

Lecture slides (CT4201/EC4215 – Computer Graphics)

# Ray Tracing

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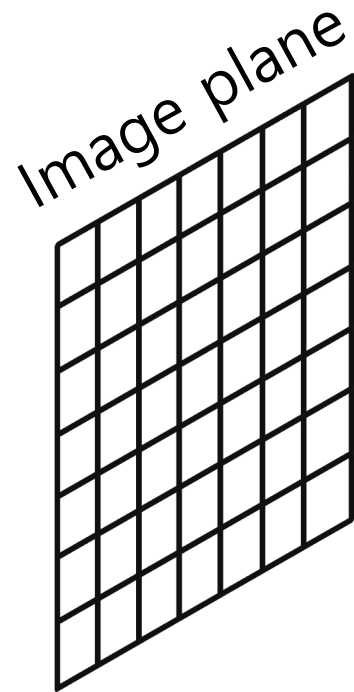
# Ray Tracing

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- A rendering technique:
  - Produce a 2D image from a scene (models)
- Image-order rendering:
  - Loop over pixels to decide pixel colors
- Object-order rendering:
  - Iterate objects and compute some pixel colors related to each object

# Rendering

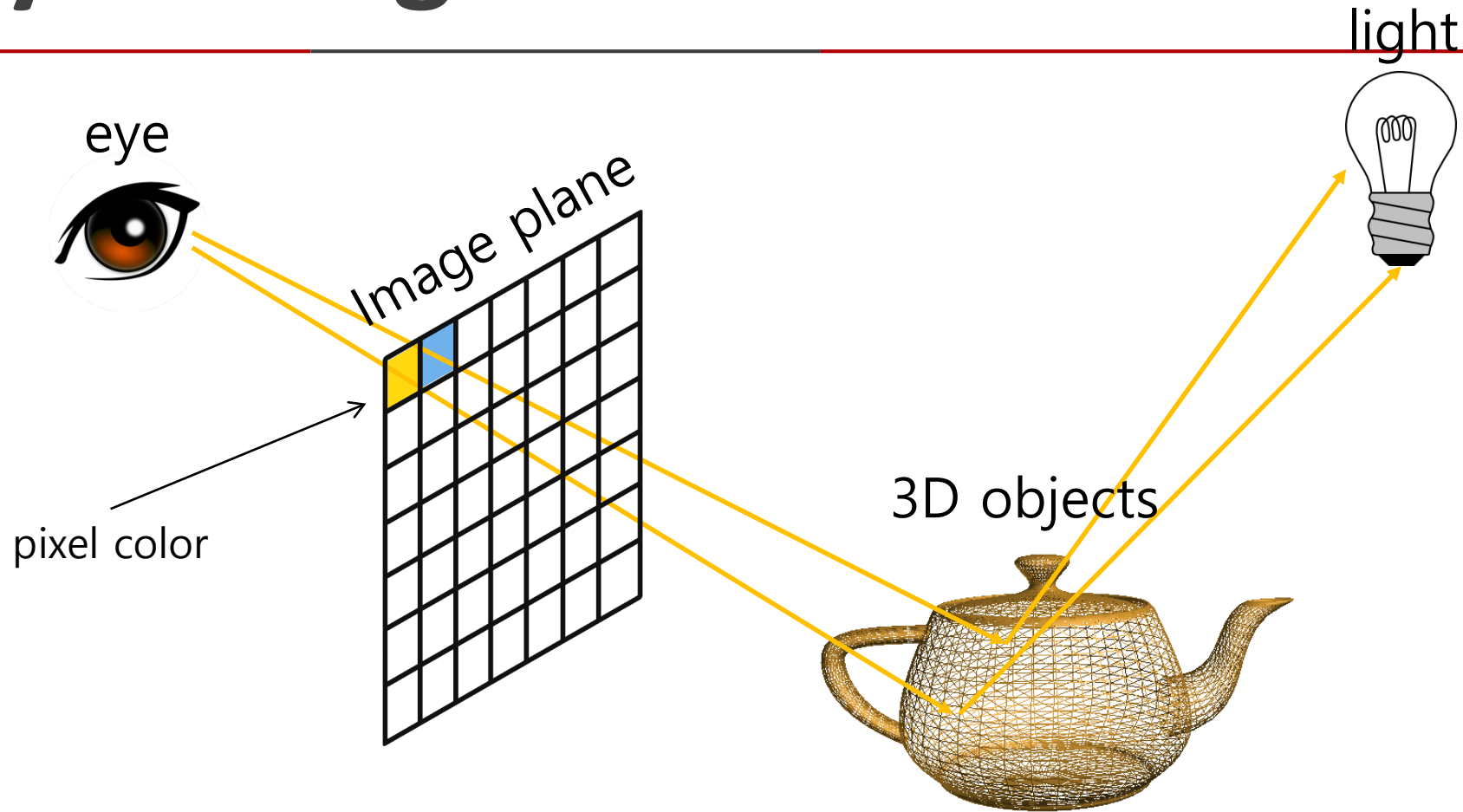
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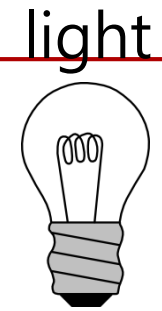
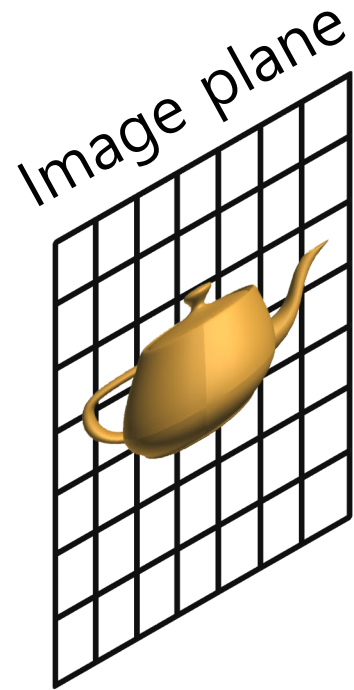
3D objects



