

Lecture slides (CT4201/EC4215 – Computer Graphics)

# Whitted Ray Tracing

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Lecturer: Bochang Moon

# Whitted Ray Tracing

- Recursively perform the ray casting to simulate refraction and reflection



Whitted (1980)

# An Improved Illumination Model [Whitted 80]

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- Phong illumination model

$$\circ I = \sum_{i=1}^{\# \text{ of lights}} \left\{ L_a^i k_a + L_d^i k_d \max(0, \mathbf{n} \cdot \mathbf{l}^i) + L_s^i k_s \max(0, \mathbf{r}^i \cdot \mathbf{v})^s \right\}$$

- Whitted illumination model

$$\circ I = \sum_{i=1}^{\# \text{ of lights}} \left\{ L_a^i k_a + L_d^i k_d \max(0, \mathbf{n} \cdot \mathbf{l}^i) \right\} + k_s S + k_t T$$

○ S: intensity of light coming from a reflection ray

○ T: intensity of light that comes from a transmission ray

○  $k_s, k_t$ : coefficients for specular and transmission

# An Improved Illumination Model [Whitted 80]

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- Whitted illumination model

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- S: intensity of light coming from a reflection ray
    - T: intensity of light that comes from a transmission ray
    - $k_s, k_t$ : coefficients for specular and transmission
  - Q. how we can compute the rays?
    - Note that we need to generate the secondary rays to find S and T

# Specular Reflection

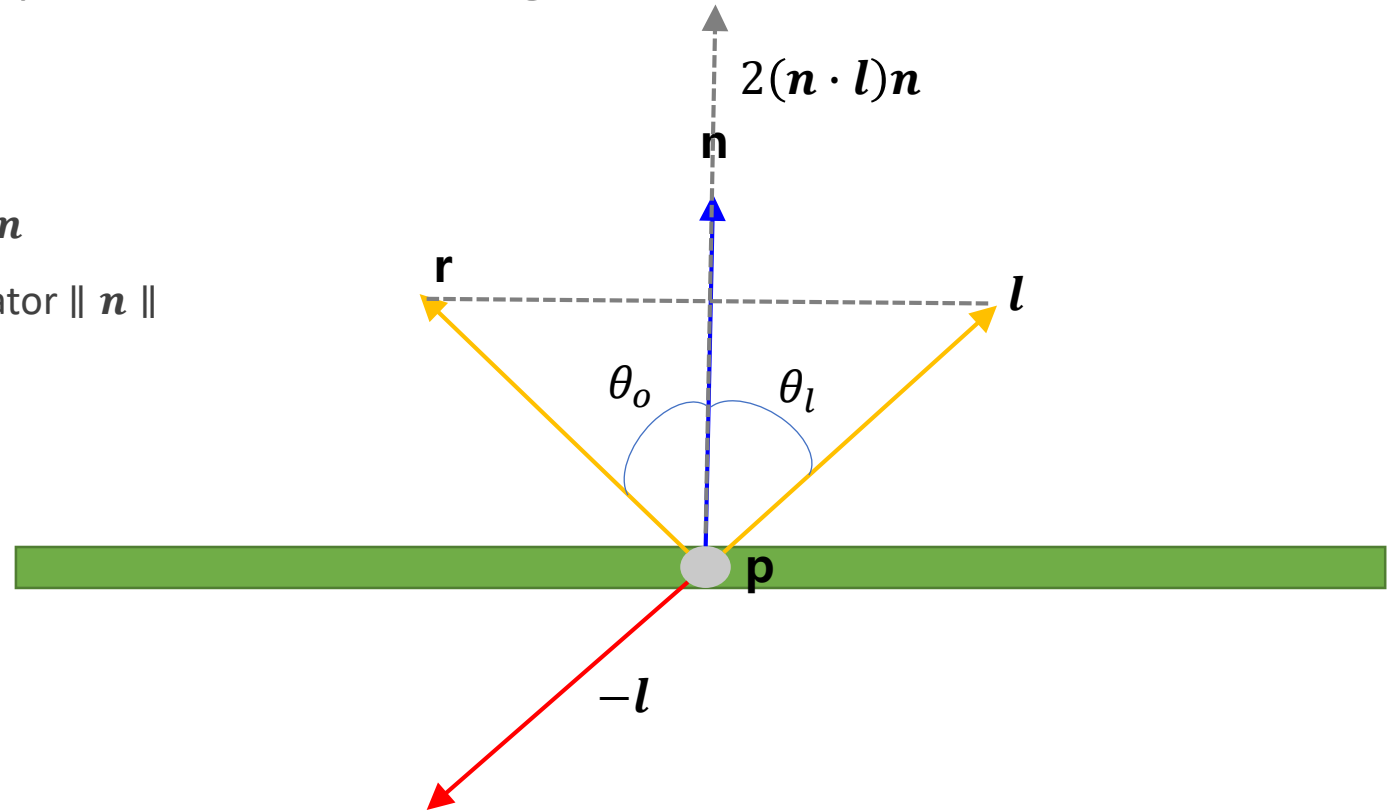
- The reflection vector,  $r$ , can be computed as the following:

- $r = 2(n \cdot l)n - l$

- $n \cdot l$ : length of the projected vector onto  $n$

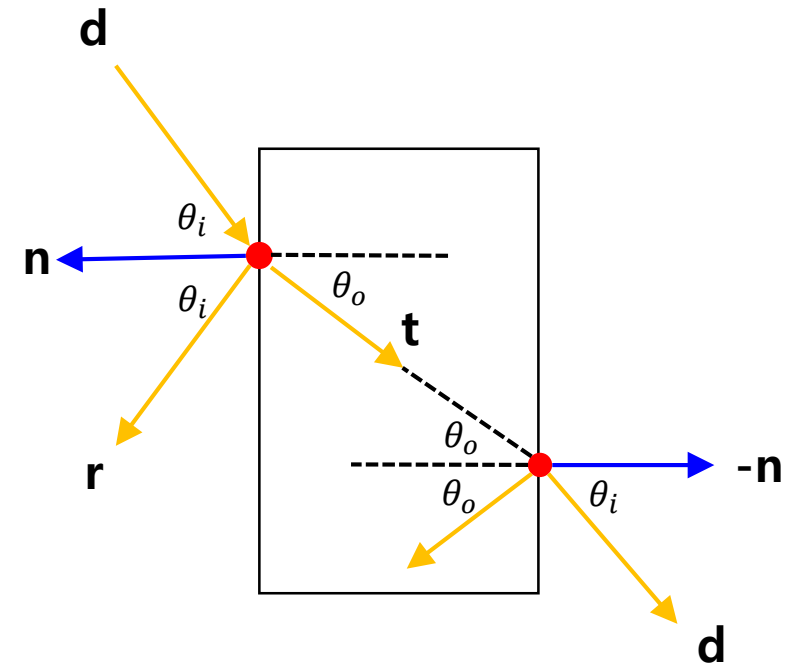
- Note: we don't need to use the denominator  $\|n\|$

- e.g., mirror



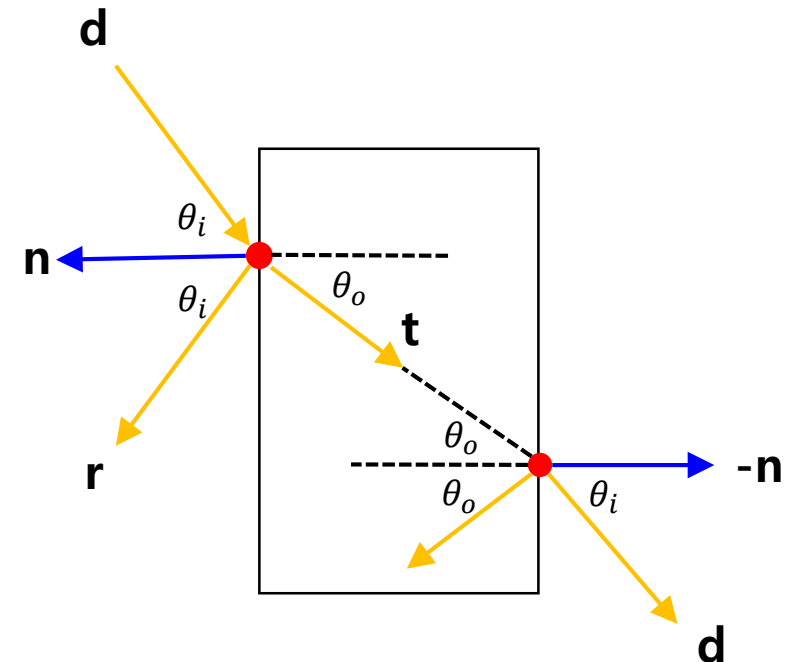
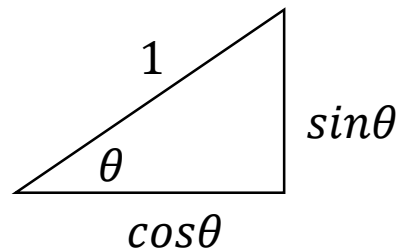
# Specular Refraction

