Lecture slides (AI3501/CT4201/EC4215 – Computer Graphics)

Ray Tracing

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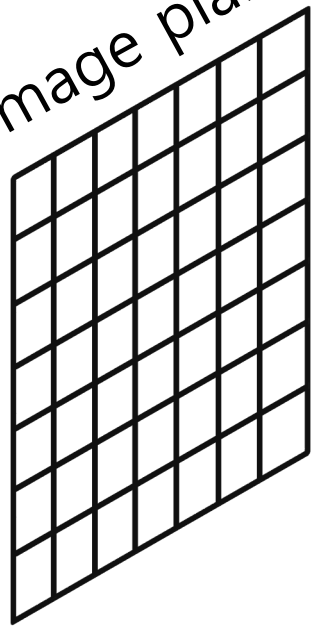
Ray Tracing

- A rendering technique:
  - Produce a 2D image from a scene (models)
- Image-order rendering:
  - Loop over pixels to decide pixel colors
- Object-order rendering:
  - Iterate objects and compute some pixel colors related to each object

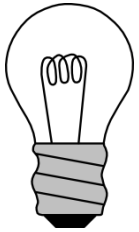
Rendering

eye

Image plane

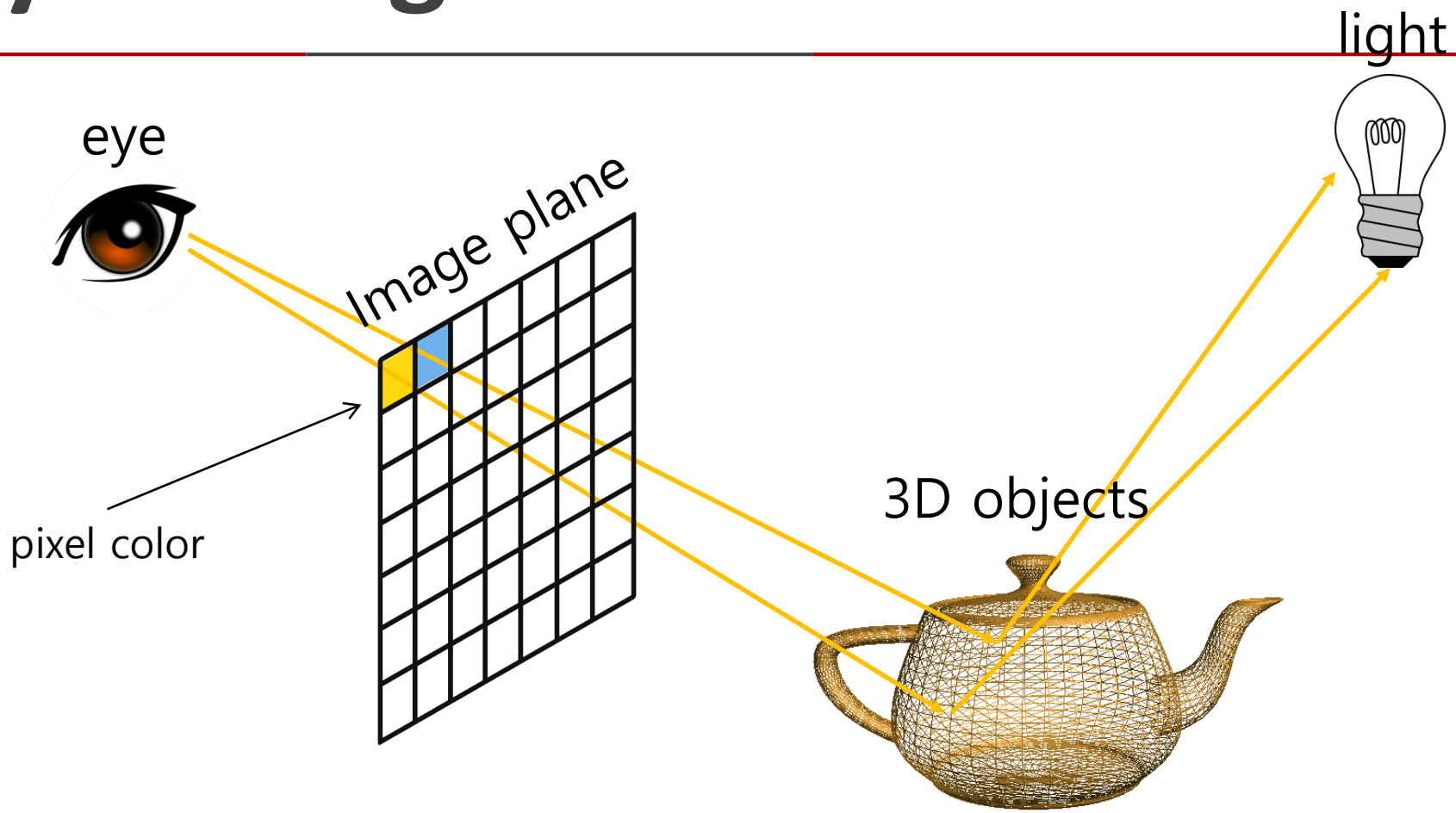


light



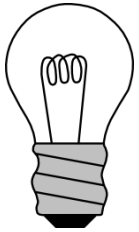
3D objects

Ray Tracing



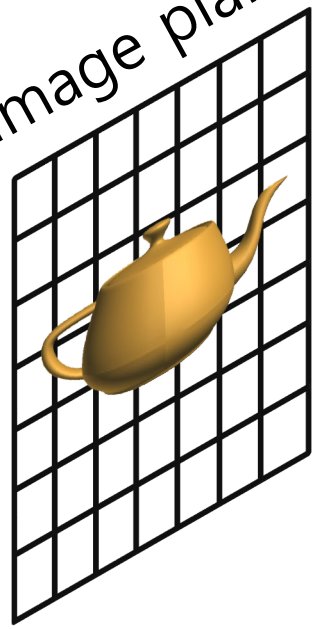
Ray Tracing

light



eye

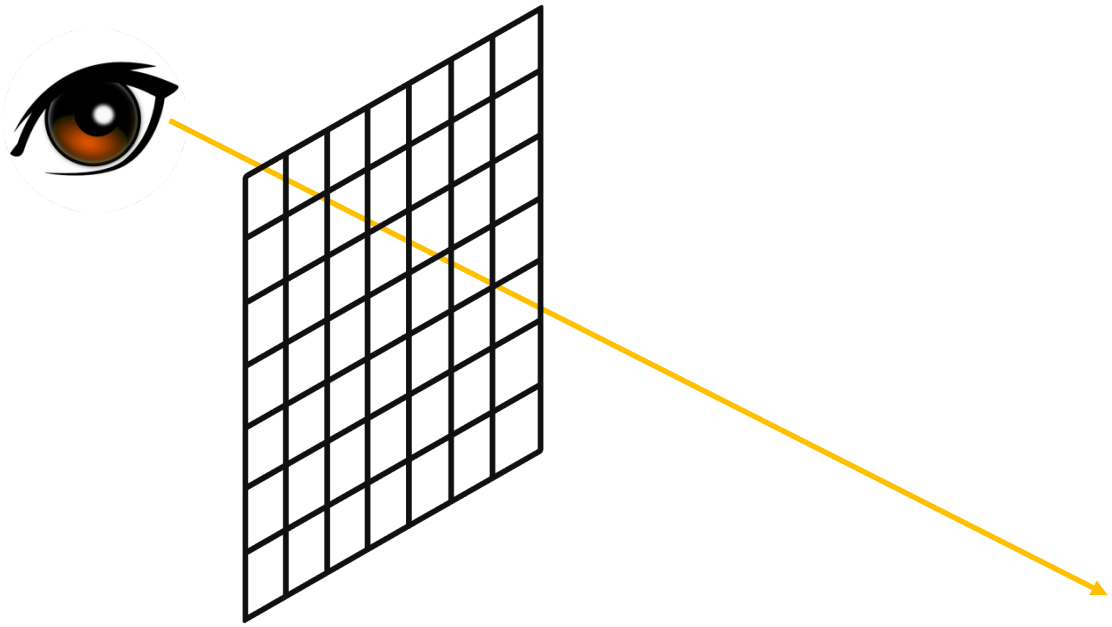
Image plane



3D objects

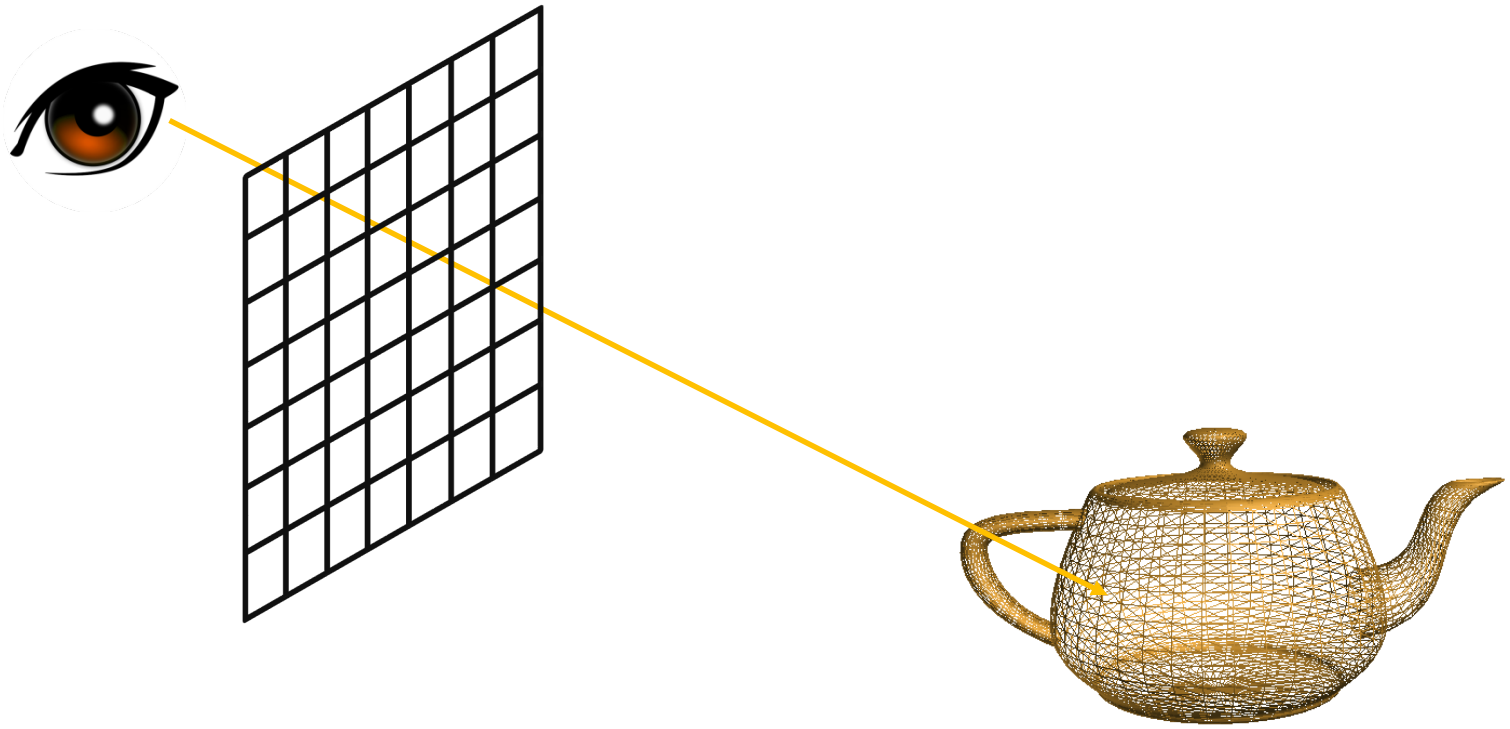
Basic Ray Tracer