# Ray Tracing



# Ray Tracing



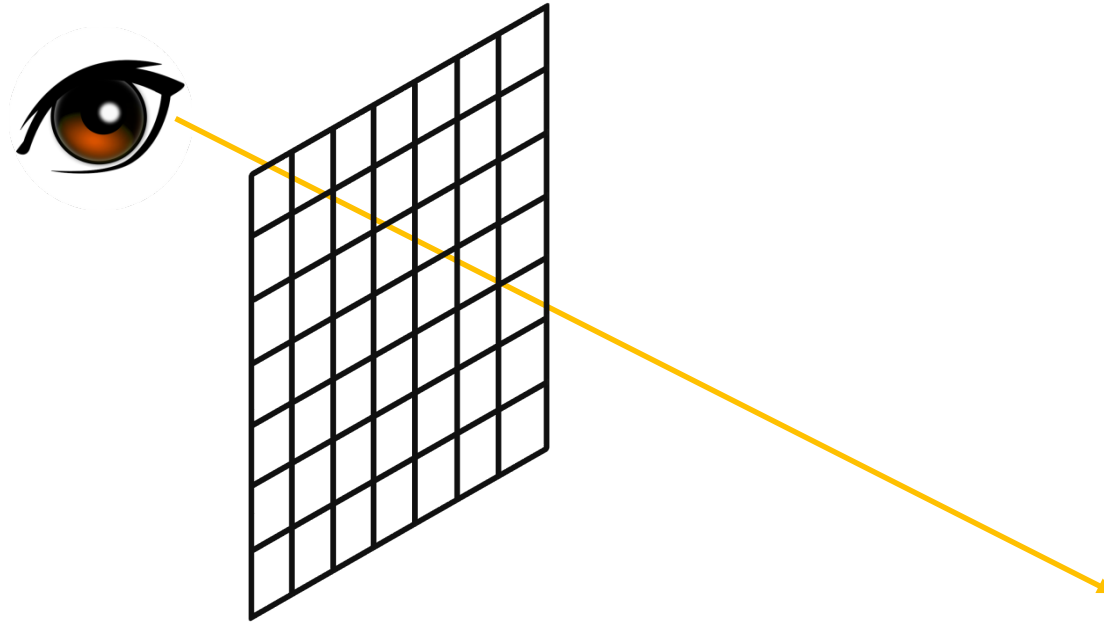
3D objects



# Basic Ray Tracer

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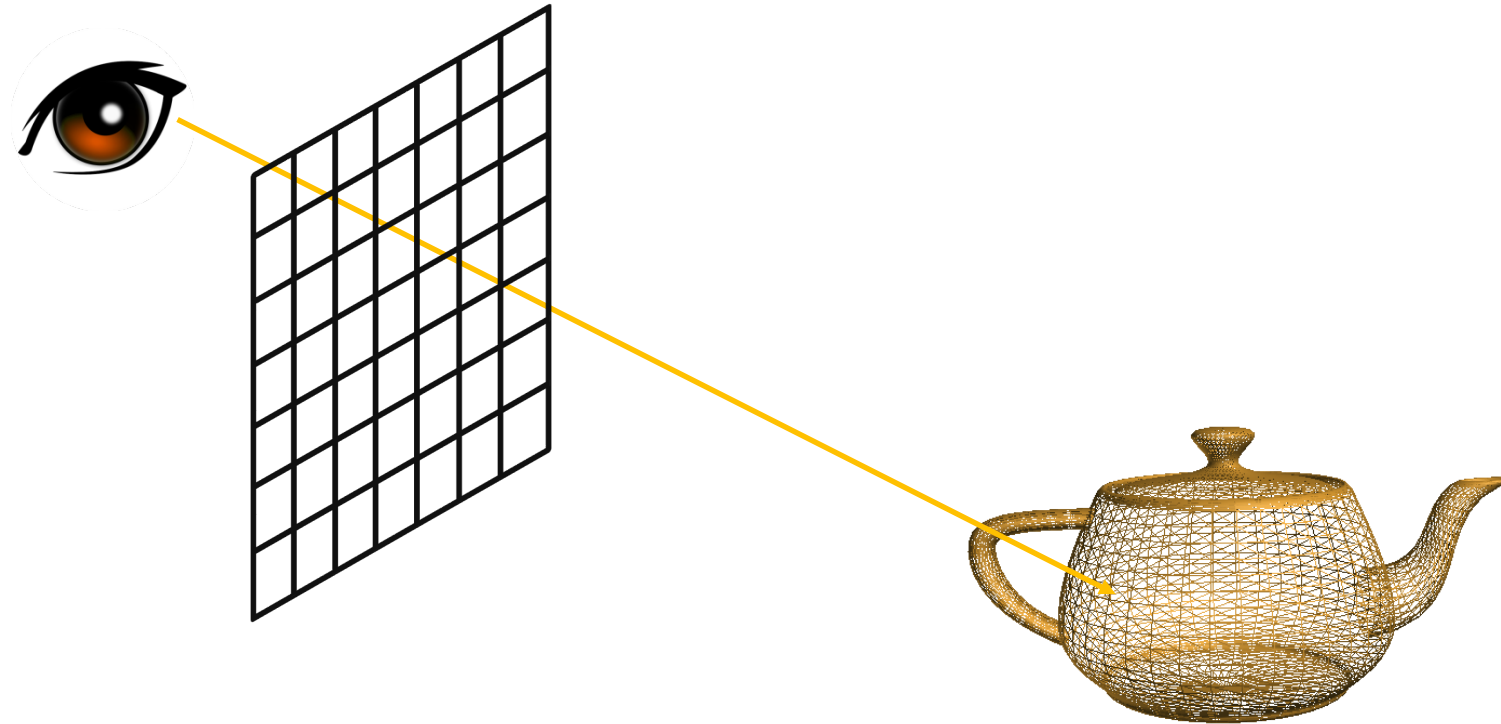
- Ray generation
  - Compute the origin and direction of a ray per pixel, by considering the camera and image plane