- Dielectric materials (e.g., diamonds, glass, water, and air)
  - Transparent objects that refract light
- Snell's law
  - $n_i \sin \theta_i = n_o \sin \theta_o$ 
    - $n_i, n_o$ : indices of refraction
  - $n_i^2 (1 - \cos^2 \theta_i) = n_o^2 (1 - \cos^2 \theta_o)$ 
    - $\therefore$  trigonometric identity ( $\sin^2 \theta + \cos^2 \theta = 1$ )
  - $\cos^2 \theta_o = 1 - \frac{n_i^2 (1 - \cos^2 \theta_i)}{n_o^2}$



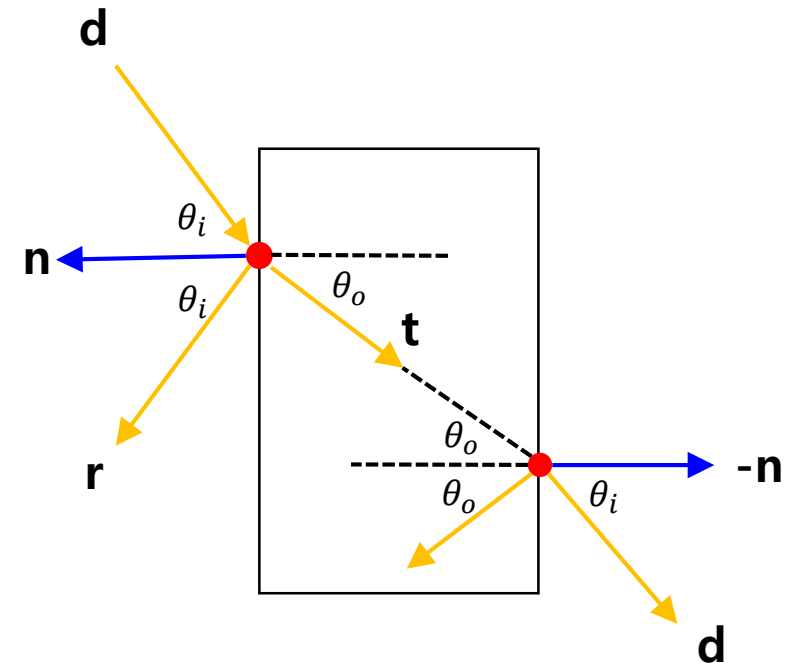
# Specular Refraction

- Vectors  $\mathbf{n}$  and  $\mathbf{b}$  forms an orthonormal basis
- $\mathbf{t} = \sin\theta_o \mathbf{b} - \cos\theta_o \mathbf{n}$ 
  - Unknowns:  $\mathbf{t}$ ,  $\mathbf{b}$
- $\mathbf{d} = \sin\theta_i \mathbf{b} - \cos\theta_i \mathbf{n}$ ,
- $\mathbf{b} = \frac{\mathbf{d} + \mathbf{n} \cos\theta_i}{\sin\theta_i}$
- Trigonometric functions