- Ray generation
  - Compute the origin and direction of a ray per pixel, by considering the camera and image plane



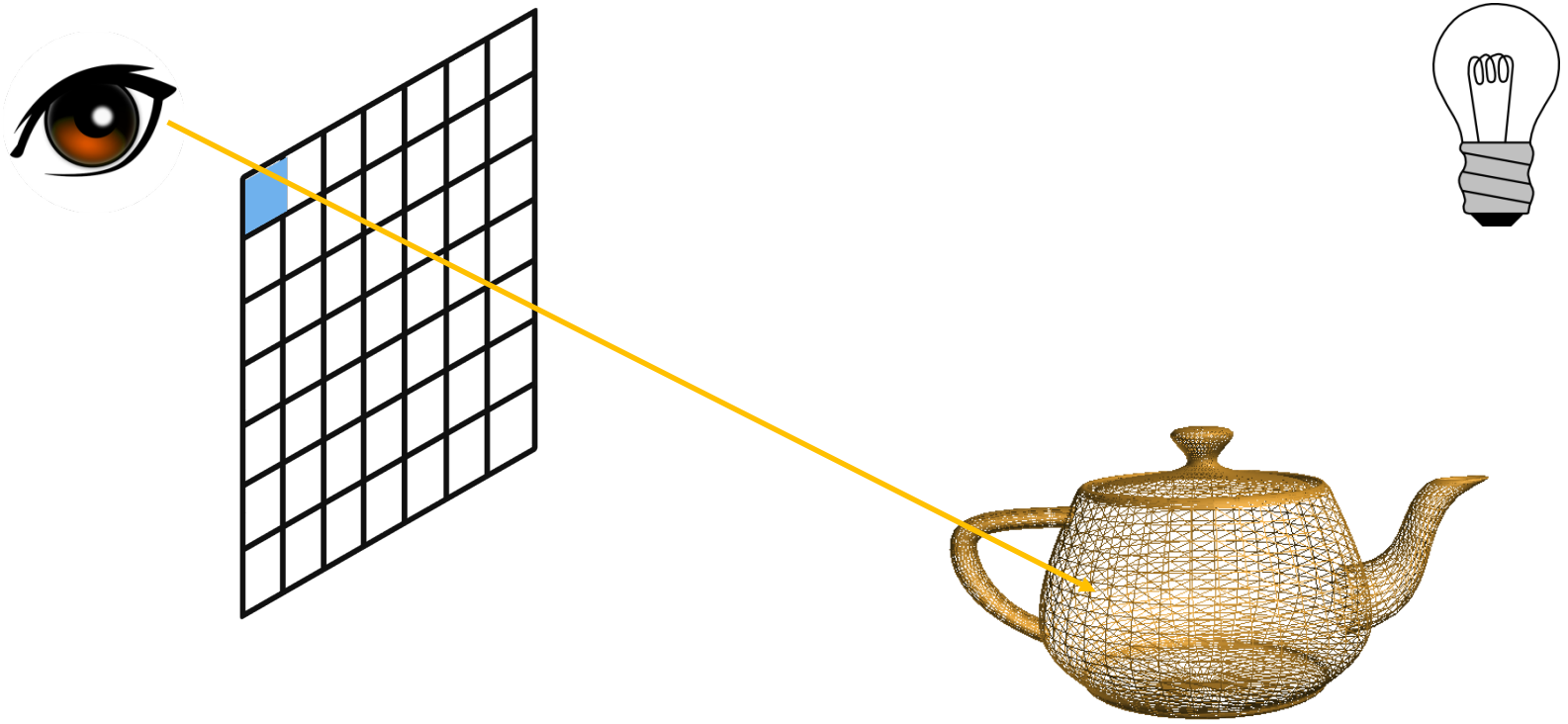
Basic Ray Tracer

- Ray intersection
  - Find the closest intersection point between the ray and objects



Basic Ray Tracer

- Shading
  - Compute the pixel color using the geometry, material, and lights at the intersection point



