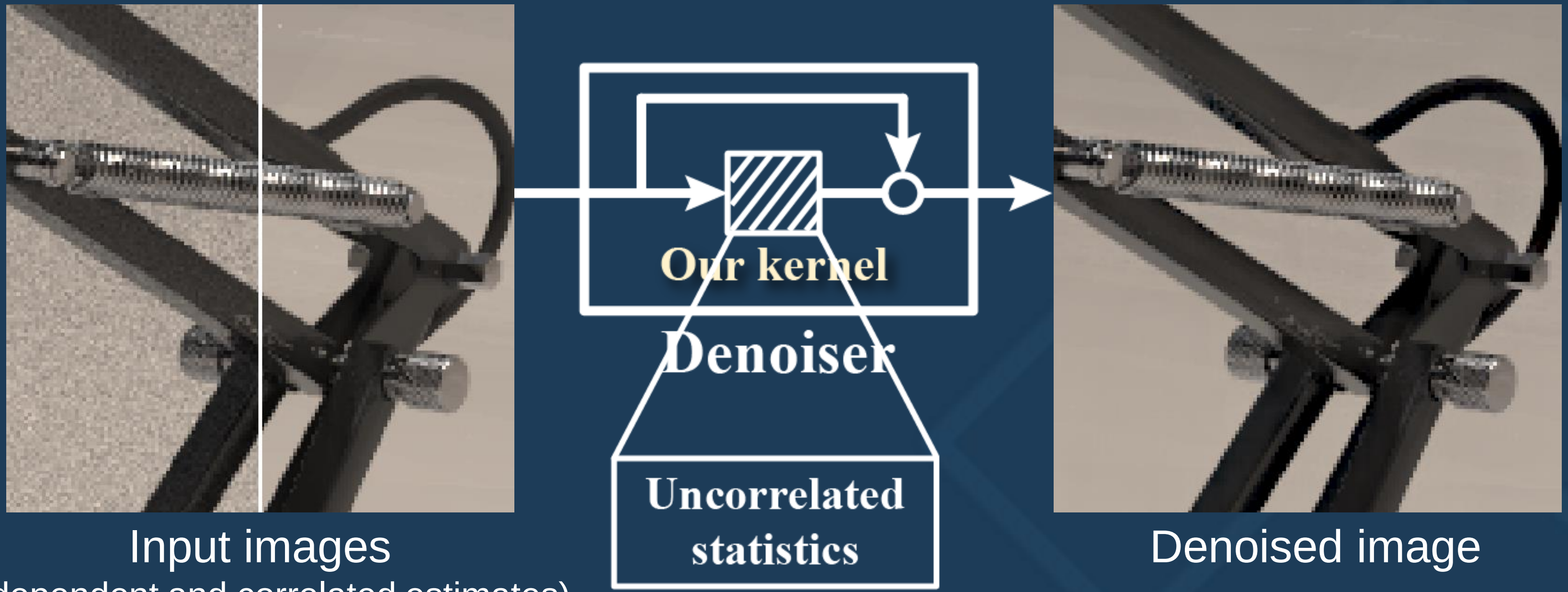
Our Contributions

- We aim at satisfying **input-dependent and uncorrelated weighting** at the same time.
  - Revisiting uncorrelated statistics to devise an input-dependent and uncorrelated kernel
  - Showing that denoising with the kernel becomes unbiased under an assumption
  - Presenting a practical implementation of the kernel as a proof of concept

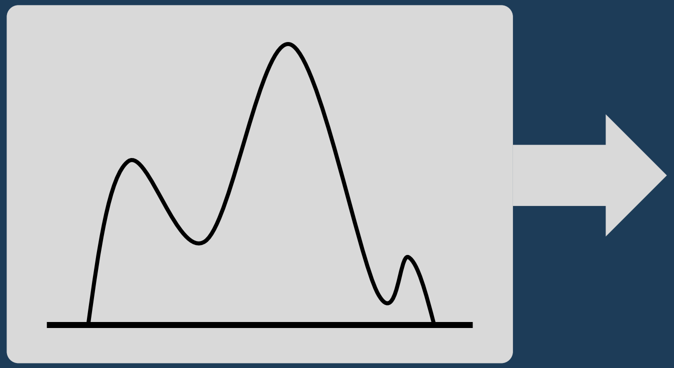


Input images
(independent and correlated estimates)

Denoised image

Uncorrelated Statistics

- General case
  - Dependent random variables are correlated in general.



