Lecture slides (CT4201/EC4215 – Computer Graphics)

Global Illumination Methods

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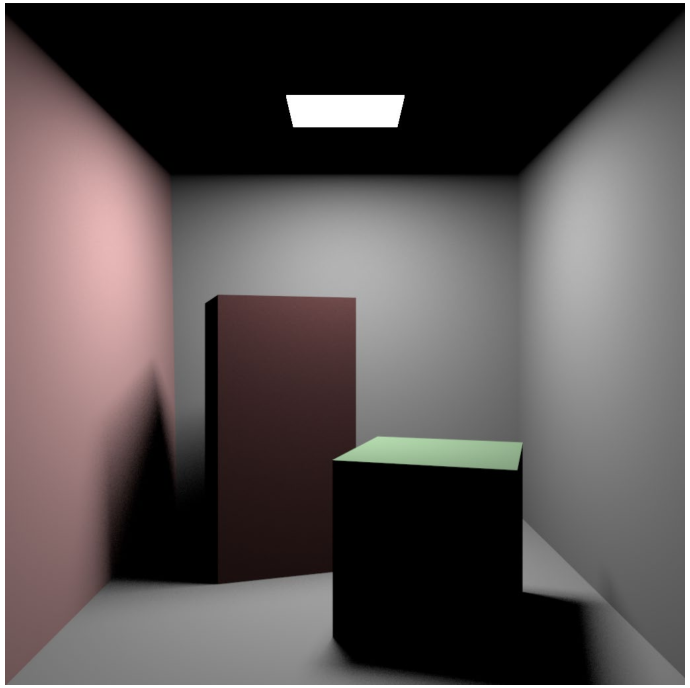
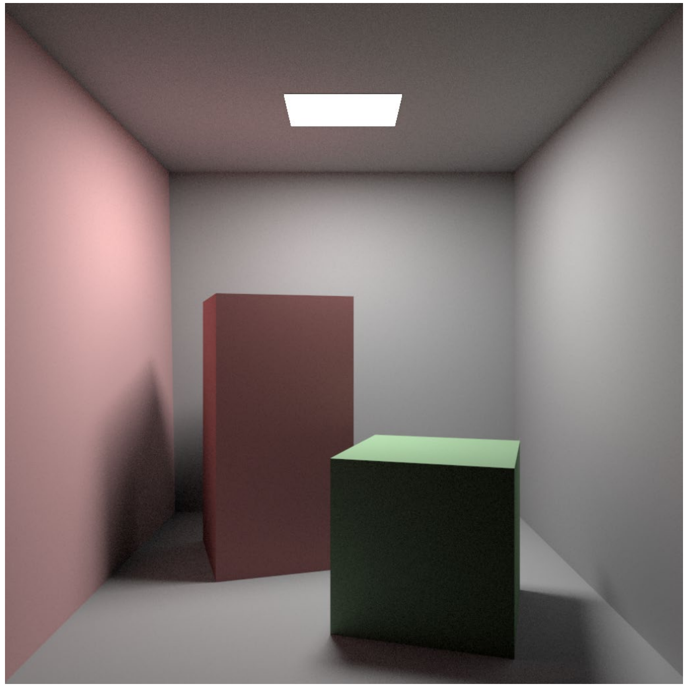
Global Illumination

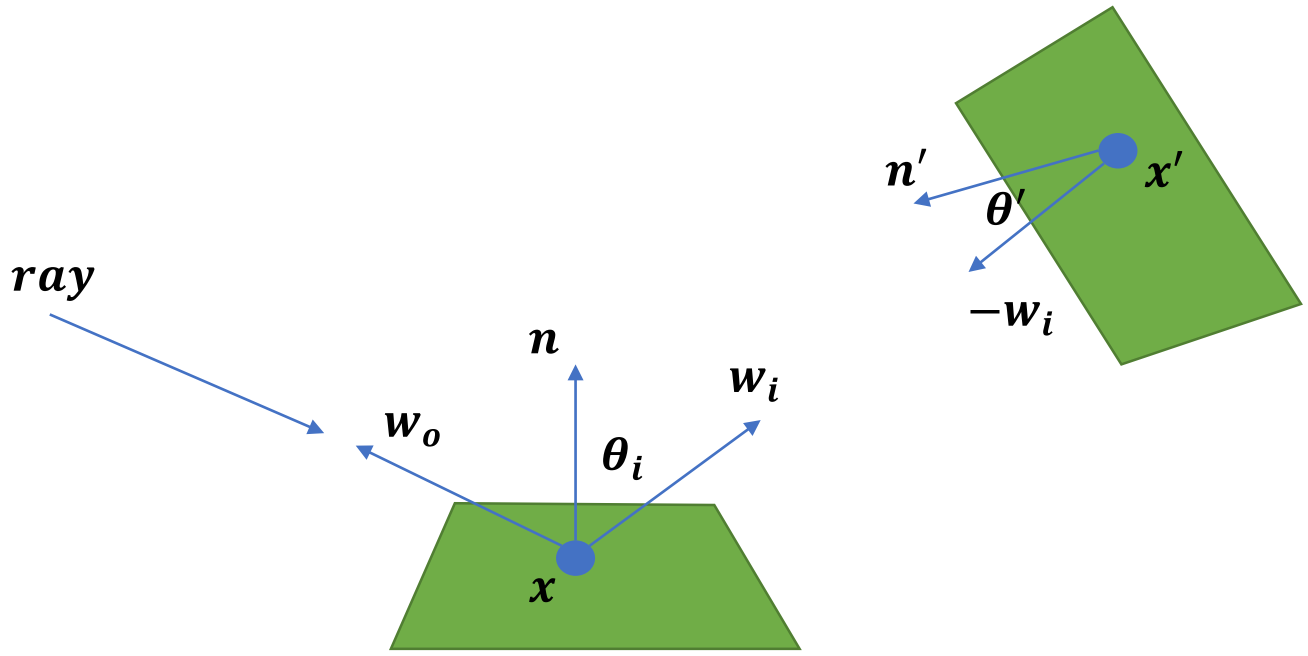
- Distributed ray tracing introduces a noticeable improvement on rendering quality, but it still misses some rendering effects