Basic Ray Tracer

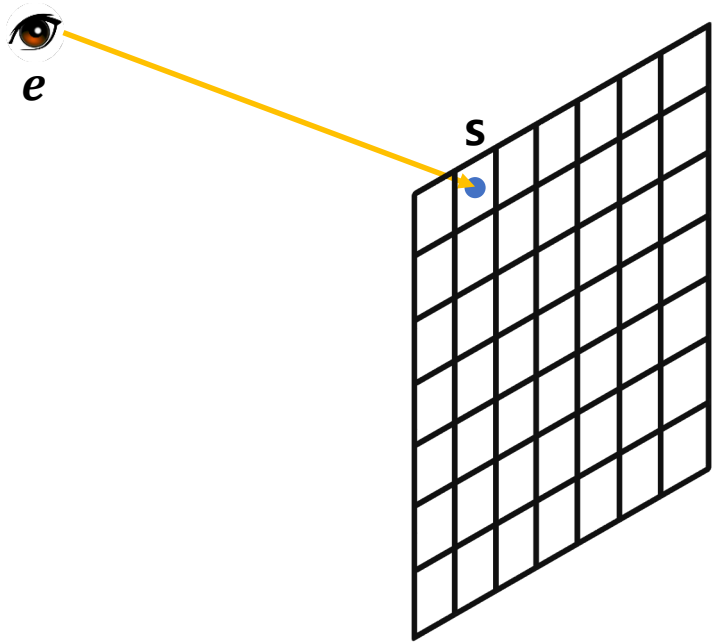
- For each pixel do
  - Compute a primary ray (viewing ray)
  - Find the closest intersection point between the ray and a scene
  - Determine a pixel color

Primary Ray Generation

- Mathematical representation for a ray
  - 3D parametric line: $\boldsymbol{p}(t) = \boldsymbol{e} + t(\boldsymbol{s} - \boldsymbol{e})$

- Properties
  - $\boldsymbol{p}(0) = \boldsymbol{e}, \quad \boldsymbol{p}(1) = \boldsymbol{s}$
  - $\boldsymbol{p}(t_1)$ is closer to the eye than $\boldsymbol{p}(t_2)$ when $0 < t_1 < t_2$
  - When $t < 0, \boldsymbol{p}(t)$ is behind the eye
  - $\boldsymbol{e}$ is a given value

- Q. How can we compute **s**?

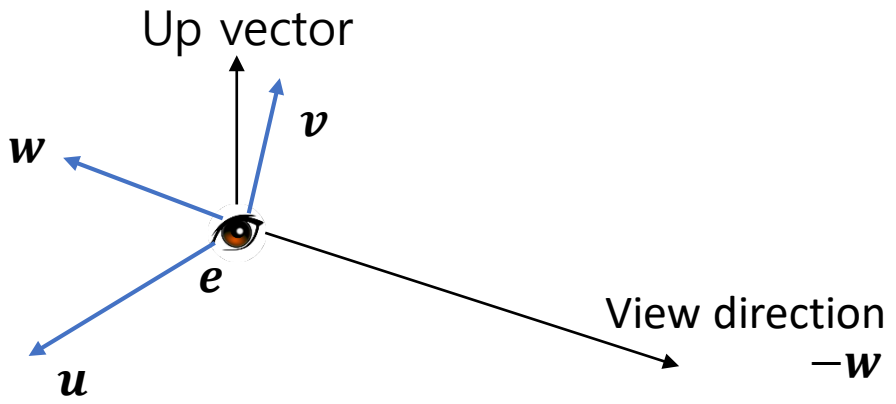


Primary Ray Generation

- Mathematical representation for a ray
  - 3D parametric line: $\boldsymbol{p}(t) = \boldsymbol{e} + t(\boldsymbol{s} - \boldsymbol{e})$

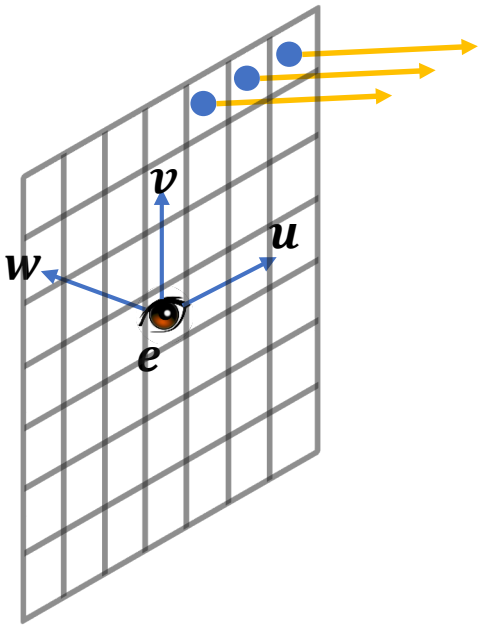
- $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ forms a right-handed coordinate system

- Two kinds of views
  - Orthographic view
  - Perspective view



Orthographic Views

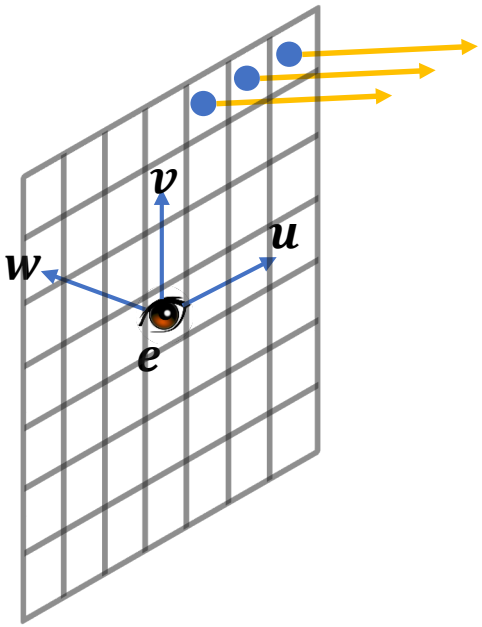
- All primary rays have the same direction, ***-w***
- The primary ray starts on the image plane defined by ***e, u, v***
- The image plane is defined with four numbers:
  - l, r: positions of left and right edges of the image plane
  - b, t: positions of bottom and top edges