# Basic Ray Tracer

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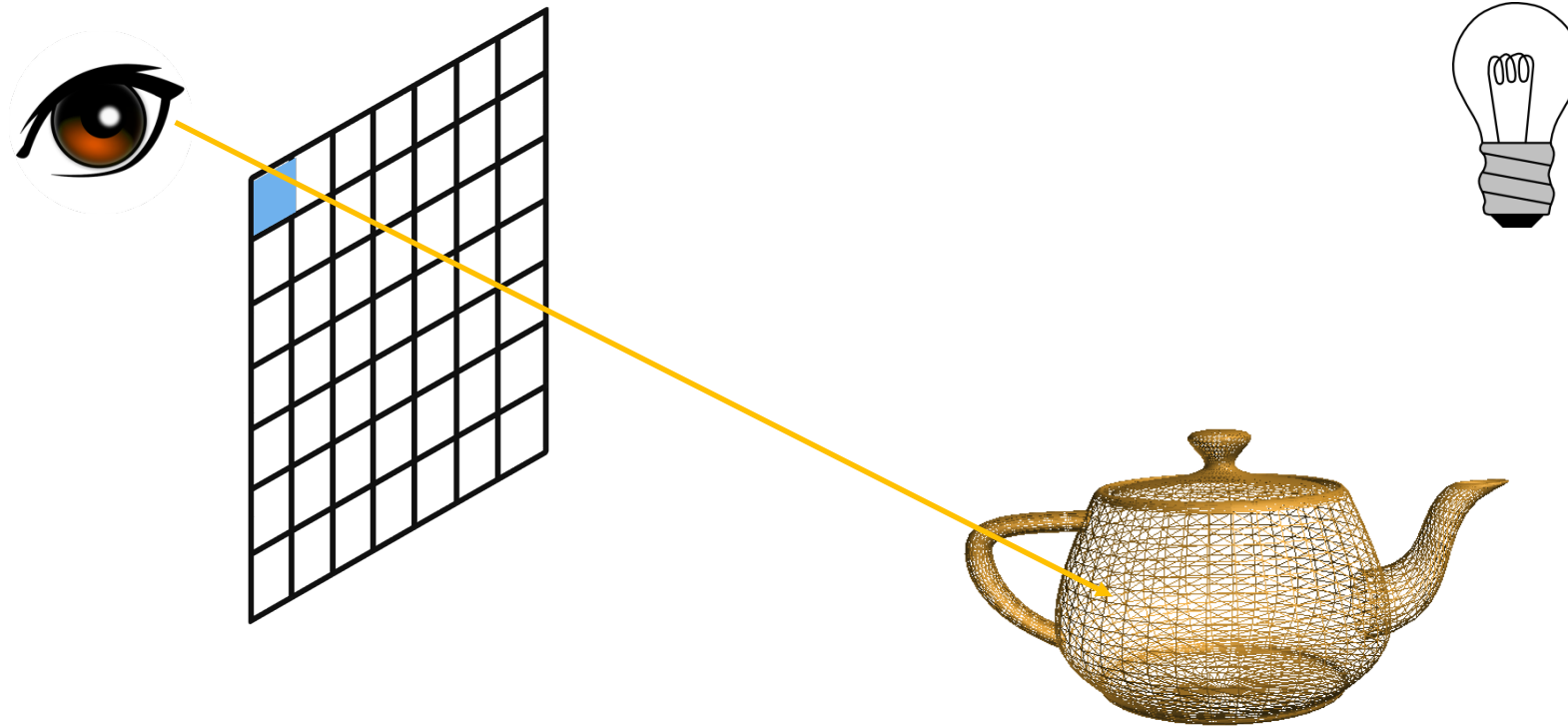
- Ray intersection
  - Find the closest intersection point between the ray and objects



# Basic Ray Tracer

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- Shading
  - Compute the pixel color using the geometry, material, and lights at the intersection point