# Specular Refraction

- $\mathbf{t} = \sin\theta_o \mathbf{b} - \cos\theta_o \mathbf{n}$ 
  - Unknowns:  $\mathbf{t}, \mathbf{b}$
- $\mathbf{b} = \frac{\mathbf{d} + \mathbf{n} \cos\theta_i}{\sin\theta_i}$
- By plug-in  $\mathbf{b}$  into the equation for  $\mathbf{t}$ :
- $\mathbf{t} = \sin\theta_o \frac{\mathbf{d} + \mathbf{n} \cos\theta_i}{\sin\theta_i} - \cos\theta_o \mathbf{n}$
- $= \frac{n_i(\mathbf{d} + \mathbf{n} \cos\theta_i)}{n_o} - \cos\theta_o \mathbf{n}$  (by Snell's law)
- $= \frac{n_i(\mathbf{d} - \mathbf{n}(\mathbf{d} \cdot \mathbf{n}))}{n_o} - \mathbf{n} \sqrt{1 - \frac{n_i^2(1 - (\mathbf{d} \cdot \mathbf{n})^2)}{n_o^2}}$ 
  - $\cos^2 \theta_o = 1 - \frac{n_i^2(1 - \cos^2 \theta_i)}{n_o^2}$



# Specular Refraction

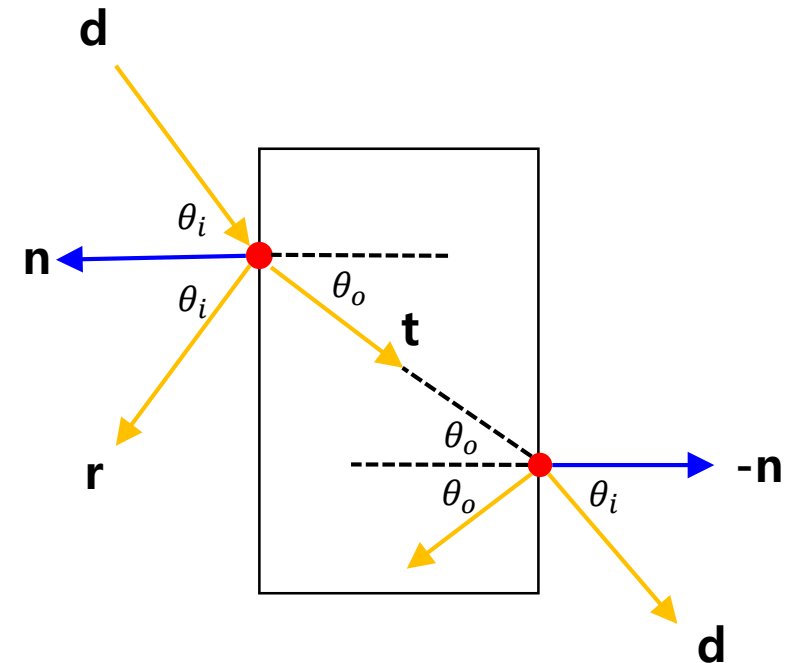
- The transmitting ray's direction  $t$

- $$t = \frac{n_i(d-n(d \cdot n))}{n_o} - n \sqrt{1 - \frac{n_i^2(1-(d \cdot n)^2)}{n_o^2}}$$

- $$1 - \frac{n_i^2(1-(d \cdot n)^2)}{n_o^2} < 0$$

- No refracted ray (i.e., all energy is reflected)

- Q. what are the amount of reflected and refracted energy (i.e., reflectivity)?