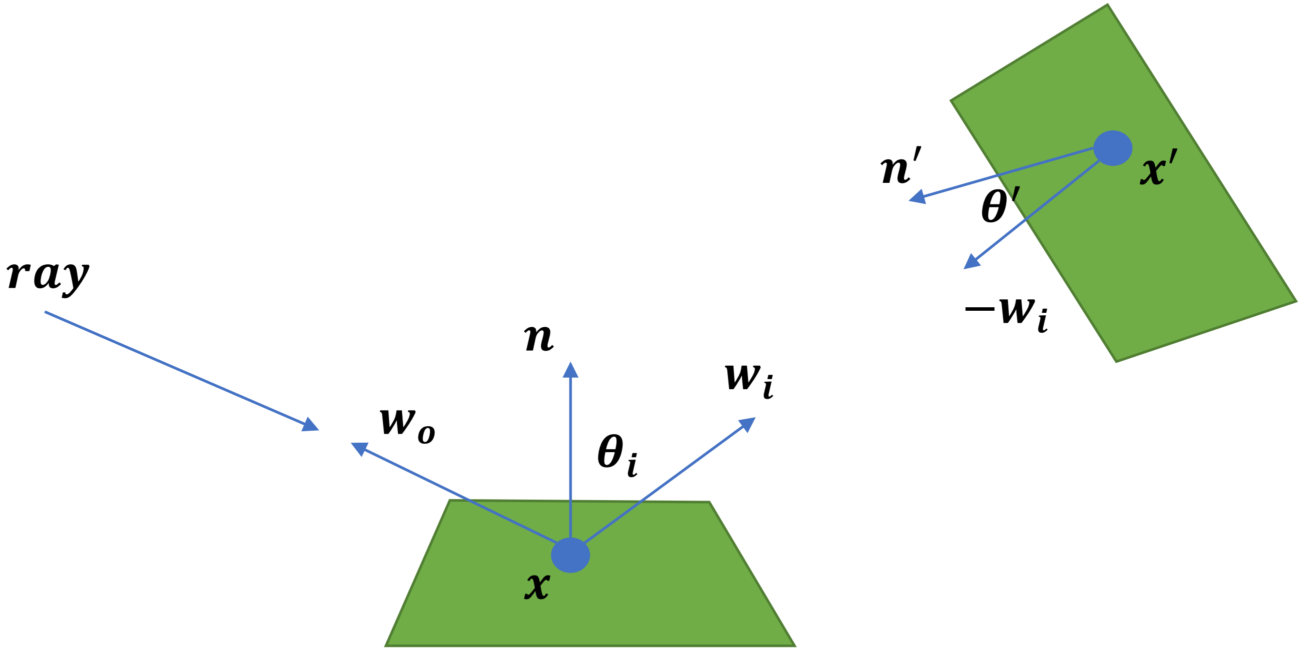
Global Illumination

- Rendering equation [Kajiya 86] (another form: [Immel and Cohen 86])
- $L_s(\boldsymbol{x}, w_o) = \int_{all\ \boldsymbol{x'}} \frac{\rho(\boldsymbol{w_i}, \boldsymbol{w_o}) L_S(\boldsymbol{x'}, \boldsymbol{x}-\boldsymbol{x'}) v(\boldsymbol{x}, \boldsymbol{x'}) cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x'}\|^2} dA$