# Basic Ray Tracer

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- For each pixel do
  - Compute a primary ray (viewing ray)
  - Find the closest intersection point between the ray and a scene
  - Determine a pixel color

# Primary Ray Generation

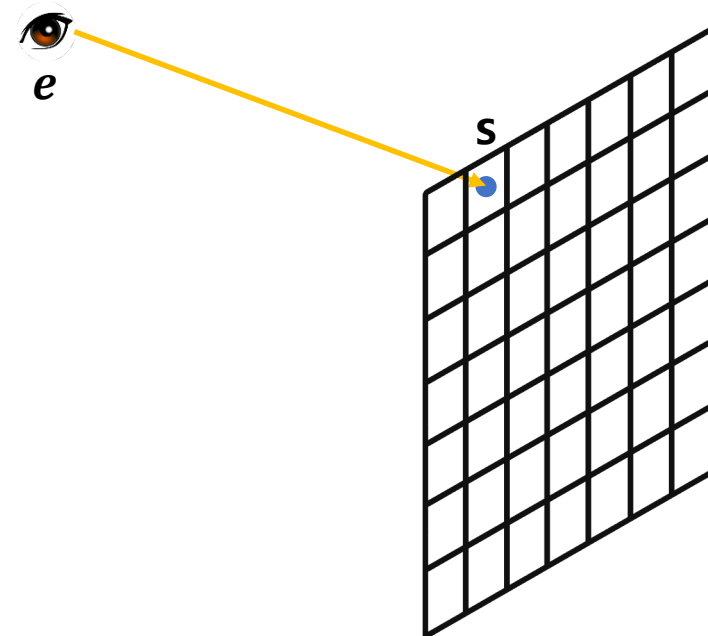
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- Mathematical representation for a ray
  - 3D parametric line:  $p(t) = e + t(s - e)$

- Properties

- $p(0) = e$ ,  $p(1) = s$
- $p(t_1)$  is closer to the eye than  $p(t_2)$  when  $0 < t_1 < t_2$
- When  $t < 0$ ,  $p(t)$  is behind the eye
- $e$  is a given value

- Q. How can we compute  $s$ ?



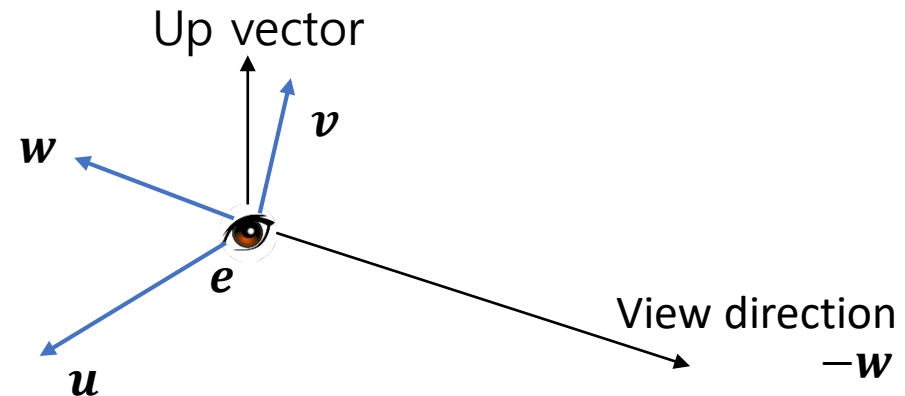
# Primary Ray Generation

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- Mathematical representation for a ray
  - 3D parametric line:  $p(t) = e + t(s - e)$

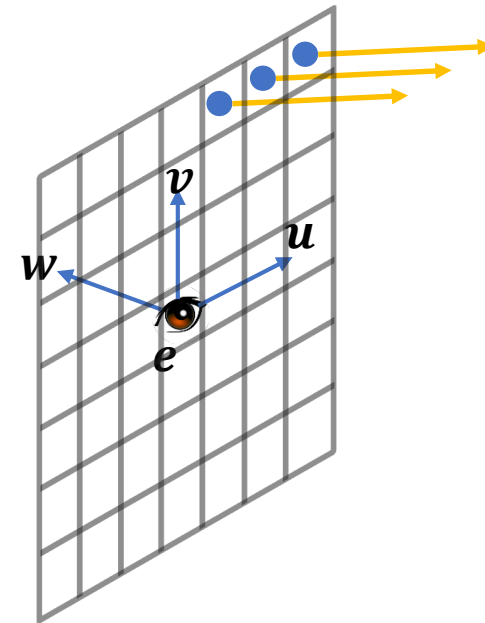
- $u, v, w$  forms a right-handed coordinate system

- Two kinds of views
  - Orthographic view
  - Perspective view



# Orthographic Views

- All primary rays have the same direction,  $-w$
- The primary ray starts on the image plane defined by  $e, u, v$
- The image plane is defined with four numbers:
  - $l, r$ : positions of left and right edges of the image plane
  - $b, t$ : positions of bottom and top edges
- To make an image with  $n_x \times n_y$ 
  - Pixels are spaced as the following:
    - $\frac{r-l}{n_x}$  horizontally,  $\frac{t-b}{n_y}$  vertically
- Position  $(\alpha, \beta)$  in the image plane is corresponding to a pixel  $(i, j)$  in the raster image:
  - $\alpha = l + \frac{(r-l)(i+0.5)}{n_x}$
  - $\beta = b + \frac{(t-b)(j+0.5)}{n_y}$
  - $(\alpha, \beta)$  are the coordinates of the pixel's position on the image plane