Probability density function

$x_1 \quad x_2 \quad \cdots \quad x_n$

n random samples

Uncorrelated Statistics

- General case
  - Dependent random variables are correlated in general.

$$f(x_1, x_2, \cdots, x_n) \qquad g(x_1, x_2, \cdots, x_n)$$

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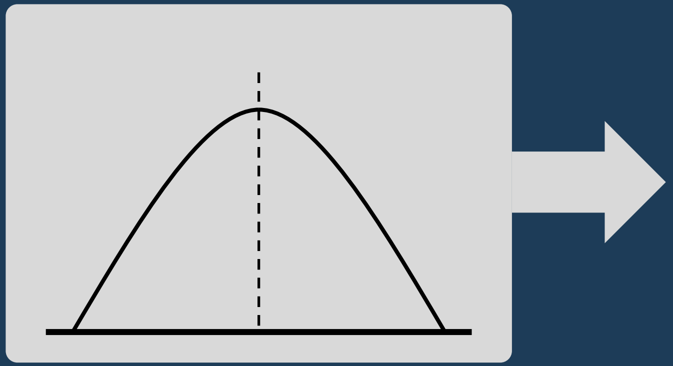
Uncorrelated Statistics

- General case
  - Dependent random variables are correlated in general.

$$\boldsymbol{Cov}\left(\; f(x_1, x_2, \cdots, x_n) \; , \; g(x_1, x_2, \cdots, x_n) \; \right) \neq \mathbf{0}$$

Uncorrelated Statistics

- Hogg's Theorem [Hogg 1960]
  - Under certain conditions, dependent variables can be uncorrelated.



Probability density function
(symmetric distribution)

$x_1 \quad x_2 \quad \cdots \quad x_n$

n random samples

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Uncorrelated Statistics

- Hogg's Theorem [Hogg 1960]
  - Under certain conditions, dependent variables can be uncorrelated.

$$f(x_1, x_2, \cdots, x_n) \qquad g(x_1, x_2, \cdots, x_n)$$

$$f(x_1+\alpha, \cdots, x_n+\alpha) = f(x_1, \cdots, x_n) + \alpha$$

$$f(-x_1, \cdots, -x_n) = -f(x_1, \cdots, x_n)$$

where α is an arbitrary value.

$$g(x_1+\alpha, \cdots, x_n+\alpha) = g(x_1, \cdots, x_n)$$

$$g(-x_1, \cdots, -x_n) = g(x_1, \cdots, x_n)$$

where α is an arbitrary value.

Uncorrelated Statistics

- Hogg's Theorem [Hogg 1960]
  - Under certain conditions, dependent variables can be uncorrelated.

$$\boldsymbol{Cov}\left(f(x_1, x_2, \cdots, x_n) \quad , \quad g(x_1, x_2, \cdots, x_n) \right) = \mathbf{0}$$

Two statistics are dependent but uncorrelated!

$$f(x_1 + \alpha, \cdots, x_n + \alpha) = f(x_1, \cdots, x_n) + \alpha$$
$$f(-x_1, \cdots, -x_n) = -f(x_1, \cdots, x_n)$$
$$g(x_1 + \alpha, \cdots, x_n + \alpha) = g(x_1, \cdots, x_n)$$
$$g(-x_1, \cdots, -x_n) = g(x_1, \cdots, x_n)$$

where α is an arbitrary value.

[Conditions]

Input-Dependent Kernel using Uncorrelated Statistics

- Denoising formula transformed from combination function

$$\hat{\mu}_c = y_c + \sum_{i\in\Omega_c} k_i \left(\Delta z_{ci} - \Delta y_{ci}\right)$$

Simplified version of combination function

$k_c = 1 - \sum_{i\in\Omega_c} k_i$ from $k_c + \sum_{i\in\Omega_c} k_i = 1$

$\Delta y_{ci} \equiv y_c - y_i, \quad \Delta z_{ci} \equiv z_c - z_i$

$$\hat{\mu}_c = k_c y_c + \sum_{i\in\Omega_c} k_i y_i + \sum_{i\in\Omega_c} k_i (z_c - z_i)$$