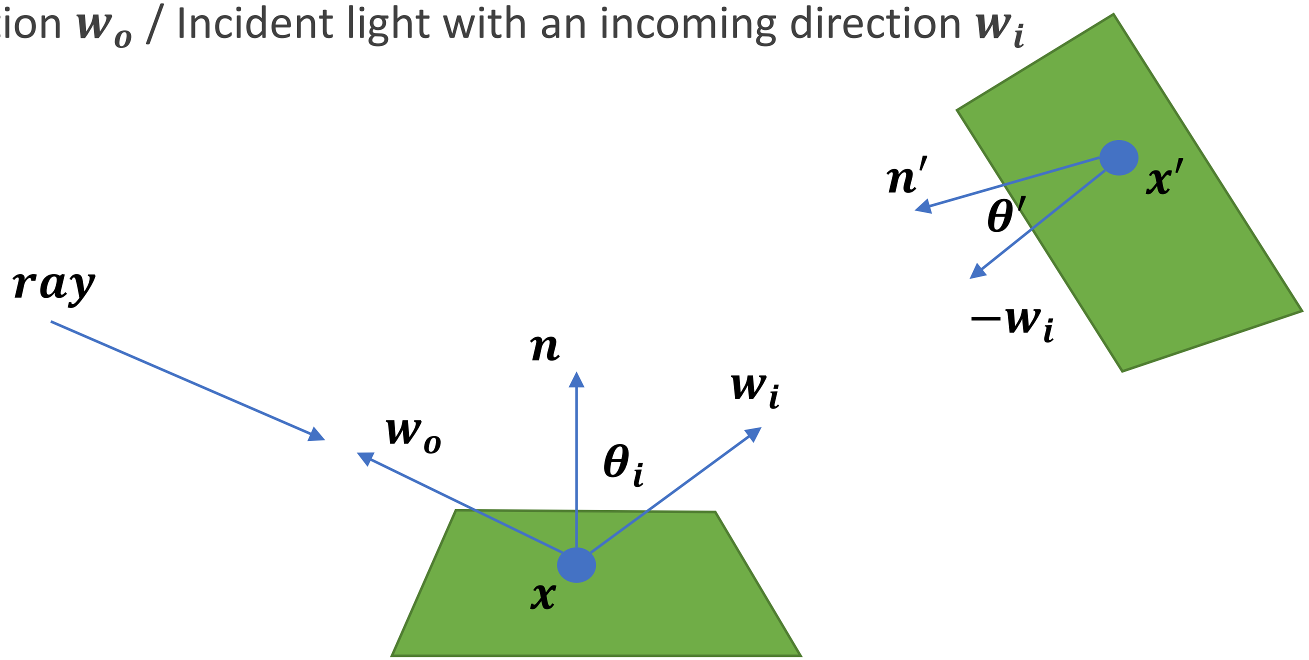
Global Illumination

- $L_s(\boldsymbol{x}, w_o) = \int_{all\ \boldsymbol{x'}} \frac{\rho(\boldsymbol{w_i},\boldsymbol{w_o}) L_S(\boldsymbol{x'},\boldsymbol{x}-\boldsymbol{x'}) v(\boldsymbol{x},\boldsymbol{x'}) cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x'}\|^2} dA$
- $v(\boldsymbol{x}, \boldsymbol{x'})$: visibility term
  - 1 if two points are mutually visible
  - 0 otherwise



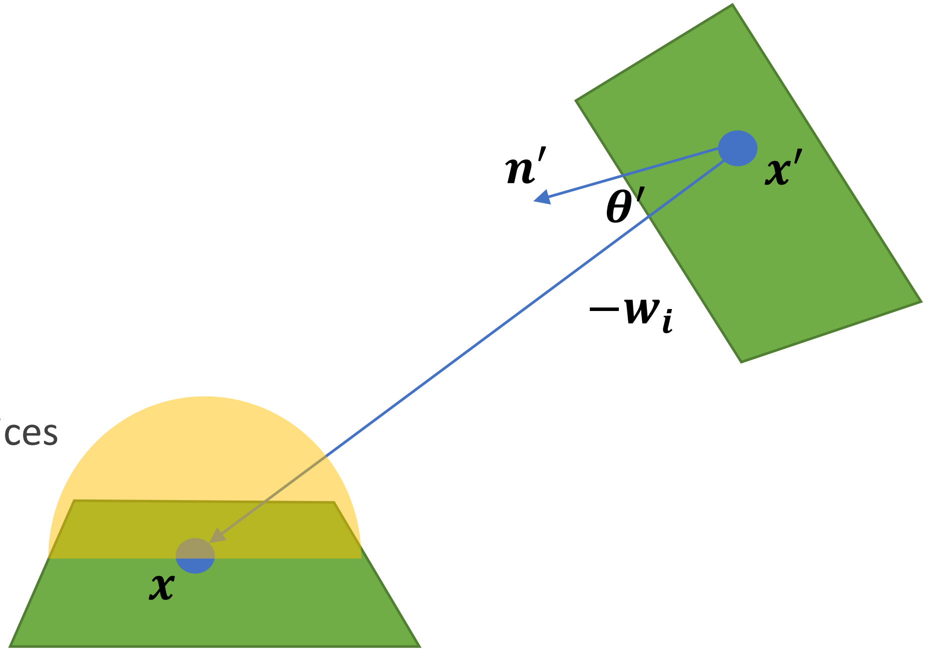
Global Illumination

- $L_S(\boldsymbol{x}, w_o) = \int_{all\ \boldsymbol{x'}} \frac{\rho(\boldsymbol{w_i},\boldsymbol{w_o})L_S(\boldsymbol{x'},\boldsymbol{x}-\boldsymbol{x'})v(\boldsymbol{x},\boldsymbol{x'})cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x'}\|^2} dA$
- $\rho(\boldsymbol{w_i}, \boldsymbol{w_o})$: bidirectional reflectance distribution function (BRDF)
  - Reflected light with an outgoing direction $\boldsymbol{w_o}$ / Incident light with an incoming direction $\boldsymbol{w_i}$
  - Determine appearance of a surface



Global Illumination

- $L_s(\boldsymbol{x}, w_o) = \int_{all\ \boldsymbol{x'}} \frac{\rho(\boldsymbol{w_i},\boldsymbol{w_o}) L_s(\boldsymbol{x'}, \boldsymbol{x}-\boldsymbol{x'}) v(\boldsymbol{x},\boldsymbol{x'}) cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x'}\|^2} dA$
- $\rho(\boldsymbol{w_i}, \boldsymbol{w_o})$: bidirectional reflectance distribution function (BRDF)
  - Reflected light with an outgoing direction $\boldsymbol{w_o}$ / Incident light with an incoming direction $\boldsymbol{w_i}$
  - Determine appearance of a surface
  - e.g. ideal diffuse surface
    - $\rho(\boldsymbol{w_i}, \boldsymbol{w_o}) = \rho$ (constant)
  - e.g. other surfaces?
    - Utilize a mathematical form for well-known materials
    - Build the 4D function by measuring the fraction using actual devices



Global Illumination

- $L_s(\boldsymbol{x}, w_o) = \int_{all\ \boldsymbol{x}\prime} \frac{\rho(\boldsymbol{w_i}, \boldsymbol{w_o}) L_S(\boldsymbol{x}', \boldsymbol{x}-\boldsymbol{x}') v(\boldsymbol{x}, \boldsymbol{x}') cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x}'\|^2} dA$

- How can we solve the integral?