# Orthographic Views

- Procedure to generate orthographic viewing rays

- Compute  $\alpha$  and  $\beta$

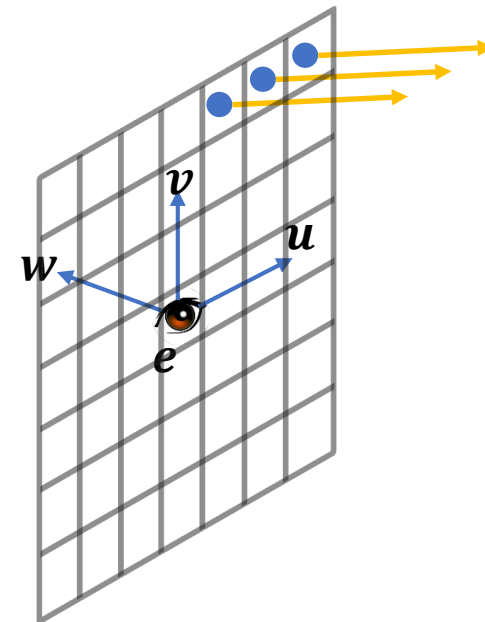
- $\alpha = l + \frac{(r-l)(i+0.5)}{n_x}$
- $\beta = b + \frac{(t-b)(j+0.5)}{n_y}$

- $ray.direction := -w$

- $ray.origin := e + \alpha u + \beta v$

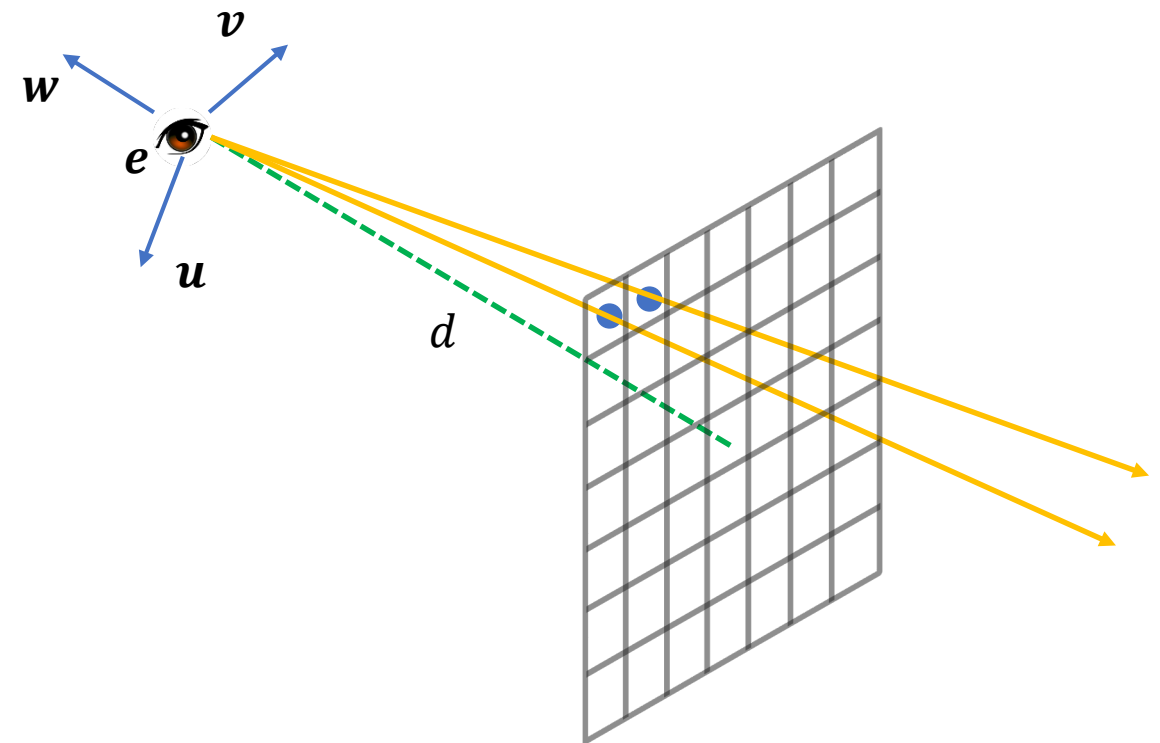
- Properties

- Same direction for all rays
- Different origins for rays



# Perspective Views

- All rays have the same origin,  $e$ , but have different directions
- The image plane is placed with a distance,  $d$ , in front of  $e$ 
  - $d$ : image plane distance (called the focal length)
- Procedure to generate perspective viewing rays
  - Compute  $\alpha$  and  $\beta$ 
    - $\alpha = l + \frac{(r-l)(i+0.5)}{n_x}$
    - $\beta = b + \frac{(t-b)(j+0.5)}{n_y}$
- $ray.direction := -dw + \alpha u + \beta v$
- $ray.origin := e$