Combination function

y_i : independent pixel estimate at pixel i

z_i : correlated pixel estimate at pixel i

k_i : normalized weight for pixel i

Ω_c : set of neighboring pixels nearby pixel c

Input-Dependent Kernel using Uncorrelated Statistics

- Denoising formula transformed from combination function

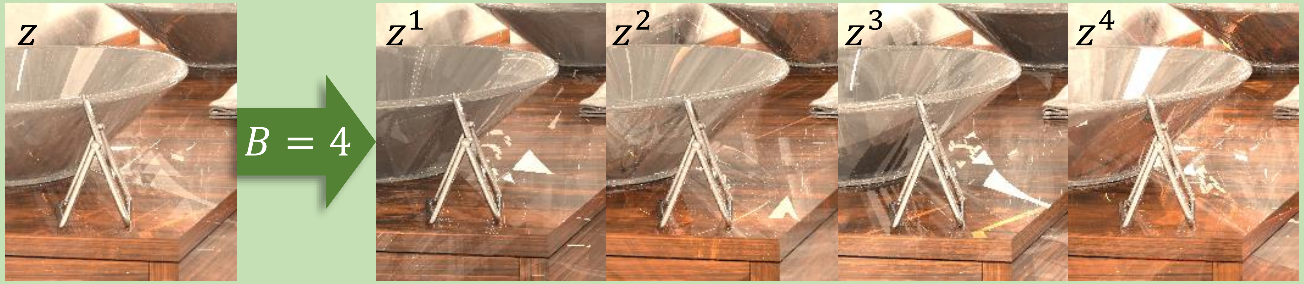
$$\hat{\mu}_c = y_c + \sum_{i \in \Omega_c} k_i \left(\boxed{\Delta z_{ci}} - \Delta y_{ci} \right)$$

Simplified version of combination function

Introducing an average function f with B sub-averages

$$\Delta z_{ci} = f\left(\Delta z_{ci}^1, \cdots, \Delta z_{ci}^B\right) = \frac{1}{B} \sum_{j=1}^{B} \Delta z_{ci}^j$$

Example)



y_i : independent pixel estimate at pixel i

z_i : correlated pixel estimate at pixel i

k_i : normalized weight for pixel i

Ω_c : set of neighboring pixels nearby pixel c

Input-Dependent Kernel using Uncorrelated Statistics

- Denoising formula transformed from combination function

$$\hat{\mu}_c = y_c + \sum_{i \in \Omega_c} k_i \left(\boxed{f\left(\Delta z_{ci}^1, \cdots, \Delta z_{ci}^B\right)} - \Delta y_{ci} \right)$$

Simplified version of combination function

y_i : independent pixel estimate at pixel i
z_i : correlated pixel estimate at pixel i
k_i : normalized weight for pixel i
Ω_c : set of neighboring pixels nearby pixel c

$$f\left(\Delta z_{ci}^1 + \alpha, \cdots, \Delta z_{ci}^B + \alpha\right) = f\left(\Delta z_{ci}^1, \cdots, \Delta z_{ci}^B\right) + \alpha$$
$$f\left(-\Delta z_{ci}^1, \cdots, -\Delta z_{ci}^B\right) = -f\left(\Delta z_{ci}^1, \cdots, \Delta z_{ci}^B\right)$$

where α is an arbitrary value.

Input-Dependent Kernel using Uncorrelated Statistics

- Denoising formula transformed from combination function

$$\hat{\mu}_c = y_c + \sum_{i\in\Omega_c} \boxed{k_i} \left(f\left(\Delta z_{ci}^1, \cdots, \Delta z_{ci}^B\right) - \Delta y_{ci} \right)$$

Simplified version of combination function

Replacing the kernel k_i into a bounded function k of estimates Δz_{ci}^j

y_i : independent pixel estimate at pixel i

z_i : correlated pixel estimate at pixel i

k_i : normalized weight for pixel i

Ω_c : set of neighboring pixels nearby pixel c

Input-Dependent Kernel using Uncorrelated Statistics

- Denoising formula transformed from combination function

$$\hat{\mu}_c = y_c + \sum_{i \in \Omega_c} k\left(\Delta z_{ci}^1, \cdots, \Delta z_{ci}^B\right) \left(f\left(\Delta z_{ci}^1, \cdots, \Delta z_{ci}^B\right) - \Delta y_{ci} \right)$$