Monte Carlo Integration

- A sampling method based on probability
- Draw random numbers and approximate an integral
- e.g., evaluate a 1D integral: $\int_a^b f(x)dx$
- $F_n = \frac{b-a}{n}\sum_{i=1}^{n} f(x_i)$
- $x_i \in [a, b]$: uniform random variable, $p(x) = \frac{1}{b-a}$
- $E[F_n] = \frac{b-a}{n}\sum_{i=1}^{n} E[f(x_i)] = \frac{b-a}{n}\sum_{i=1}^{n}\int_a^b f(x)p(x)dx = \frac{1}{n}\sum_{i=1}^{n}\int_a^b f(x)dx$
- $= \int_a^b f(x)dx$

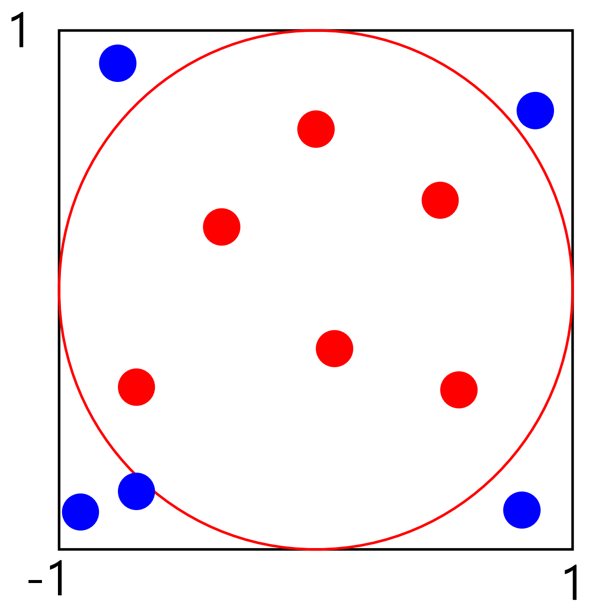
Monte Carlo Integration

- A sampling method based on probability
- Draw random numbers and approximate an integral
- e.g., evaluate a 1D integral: $\int_a^b f(x)dx$
- $F_n = \frac{b-a}{n}\sum_{i=1}^{n} f(x_i)$
- $x_i \in [a, b]$: from a probability density function $p(x)$
  - $F_n = \frac{1}{n}\sum_{i=1}^{n} \frac{f(x_i)}{p(x_i)}$
  - Indicate that we can control sampling density by adjusting $p(x)$
  - Require a good sampling strategy (e.g., *important sampling* – a research area)
  - The approximation error decreases as the number of samples increases

Monte Carlo Integration

- $f(x, y) = 1$ if $x^2 + y^2 \leq 1$
- $= 0$ otherwise

- $I = \int_{\Omega} f(x, y) dxdy = \pi$
- $\Omega = [-1, -1] \times [1,1]$
- $I \approx \frac{V}{n} \sum_{i=1}^{n} f(x_i, y_i)$
- V is the volume of the Ω
  - V = 4
- (x_i, y_i) has a uniform pdf