# Intersection between Ray and Object

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- Generated ray:  $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$
- The next task is to find the closest intersection point between a ray and objects
  - i.e., need to find a  $t$  in the interval  $[t_0, t_1]$  (e.g.,  $[0, +\infty)$ )
- Objects
  - Sphere
  - Triangle
  - Multiple objects

# Intersection between Ray and Sphere

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- Ray:  $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$
- Implicit surface:  $f(\mathbf{p}) = 0$
- Intersection points should satisfy both equations
  - $f(\mathbf{p}(t)) = f(\mathbf{e} + t\mathbf{d}) = 0$
- Let's define a sphere with center  $\mathbf{c} = (x_c, y_c, z_c)$  and radius  $r$ 
  - $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - r^2 = 0$
  - $(\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - r^2 = 0$  (vector form)
  - A point  $\mathbf{p}$  that satisfies this equation is on the sphere
- By plug-in the parametric ray equation,
  - $(\mathbf{e} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{e} + t\mathbf{d} - \mathbf{c}) - r^2 = 0$
  - By rearranging terms with respect to  $t$  (unknown value):
  - $(\mathbf{d} \cdot \mathbf{d})t^2 + 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$



# Intersection between Ray and Sphere

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- A quadratic equation in  $t$

- $(\mathbf{d} \cdot \mathbf{d})t^2 + 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$

- The solutions for  $at^2 + bt + c = 0$

- $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

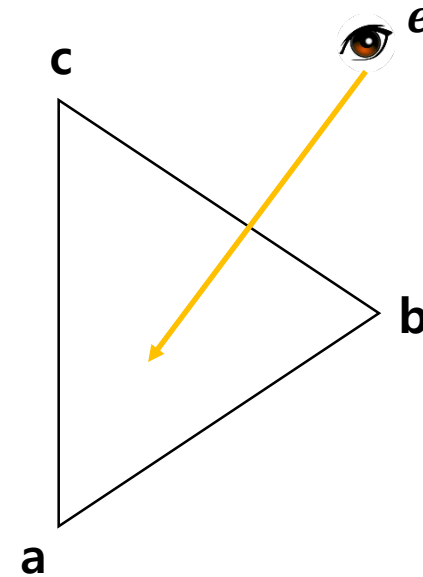
- $b^2 - 4ac$  (called discriminant)

- When  $b^2 - 4ac < 0$ , there is no solution (the ray does not intersect with the sphere)
    - When  $b^2 - 4ac = 0$ , a solution exists (the ray touches the sphere)
    - When  $b^2 - 4ac > 0$ , two solutions exist (the ray enters and leaves the sphere)

- $t = \frac{-\mathbf{d} \cdot (\mathbf{e} - \mathbf{c}) \pm \sqrt{(\mathbf{d} \cdot (\mathbf{e} - \mathbf{c}))^2 - (\mathbf{d} \cdot \mathbf{d})((\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2)}}{(\mathbf{d} \cdot \mathbf{d})}$

# Intersection between Ray and Triangle

- Ray:  $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$
- Intersection point:
  - $\mathbf{e} + t\mathbf{d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$
- Solving the equation for  $t, \beta, \gamma$ :
  - $x_e + tx_d = x_a + \beta(x_b - x_a) + \gamma(x_c - x_a)$
  - $y_e + ty_d = y_a + \beta(y_b - y_a) + \gamma(y_c - y_a)$
  - $z_e + tz_d = z_a + \beta(z_b - z_a) + \gamma(z_c - z_a)$
  - Can be rewritten:
    - $$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$



# Intersection between Ray and Triangle

- Cramer's rule can be utilized to solve the 3 x 3 linear system

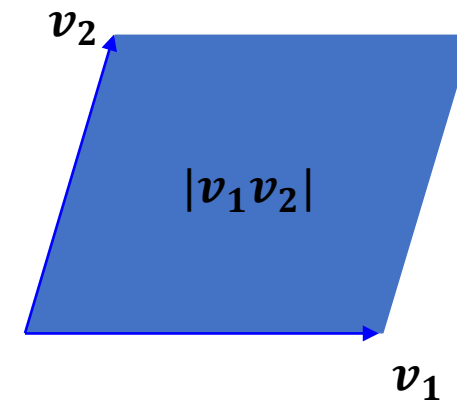
$$\circ \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\circ x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{|A|}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{|A|}, \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{|A|}$$

$$\circ \text{ where } |A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \text{ } |\cdot| \text{ is the determinant}$$

$$\circ |A| = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$\circ \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} = b_2 c_3 - c_2 b_3$$



# Intersection between Ray and Triangle

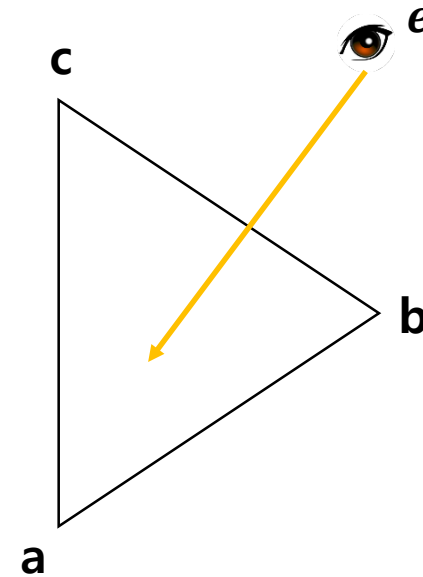
$$\circ \begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

$$\circ \beta = \frac{\begin{vmatrix} x_a - x_e & x_a - x_c & x_d \\ y_a - y_e & y_a - y_c & y_d \\ z_a - z_e & z_a - z_c & z_d \end{vmatrix}}{|A|}$$

$$\circ \gamma = \frac{\begin{vmatrix} x_a - x_b & x_a - x_e & x_d \\ y_a - y_b & y_a - y_e & y_d \\ z_a - z_b & z_a - z_e & z_d \end{vmatrix}}{|A|}$$