- To make an image with $n_x \times n_y$
  - Pixels are spaced as the following:
    - $\frac{r-l}{n_x}$ horizontally, $\frac{t-b}{n_y}$ vertically
- Position (α, β) in the image plane is corresponding to a pixel (i, j) in the raster image:
  - $\alpha = l + \frac{(r-l)(i+0.5)}{n_x}$
  - $\beta = b + \frac{(t-b)(j+0.5)}{n_y}$
  - (α, β) are the coordinates of the pixel's position on the image plane

Orthographic Views

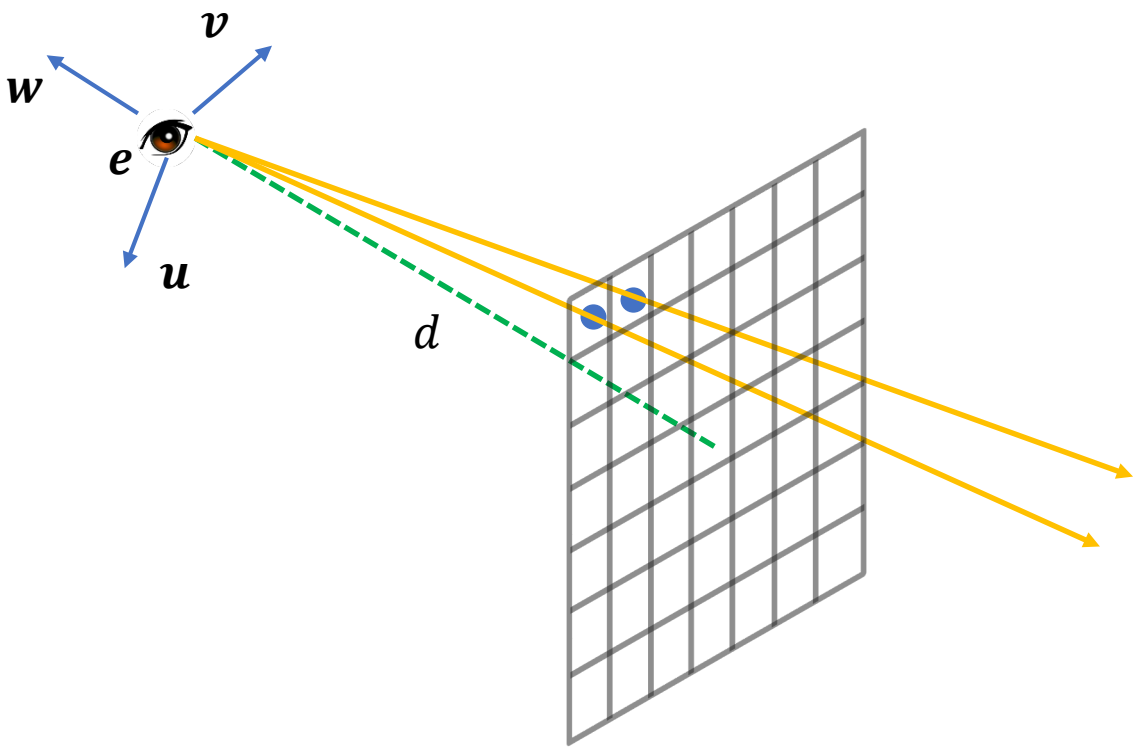
- Procedure to generate orthographic viewing rays
  - Compute α and β
    - $\alpha = l + \frac{(r-l)(i+0.5)}{n_x}$
    - $\beta = b + \frac{(t-b)(j+0.5)}{n_y}$
- $ray.direction := -\boldsymbol{w}$
- $ray.origin := \boldsymbol{e} + \alpha\boldsymbol{u} + \beta\boldsymbol{v}$



- Properties
  - Same direction for all rays
  - Different origins for rays

Perspective Views

- All rays have the same origin, ***e***, but have different directions
- The image plane is placed with a distance, *d*, in front of ***e***
  - *d: image plane distance (called the focal length)*
- Procedure to generate perspective viewing rays
  - Compute α and β
    - $\alpha = l + \frac{(r-l)(i+0.5)}{n_x}$
    - $\beta = b + \frac{(t-b)(j+0.5)}{n_y}$
- $ray.direction := -d\boldsymbol{w} + \alpha\boldsymbol{u} + \beta\boldsymbol{v}$
- $ray.origin := \boldsymbol{e}$



Intersection between Ray and Object

- Generated ray: $\mathbf{p}(\mathrm{t}) = \boldsymbol{e} + t\boldsymbol{d}$
- The next task is to find the closest intersection point between a ray and objects
  - i.e., need to find a t in the interval $[t_0, t_1]$ (e.g., $[0, +\infty]$)
- Objects
  - Sphere
  - Triangle
  - Multiple objects

Intersection between Ray and Sphere

- Ray: $\boldsymbol{p}(t) = \boldsymbol{e} + t\boldsymbol{d}$
- Implicit surface: $f(\boldsymbol{p}) = 0$
- Intersection points should satisfy both equations
  - $f(\boldsymbol{p}(t)) = f(\boldsymbol{e} + t\boldsymbol{d}) = 0$