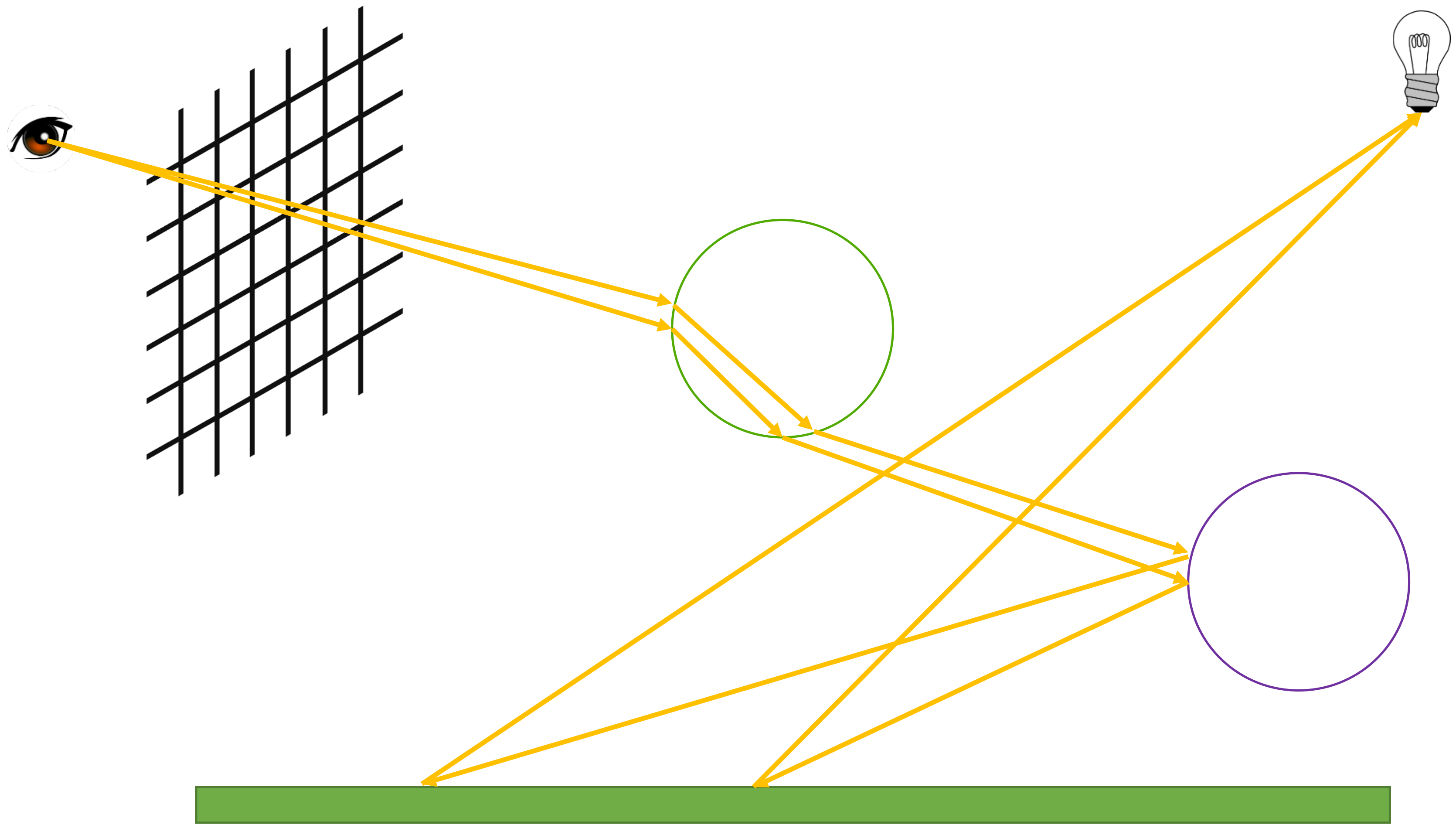


Monte Carlo Integration

- $L_s(\boldsymbol{x}, w_o) = \int_{all\ \boldsymbol{x'}} \frac{\rho(\boldsymbol{w_i},\boldsymbol{w_o}) L_s(\boldsymbol{x'},\boldsymbol{x}-\boldsymbol{x'}) v(\boldsymbol{x},\boldsymbol{x'}) cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x'}\|^2} dA$

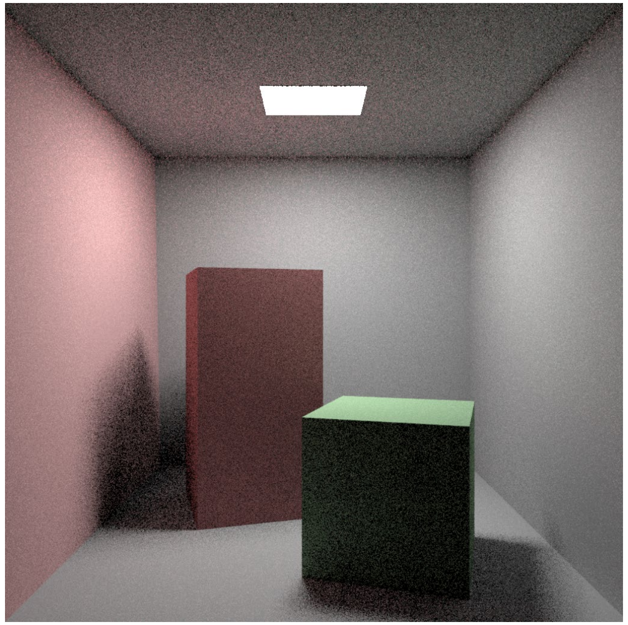
- Monte Carlo ray tracing
  - e.g., distributed ray tracing, path tracing
  - A random sample x_i is a high-dimensional vector
  - $x_i = [random\ numbers\ on\ image\ plane, lens, time, area\ lights, for\ secondary\ rays]$
  - Essentially the integral is a high-dimensional integral
  - The pixel color is an output by averaging the colors from multiple light paths

Path Tracing

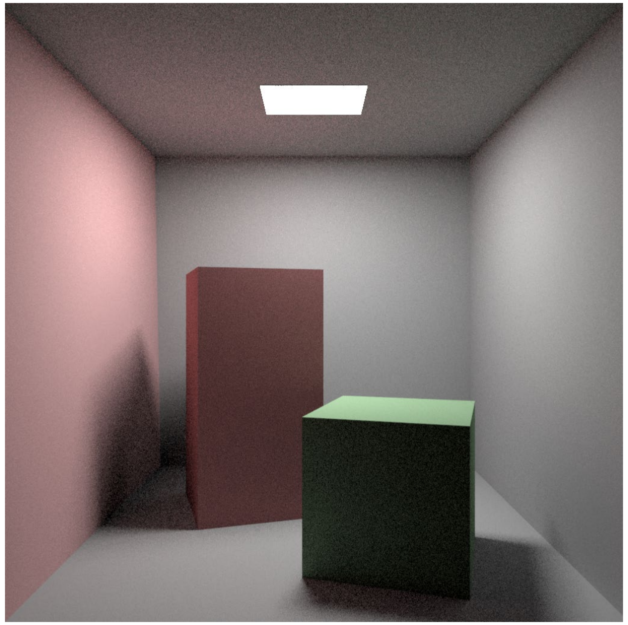


Path Tracing

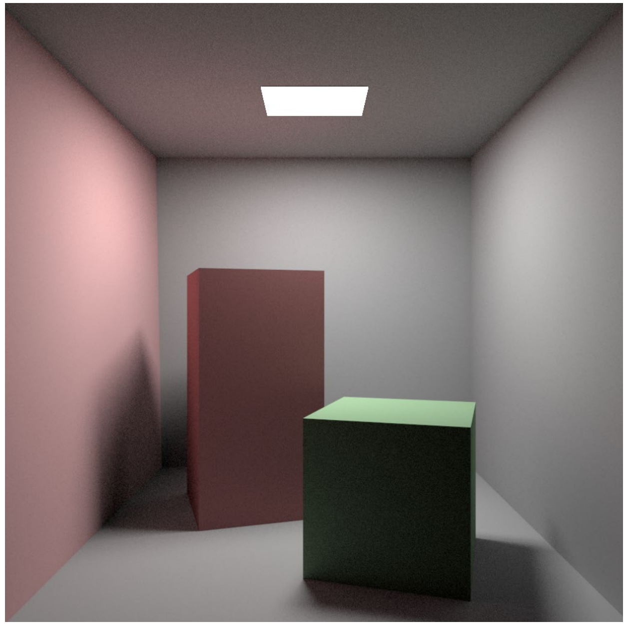
- A general rendering method that solves the full light transport equation (i.e., rendering equation)
- For each pixel color, it makes multiple ray paths, then averages the colors from the ray paths