Simplified version of combination function

Replacing the kernel k_i into a bounded function k of estimates Δz_{ci}^j

y_i : independent pixel estimate at pixel i

z_i : correlated pixel estimate at pixel i

k_i : normalized weight for pixel i

Ω_c : set of neighboring pixels nearby pixel c

Input-Dependent Kernel using Uncorrelated Statistics

- Our new theorem

Let the kernel k be a bounded function satisfying the following conditions:

$$k\left(\Delta z_{ci}^{1}+\alpha,\cdots,\Delta z_{ci}^{B}+\alpha\right)=k\left(\Delta z_{ci}^{1},\cdots,\Delta z_{ci}^{B}\right),$$
$$k\left(-\Delta z_{ci}^{1},\cdots,-\Delta z_{ci}^{B}\right)=k\left(\Delta z_{ci}^{1},\cdots,\Delta z_{ci}^{B}\right).$$

By assuming that Δz_{ci}^{j} has a symmetric distribution,

$$E[\hat{\mu}_c]=\mu_c$$

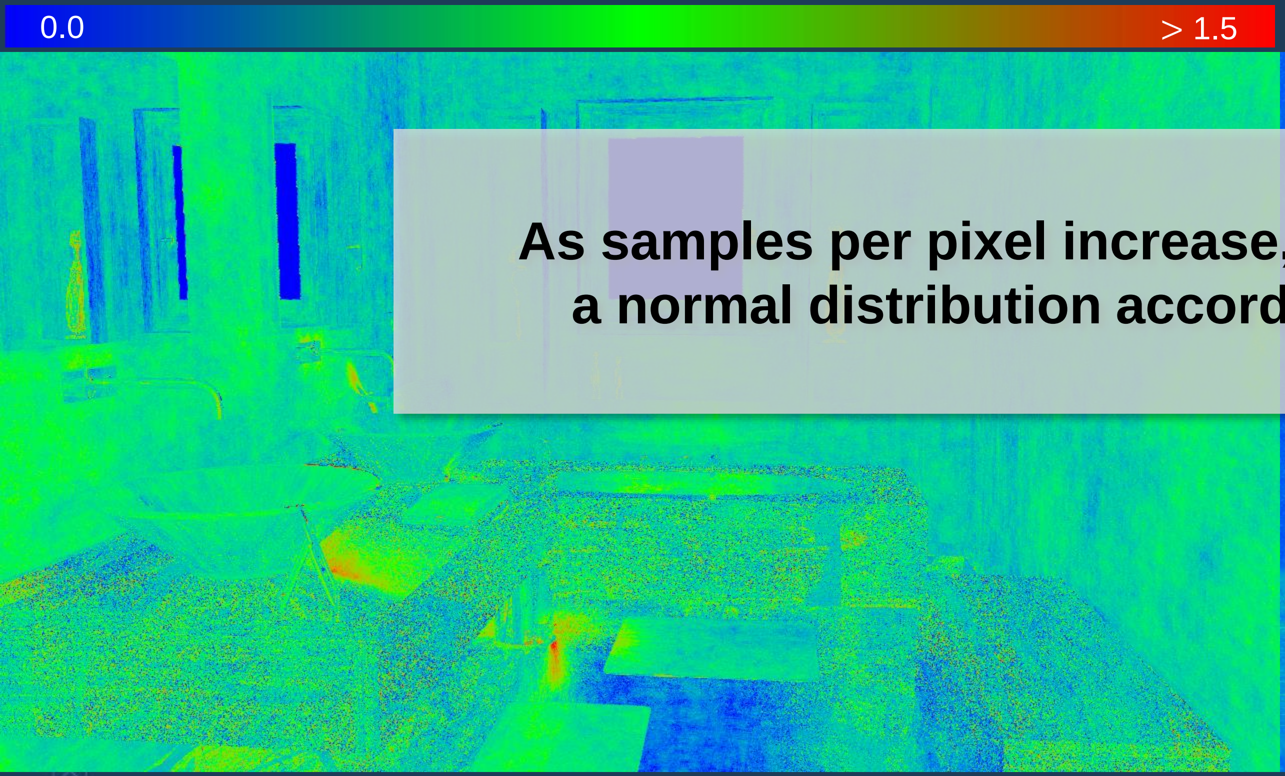
the denoised output is an unbiased estimate of μ_c.