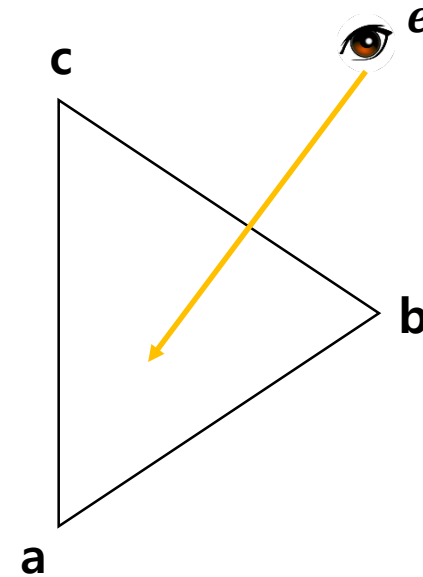
$$\circ t = \frac{\begin{vmatrix} x_a - x_b & x_a - x_c & x_a - x_e \\ y_a - y_b & y_a - y_c & y_a - y_e \\ z_a - z_b & z_a - z_c & z_a - z_e \end{vmatrix}}{|A|}$$

$$\circ \text{ where } |A| = \begin{vmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{vmatrix}$$



# Intersection between Ray and Triangle

- Procedure (with early termination) for finding the intersection:
  - Input: a ray, vertex  $a, b, c$ , interval  $[t_0, t_1]$
  - Compute  $t$
  - If  $(t < t_0)$  or  $(t > t_1)$  then
    - return false
  - Compute  $\gamma$
  - If  $(\gamma < 0)$  or  $(\gamma > 1)$  then
    - return false
  - Compute  $\beta$
  - If  $(\beta < 0)$  or  $(\beta > 1 - \gamma)$  then
    - return false
  - return true



# Intersection between Ray and Objects

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- Procedure for finding the *closest* intersection:
  - hit = false
  - For each object  $o$  do
    - If ( $o$  is intersected with the ray at a parameter  $t$  and  $t \in [t_0, t_1]$ ) then
      - hit = true
      - store some information (e.g.,  $o$ , normal, etc.) for shading
      - $t_1 = t$
  - return hit

# Basic Ray Tracer

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- For each pixel do
  - Compute a primary ray (viewing ray)
  - Find the closest intersection point between the ray and a scene
  - Determine a pixel color
    - e.g., we can apply the Phong illumination model here

# Basic Ray Tracer

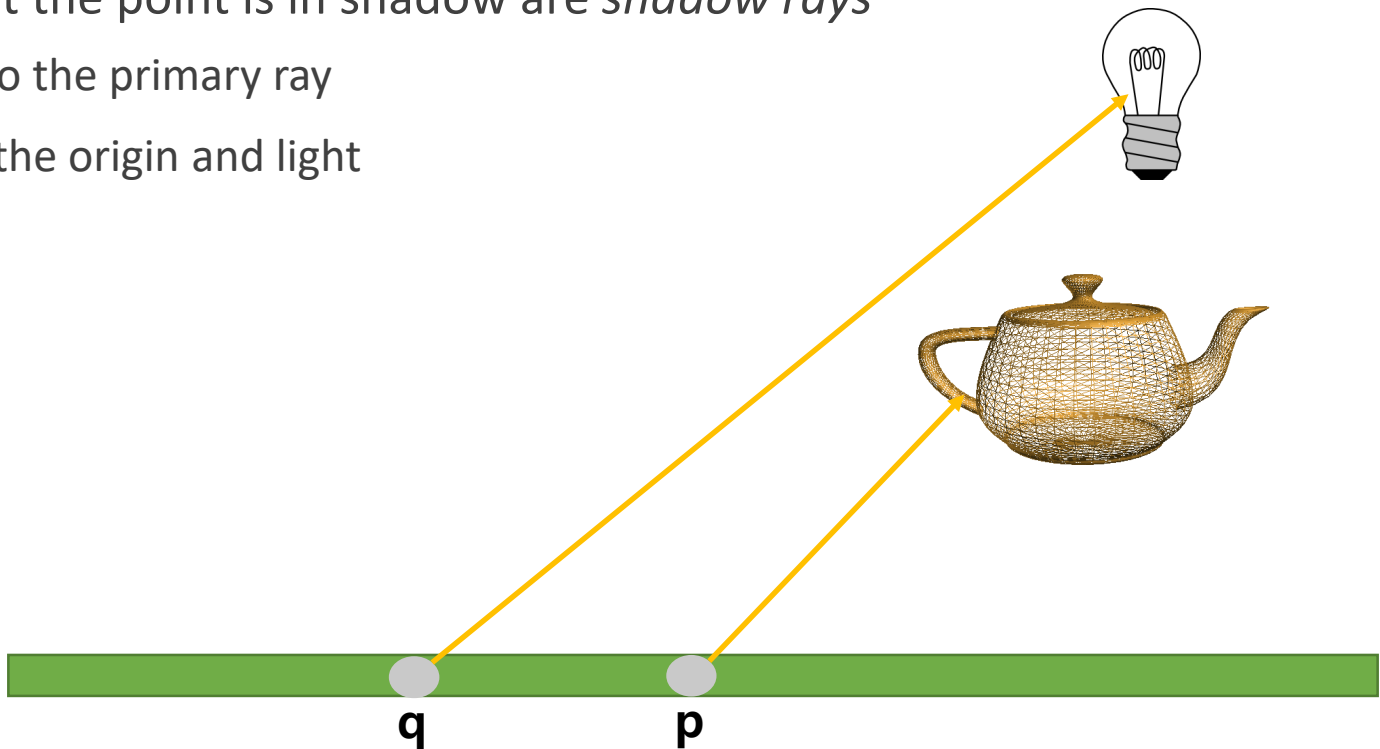
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- For each pixel do
  - Compute a primary ray (viewing ray)
  - If (ray intersects an object with  $t \in [0, \infty)$ ) then
    - Compute a hit record that contains some information (normal, materials, ...)
    - Evaluate an illumination model and set a pixel color
  - Else
    - Set a pixel color to background color