4 samples / pixel (1.25 secs)



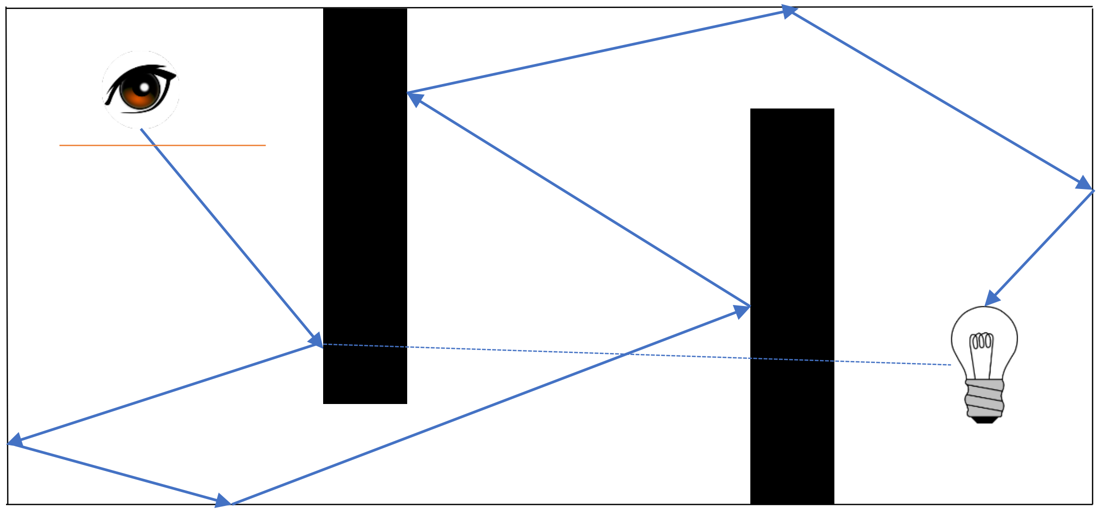
16 samples / pixel (5 secs)



64 samples / pixel (20 secs)

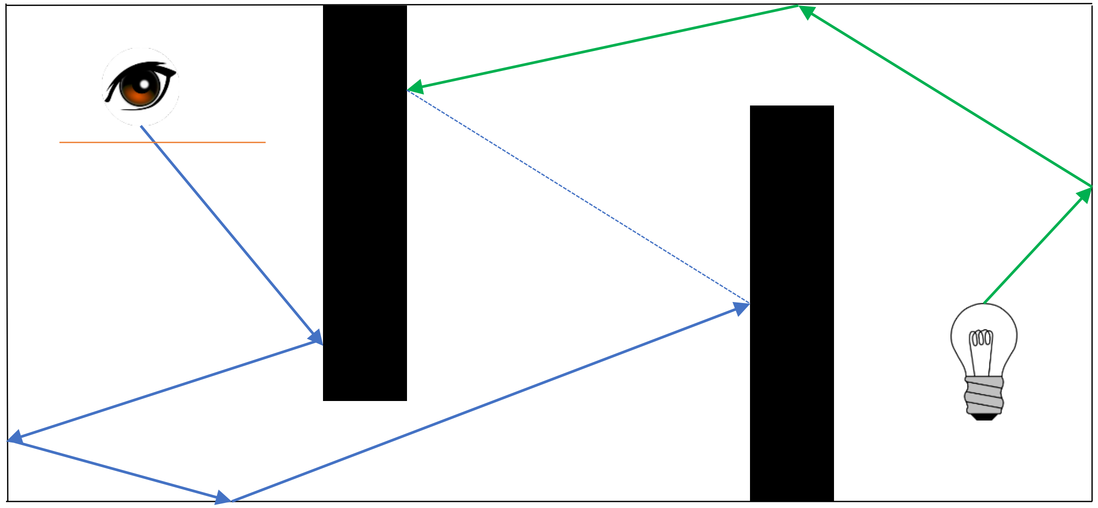
Path Tracing

- Find each light path starting from eye
  - Can be inefficient when light paths are difficult to reach from the eye



Bidirectional Path Tracing

- [Veach and Guibas 95][Lafortune and Willems 93]
- Find light subpaths from the eye and light sources, and connect the subpaths