- Let's define a sphere with center $c = (x_c, y_c, z_c)$ and radius r
  - $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - r^2 = 0$
  - $(\boldsymbol{p} - \boldsymbol{c}) \cdot (\boldsymbol{p} - \boldsymbol{c}) - r^2 = 0$ (vector form)
  - A point p that satisfies this equation is on the sphere
- By plug-in the parametric ray equation,
  - $(\boldsymbol{e} + t\boldsymbol{d} - \boldsymbol{c}) \cdot (\boldsymbol{e} + t\boldsymbol{d} - \boldsymbol{c}) - r^2 = 0$
  - By rearranging terms with respect to *t* (unknown value):
  - $(\boldsymbol{d} \cdot \boldsymbol{d})t^2 + 2\boldsymbol{d} \cdot (\boldsymbol{e} - \boldsymbol{c})t + (\boldsymbol{e} - \boldsymbol{c}) \cdot (\boldsymbol{e} - \boldsymbol{c}) - r^2 = 0$

Intersection between Ray and Sphere

- A quadratic equation in t
  - $(\boldsymbol{d} \cdot \boldsymbol{d})t^2 + 2\boldsymbol{d} \cdot (\boldsymbol{e} - \boldsymbol{c})t + (\boldsymbol{e} - \boldsymbol{c}) \cdot (\boldsymbol{e} - \boldsymbol{c}) - r^2 = 0$

- The solutions for $at^2 + bt + c = 0$
  - $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  - $b^2 - 4ac$ (called discriminant)
    - When $b^2 - 4ac < 0$, there is no solution (the ray does not intersect with the sphere)
    - When $b^2 - 4ac = 0$, a solution exists (the ray touches the sphere)
    - When $b^2 - 4ac > 0$, two solutions exist (the ray enters and leaves the sphere)

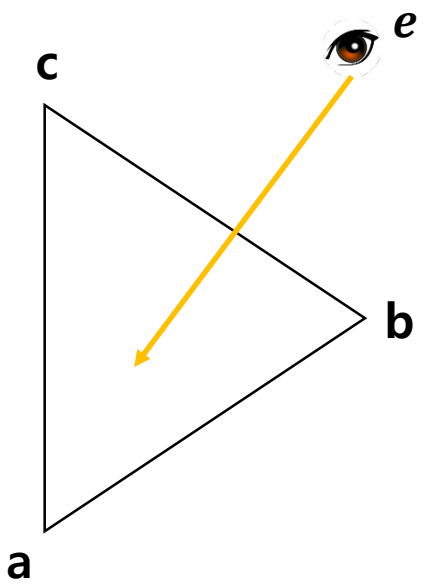
- $t = \frac{-\boldsymbol{d} \cdot (\boldsymbol{e} - \boldsymbol{c}) \pm \sqrt{\left(\boldsymbol{d} \cdot (\boldsymbol{e} - \boldsymbol{c})\right)^2 - (\boldsymbol{d} \cdot \boldsymbol{d})\left((\boldsymbol{e} - \boldsymbol{c}) \cdot (\boldsymbol{e} - \boldsymbol{c}) - r^2\right)}}{(\boldsymbol{d} \cdot \boldsymbol{d})}$

Intersection between Ray and Triangle

- Ray: $\mathbf{p}(\mathrm{t}) = \boldsymbol{e} + t\boldsymbol{d}$
- Intersection point:
  - $\boldsymbol{e} + t\boldsymbol{d} = \boldsymbol{a} + \beta(\boldsymbol{b} - \boldsymbol{a}) + \gamma(\boldsymbol{c} - \boldsymbol{a})$

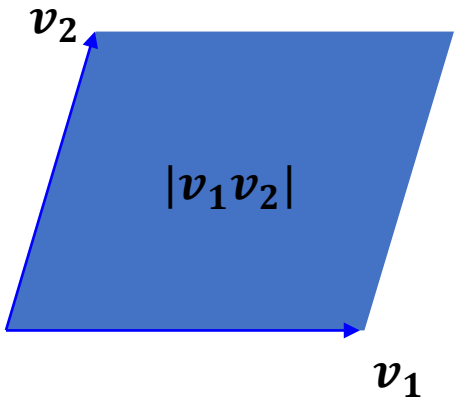
- Solving the equation for t, β, γ:
  - $x_e + tx_d = x_a + \beta(x_b - x_a) + r(x_c - x_a)$
  - $y_e + ty_d = y_a + \beta(y_b - y_a) + r(y_c - y_a)$
  - $z_e + tz_d = z_a + \beta(z_b - z_a) + r(z_c - z_a)$

  - Can be rewritten:
  - $$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$



Intersection between Ray and Triangle

- Cramer's rule can be utilized to solve the 3 x 3 linear system
  - $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$
  - $x = \dfrac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{|A|}, \quad y = \dfrac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{|A|}, \quad z = \dfrac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{|A|}$
  - where $|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $|.|$ is the determinant
  - $|A| = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$
  - $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} = b_2 c_3 - c_2 b_3$