# Shadows

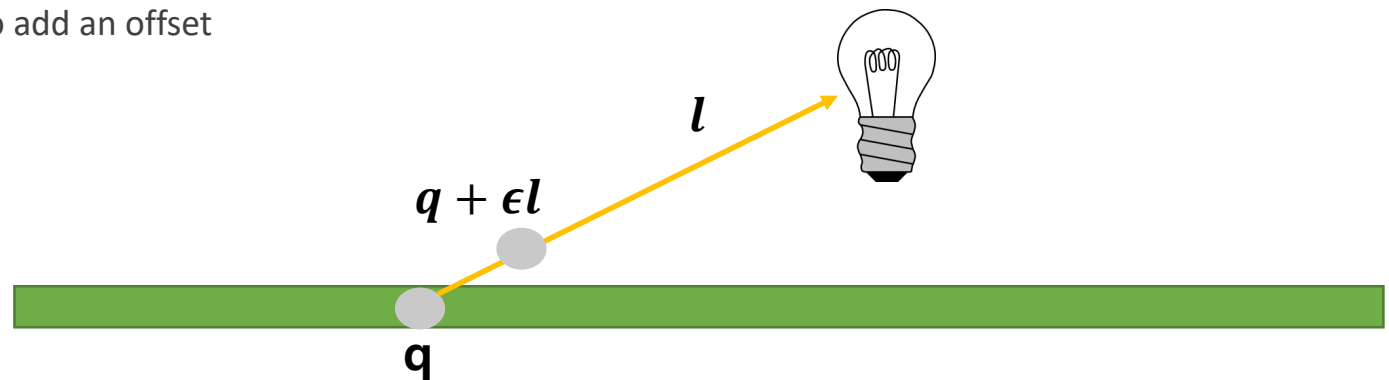
- Assume there are two intersection points,  $p$  and  $q$ 
  - $p$  is in shadow, but  $q$  is not in shadow
- Rays to determine whether or not the point is in shadow are *shadow rays*
  - Generate a shadow ray similar to the primary ray
  - Check there is any hit between the origin and light
    - $t = [t_0, t_1]$
  - e.g.,  $t = [0, t_1]$



# Shadows

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- Assume there are two intersection points,  $p$  and  $q$ 
  - $p$  is in shadow, but  $q$  is not in shadow
- Rays to determine whether or not the point is in shadow are *shadow rays*
  - Generate a shadow ray similar to the primary ray
  - Check there is any hit between the origin and light
    - $t = [t_0, t_1]$
  - Due to numerical issues, the shadow ray can intersect the surface on which the point lies
    - A naïve but common approach is to add an offset
    - $t = [\epsilon, t_1]$



# Shadows

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- Pseudocode to implement shadows (based on the Phong illumination)
- Input: a ray  $\mathbf{e} + t\mathbf{d}$ ,  $[t_0 = 0, t_1 = \infty]$
- If (there is a hit between the ray and objects) then
  - $\mathbf{p} = \mathbf{e} + t\mathbf{d}$  // p is the closest intersection from e
  - **color**  $\mathbf{c} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$
  - If (there is no hit between the shadow ray and a light) then
    - $\mathbf{c} = \mathbf{c} + k_a L_a + L_d k_d \max(0, \mathbf{n} \cdot \mathbf{l}) + L_s k_s \max(0, \mathbf{r} \cdot \mathbf{v})^s$
  - return c
- Else
  - return background color

# Some History of Ray Tracing

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- Rene Descartes (1637) used ray tracing to explain the phenomena of rainbow
- In rendering, the ray casting was presented by Arthur Appel (1968)
  - Ray casting (discussed so far) tends to be interchangeable to ray tracing
  - Ray tracing generates additional rays (e.g., secondary rays) to simulate global illumination effects
  - Ray tracing becomes popular due to the Whitted's paper (1980)
    - T. Whitted. An improved illumination model for shading display. Communications of the ACM, 23(6):343–349, 1980

# Further Readings

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- Chapter 4