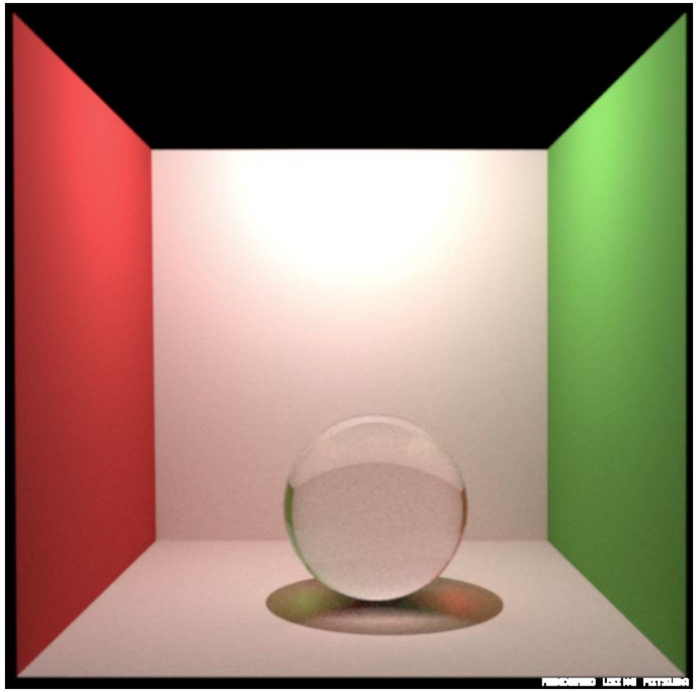
Unidirectional vs. bidirectional PT

Path tracing, 64 samples per pixel

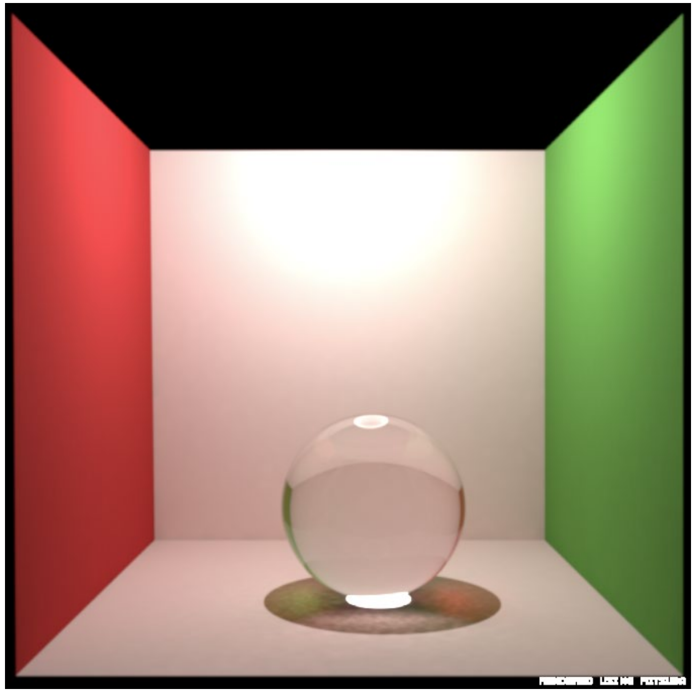
Bidirectional Path tracing, 64 samples per pixel

Photon Mapping

- [Jensen 1996]
- Motivation: it is hard to simulate some rendering effects (e.g., caustics) unless a large number of ray samples are used



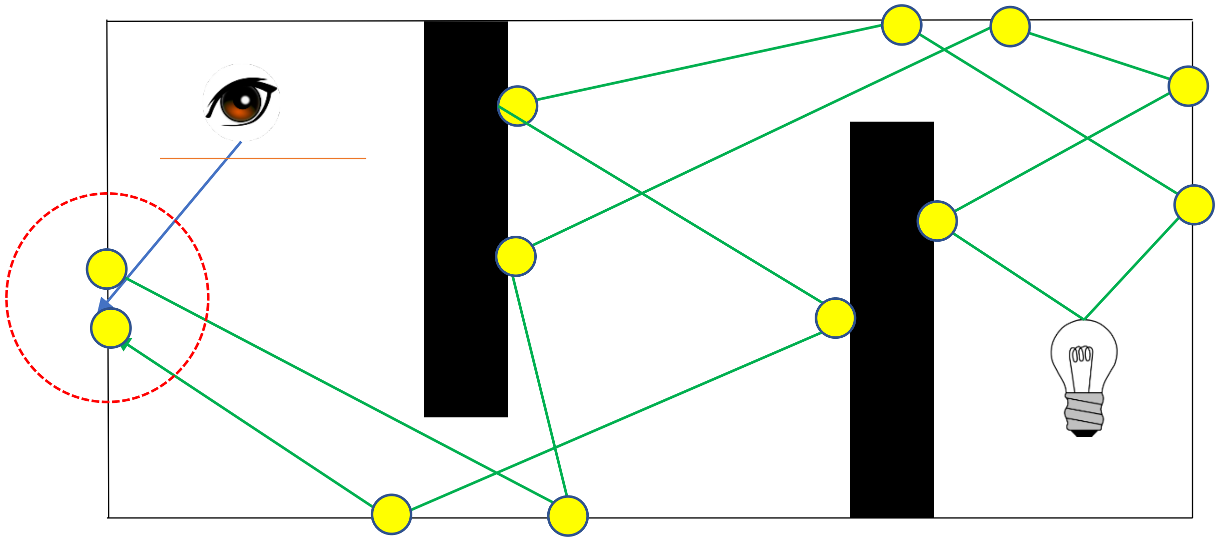
Path tracing



Photon mapping

Photon Mapping

- Generate photons from each light and store them in kd-tree
- Trace primary rays and estimate a photon density at intersected points
  - Similar to the bidirectional path tracing, but the stored photons can be utilized for all eye rays (why?)