Intersection between Ray and Triangle

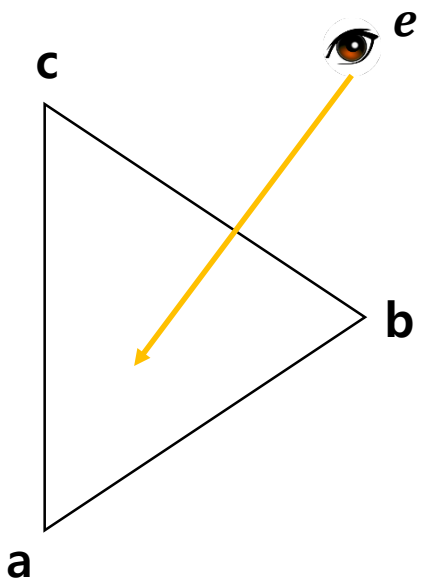
- $\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$

- $\beta = \dfrac{\begin{vmatrix} x_a - x_e & x_a - x_c & x_d \\ y_a - y_e & y_a - y_c & y_d \\ z_a - z_e & z_a - z_c & z_d \end{vmatrix}}{|A|}$

- $\gamma = \dfrac{\begin{vmatrix} x_a - x_b & x_a - x_e & x_d \\ y_a - y_b & y_a - y_e & y_d \\ z_a - z_b & z_a - z_e & z_d \end{vmatrix}}{|A|}$

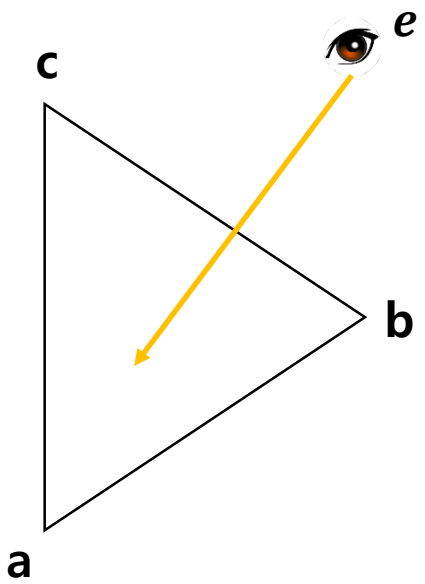
- $t = \dfrac{\begin{vmatrix} x_a - x_b & x_a - x_c & x_a - x_e \\ y_a - y_b & y_a - y_c & y_a - y_e \\ z_a - z_b & z_a - z_c & z_a - z_e \end{vmatrix}}{|A|}$

- where $|A| = \begin{vmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{vmatrix}$



Intersection between Ray and Triangle

- Procedure (with early termination) for finding the intersection:
  - Input: a ray, vertex a, b, c, interval $[t_0, t_1]$
  - Compute t
  - If $(t < t_0)$ or $(t > t_1)$ then
    - return false
  - Compute γ
  - If $(\gamma < 0)$ or $(\gamma > 1)$ then
    - return false
  - Compute β
  - If $(\beta < 0)$ or $(\beta > 1 - \gamma)$ then
    - return false
  - return true



Intersection between Ray and Objects

- Procedure for finding the *closest* intersection:
  - hit = false
  - For each object o do
    - If (o is intersected with the ray at a parameter t and $t \in [t_0, t_1]$) then
      - hit = true
      - store some information (e.g., o, normal, etc.) for shading
      - $t_1 = t$
  - return hit

Basic Ray Tracer

- For each pixel do
  - Compute a primary ray (viewing ray)
  - Find the closest intersection point between the ray and a scene
  - Determine a pixel color
    - e.g., we can apply the Phong illumination model here