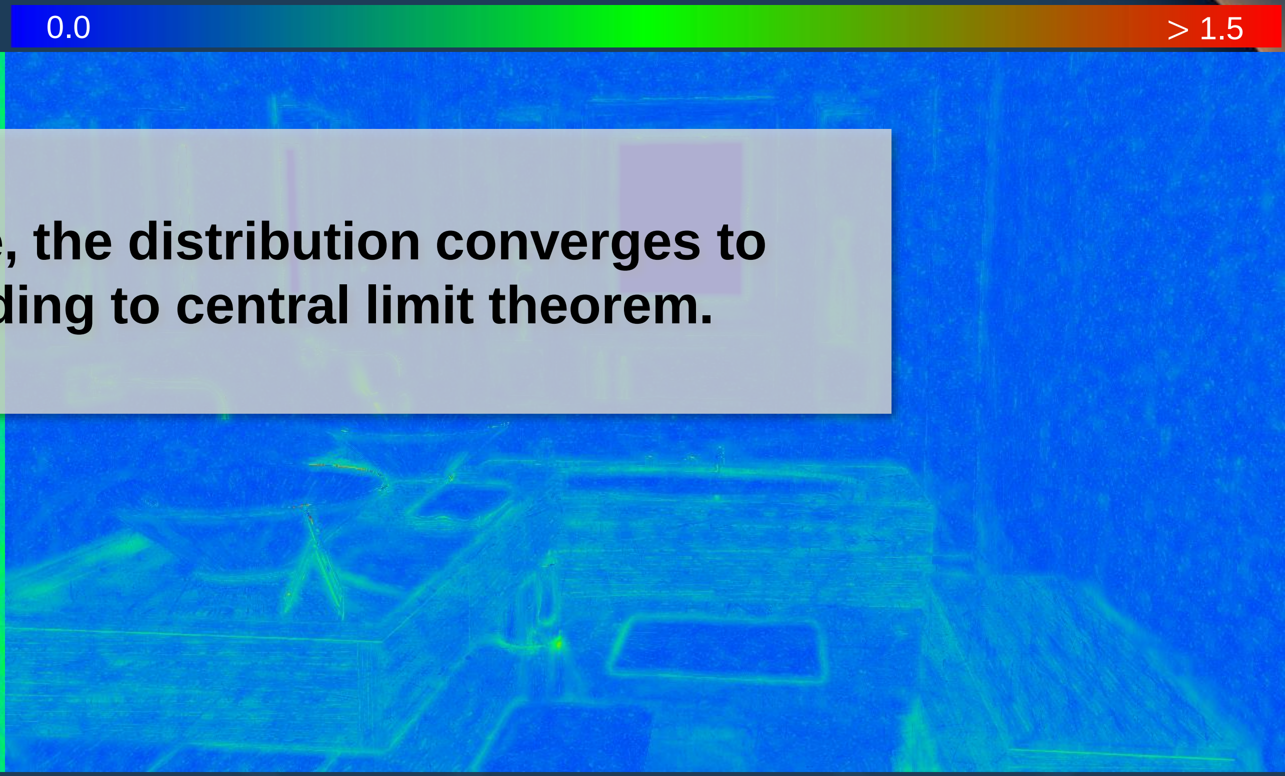
$$\hat{\mu}_c=y_c+\sum_{i\in\Omega_c}k\left(\Delta z_{ci}^{1},\cdots,\Delta z_{ci}^{B}\right)\left(f\left(\Delta z_{ci}^{1},\cdots,\Delta z_{ci}^{B}\right)-\Delta y_{ci}\right)$$

Simplified version of combination function

Input-Dependent Kernel using Uncorrelated Statistics

- Visualization of the Pearson median skewness
  - Using differences than rendered pixel estimates in our assumption





As samples per pixel increase, the distribution converges to a normal distribution according to central limit theorem.