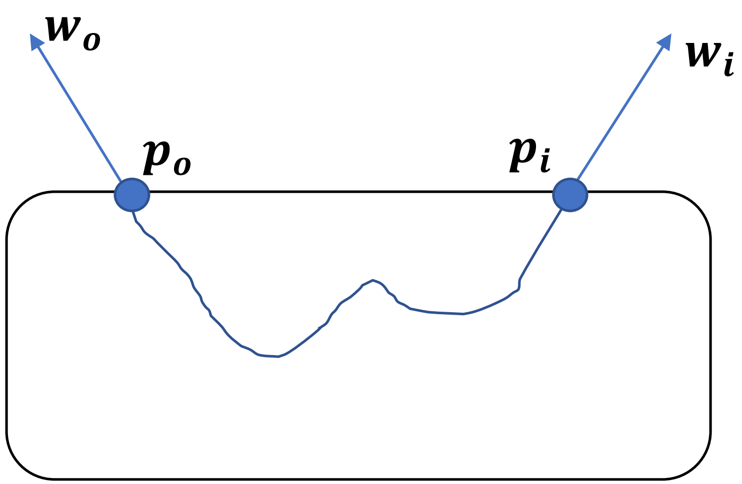


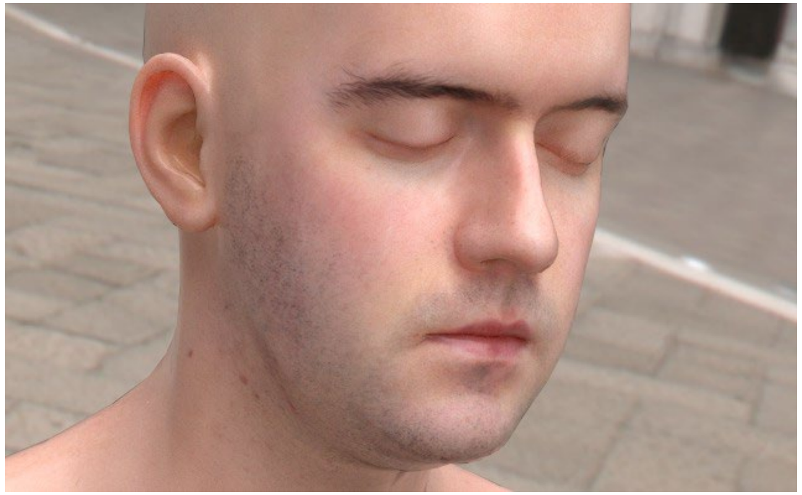
Advanced Topics

- Modeling complex materials
  - Bidirectional scattering-surface reflectance distribution function (BSSRDF)

- $\rho(\boldsymbol{w_i}, \boldsymbol{w_o})$: BRDF

- $\rho(\boldsymbol{p_i}, \boldsymbol{w_i}, \boldsymbol{p_o}, \boldsymbol{w_o})$: BSSRDF





Human head model with a BSSRDF

Model courtesy of Infinite Realities, Inc.
Image comes from pbrt.org

Advanced Topics

- Complex geometry (e.g., hair, fur) and scattering

[Jakob et al. 2010]

Model courtesy of Cem Yuksel
Image comes from pbrt.org

Advanced Topics

- Importance Sampling
  - $F_n = \frac{1}{n}\sum_{i=1}^{n}\frac{f(x_i)}{p(x_i)}$
  - Can we estimate an optimal pdf instead of the uniform distribution?

- Denoising

Path tracing,
16 samples per pixel

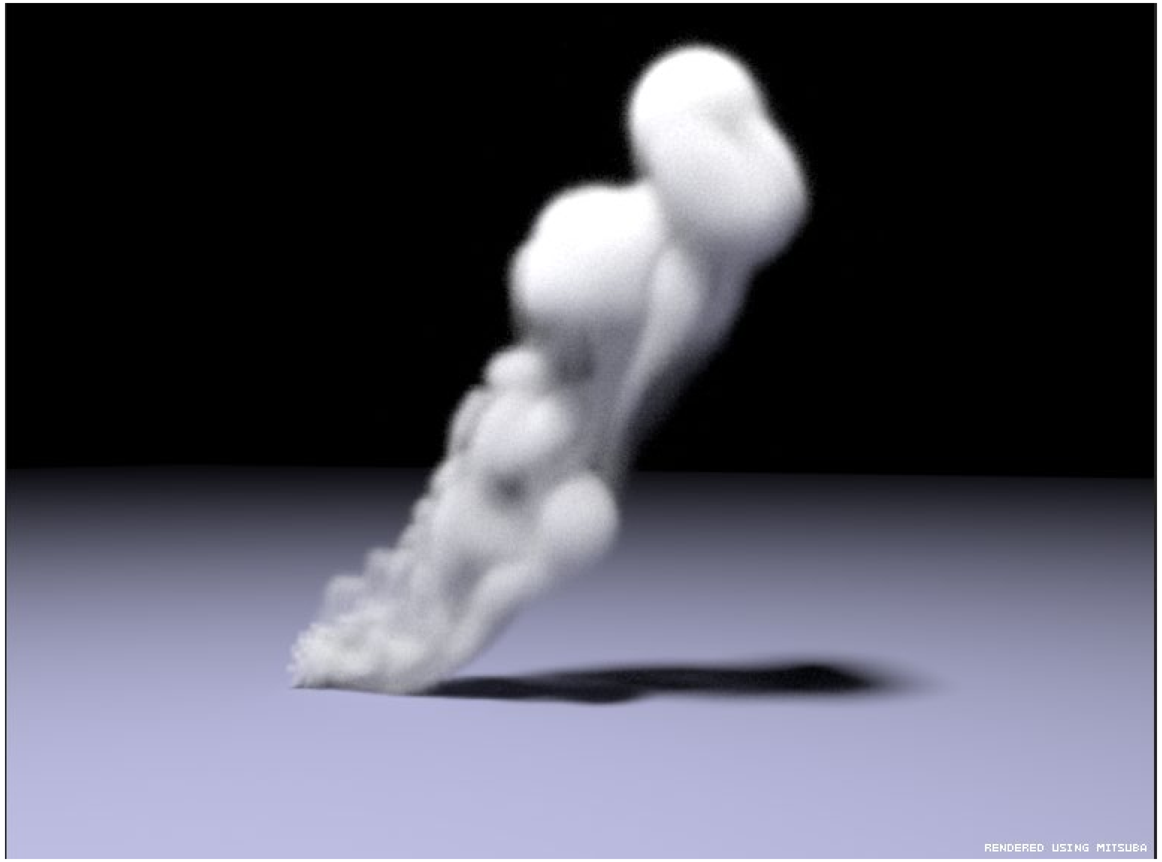
Denoised image,
16 samples per pixel

Path tracing
32K samples per pixel

Images from [Moon et al. 2016]

Advanced Topics

- Volume rendering



Graphics Topics

- Graphics Areas
  - Rendering
  - Modeling
  - Animation
  - User interaction
  - Augmented & virtual reality
  - Visualization
  - etc.

- Check out recent SIGGRAPH/SIGGRAPH Asia papers to know the active areas