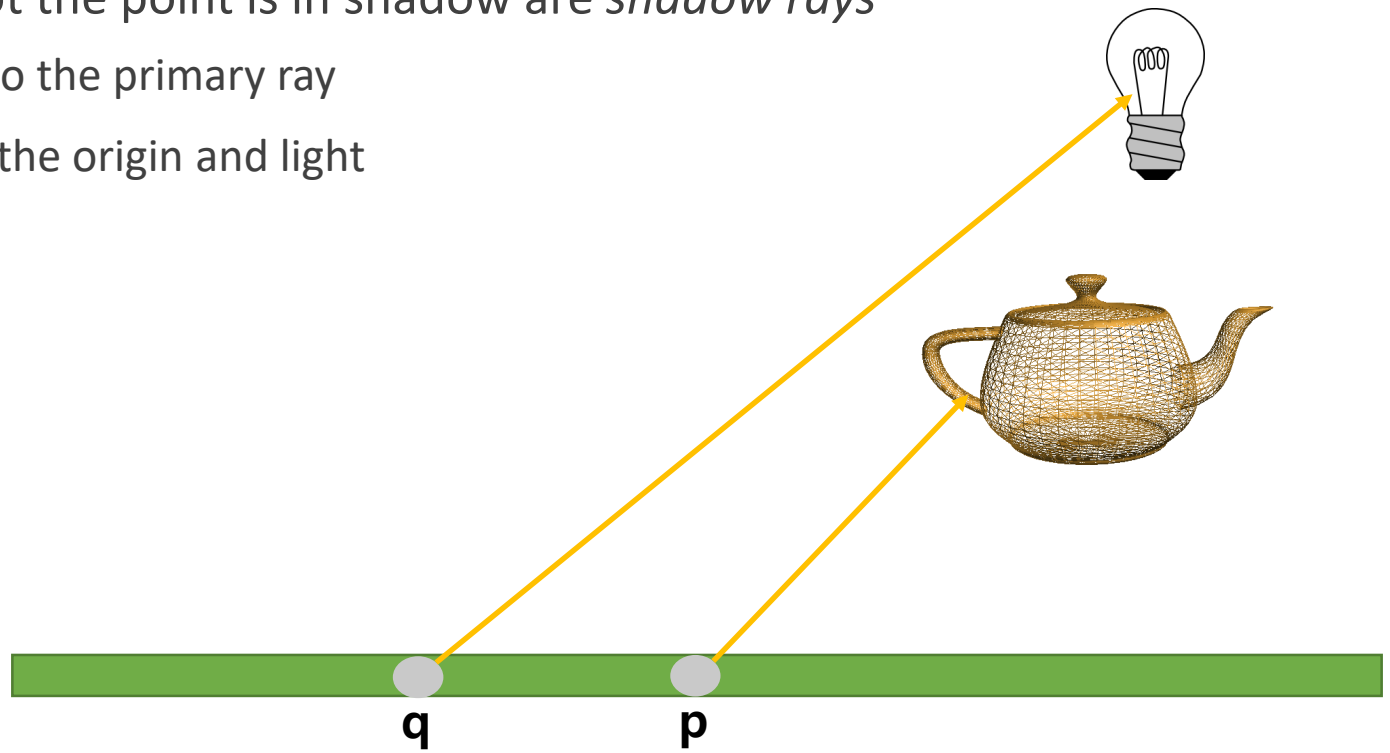
Basic Ray Tracer

- For each pixel do
  - Compute a primary ray (viewing ray)
  - If (ray intersects an object with $t \in [0, \infty)$) then
    - Compute a hit record that contains some information (normal, materials, …)
    - Evaluate an illumination model and set a pixel color
  - Else
    - Set a pixel color to background color

Shadows

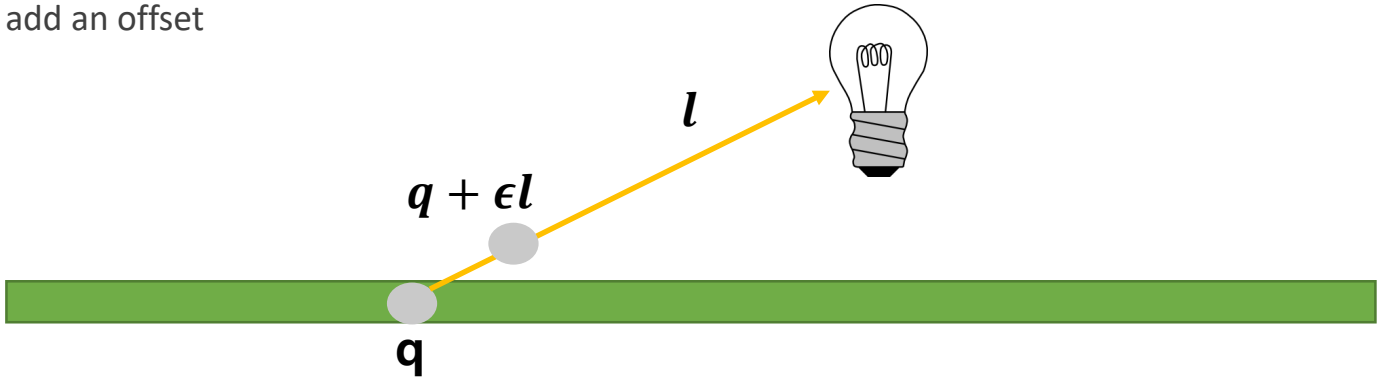
- Assume there are two intersection points, p and q
  - p is in shadow, but q is not in shadow

- Rays to determine whether or not the point is in shadow are *shadow rays*
  - Generate a shadow ray similar to the primary ray
  - Check there is any hit between the origin and light
    - $t = [t_0, t_1]$
  - e.g., $t = [0, t_1]$



Shadows

- Assume there are two intersection points, p and q
  - p is in shadow, but q is not in shadow
- Rays to determine whether or not the point is in shadow are *shadow rays*
  - Generate a shadow ray similar to the primary ray
  - Check there is any hit between the origin and light
    - $t = [t_0, t_1]$
  - Due to numerical issues, the shadow ray can intersect the surface on which the point lies
    - A naïve but common approach is to add an offset
    - $t = [\epsilon, t_1]$



Shadows

- Pseudocode to implement shadows (based on the Phong illumination)
- Input: a ray $\boldsymbol{e} + t\boldsymbol{d}$, $[t_0 = 0, t_1 = \infty]$
- If (there is a hit between the ray and objects) then
  - $\boldsymbol{p} = \boldsymbol{e} + t\boldsymbol{d}$ // p is the closest intersection from e
  - $\boldsymbol{color\ c} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$
  - If (there is no hit between the shadow ray and a light) then
    - $\boldsymbol{c} = \boldsymbol{c} + \boldsymbol{k_a L_a} + \boldsymbol{L_d k_d} \max(0, \boldsymbol{n} \cdot \boldsymbol{l}) + \boldsymbol{L_s k_s} \max(0, \boldsymbol{r} \cdot \boldsymbol{v})^s$
  - return c
- Else
  - return background color

Some History of Ray Tracing

- Rene Descartes (1637) used ray tracing to explain the phenomena of rainbow
- In rendering, the ray casting was presented by Arthur Appel (1968)
  - Ray casting (discussed so far) tends to be interchangeable to ray tracing
  - Ray tracing generates additional rays (e.g., secondary rays) to simulate global illumination effects
  - Ray tracing becomes popular due to the Whitted's paper (1980)
    - T. Whitted. An improved illumination model for shading display. Communications of the ACM, 23(6):343–349, 1980

Further Readings

- Chapter 4