0.484610
Correlated estimates

0.135028
Differences of correlated estimates

Input-Dependent Kernel using Uncorrelated Statistics

- Example denoising kernel satisfying our theorem
  - Simple variance-based weighting with $B = 2$

$$k\left(\Delta z_{ci}^{1}, \Delta z_{ci}^{2}\right) = \frac{1}{|\Omega_c|} \exp\left(-\gamma n\left(\Delta z_{ci}^{1} - \Delta z_{ci}^{2}\right)^{2}\right)$$

$\Delta z_{ci}^{1}, \Delta z_{ci}^{2}$: two sub-averages of an estimate Δz_{ci}

n : sample size for each sub-average

γ : scale parameter shared for all center pixels

Results

- Comparisons with input-independent kernels (uniform kernel and cross-weighting)

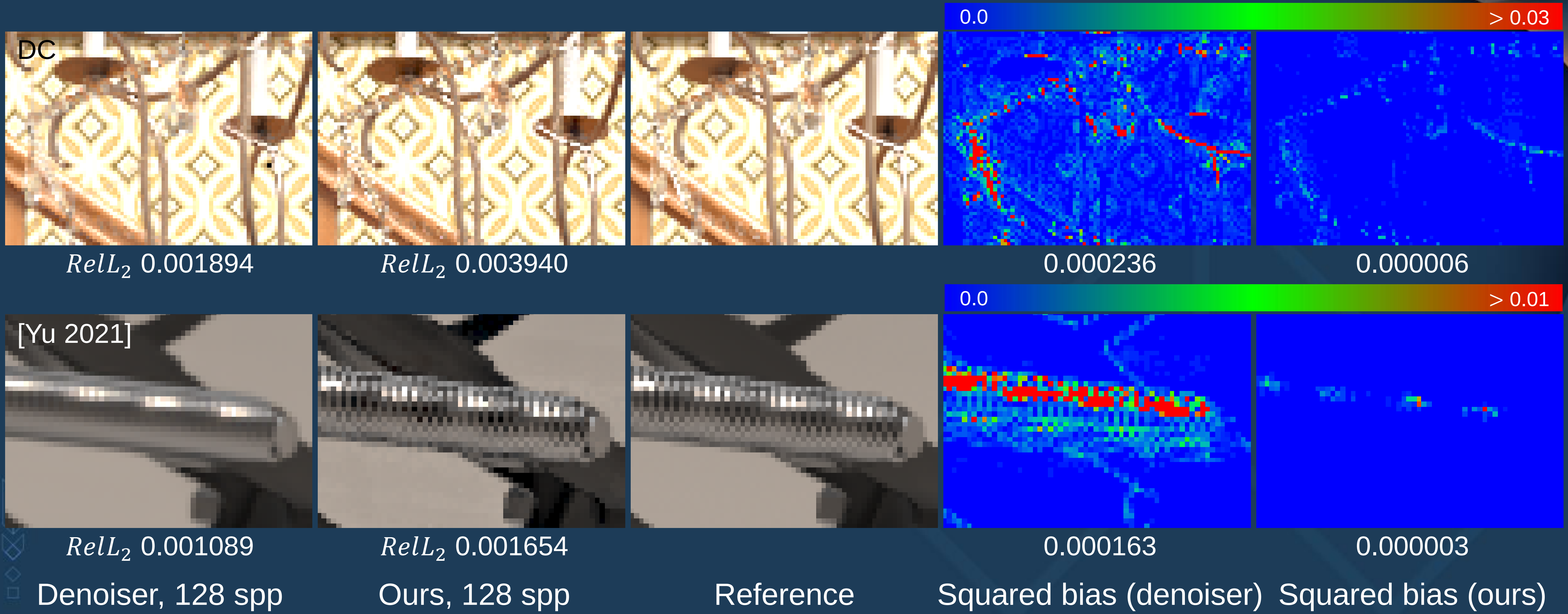
128 / 0.086589 | 128 / 0.057743 | 256 / 0.032066 | 256 / 0.027440 | 256 / 0.009671 | spp / *Relative* L_2

128 / 0.077382 | 128 / 0.068711 | 256 / 0.058965 | 256 / 0.046768 | 256 / 0.017516 | spp / *Relative* L_2

Input image (PT) | Input image (CRN) | Uni-kernel | Cross-weighting | Our kernel | Reference

Results

- Comparisons with an input-dependent kernel (DC) and a recent biased denoiser [Yu 2021]



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Conclusion

- New denoising strategy for an input-dependent kernel built upon uncorrelated statistics
  - Theoretical conditions allowing to reduce noise while limiting denoising bias
- Opening up a large design space of an input-dependent kernel via our theorem
  - Steppingstone toward achieving unbiased denoising with an input-dependent kernel

Code, interactive viewer, and presentation slides on our project page:

Acknowledgments

- Valuable comments
  - Anonymous reviewers
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