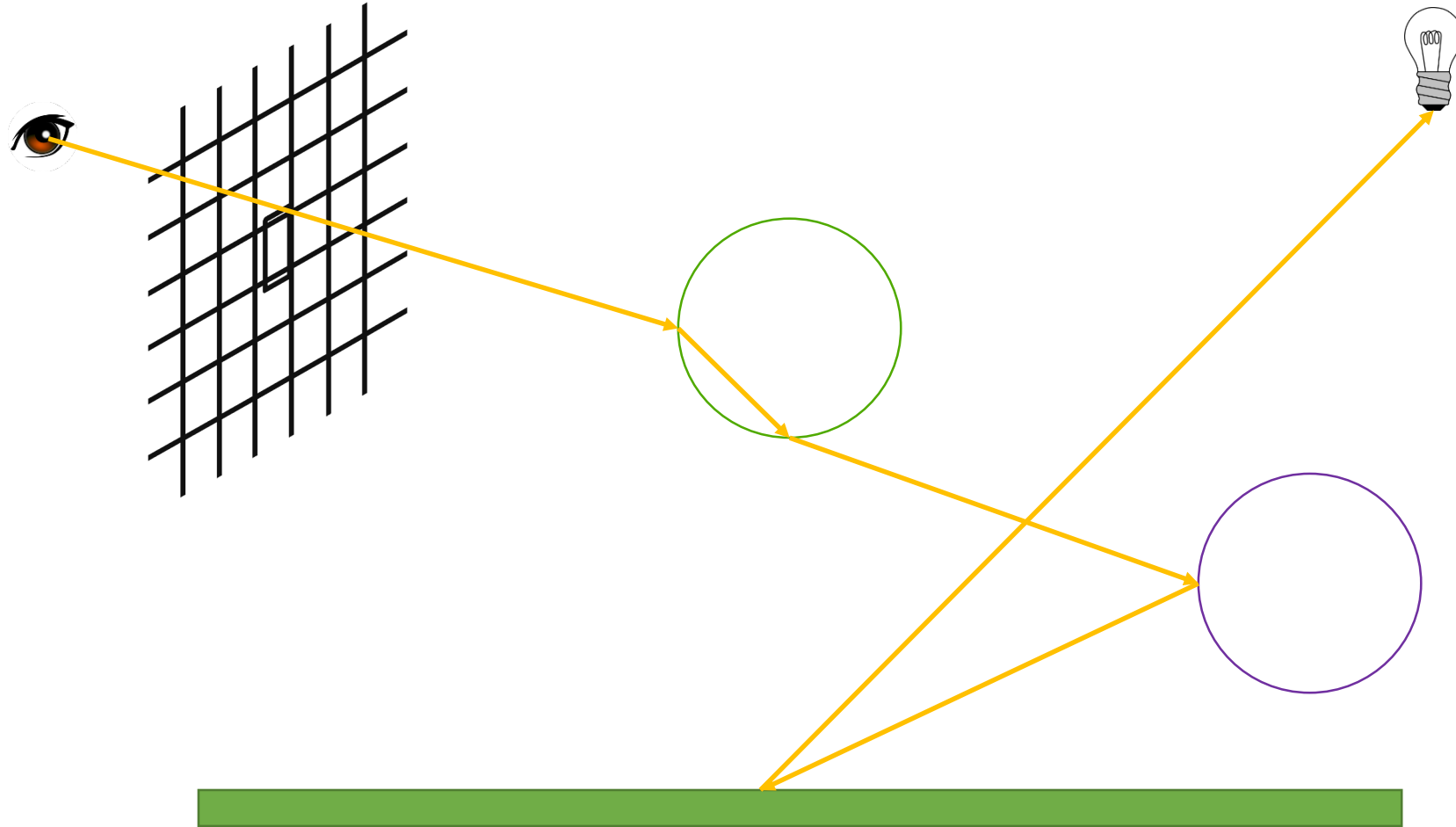
# Specular Refraction

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- The reflectivity varies with the incident direction of light and it is determined by *Fresnel equations* (from physics).
- Schlick approximation (1994) of the Fresnel equations
  - $R(\theta_i) = R_0 + (1 - R_0)(1 - \cos\theta_i)^5$
  - $R_0 = \left(\frac{n_o - 1}{n_o + 1}\right)^2$

# Whitted Ray Tracing

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# Whitted Ray Tracing

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- For each pixel
  - Color  $c = (0, 0, 0)$
  - Generate a primary ray (camera ray)
  - Find the closest intersection point between the ray and objects
  - If (there is a hit) then
    - Generate a shadow ray
    - If (there is no hit between the shadow ray and a light) then
      - $c = c + \text{shading}()$
    - Generate a secondary ray (reflection or refraction ray)
    - Go to the third line
  - Else
    - $c = c + \text{background color}$
  - Set the pixel color with  $c$
  
- Issue: cannot trace a ray path that has infinite depth

# Whitted Ray Tracing

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- For each pixel
  - Color  $c = (0, 0, 0)$
  - Generate a primary ray (with depth 0)
  - While (depth < d)
    - Find the closest intersection point between the ray and objects
    - If (there is a hit) then
      - Generate a shadow ray
      - If (there is no hit between the shadow ray and a light) then
        - $c = c + \text{shading}()$
      - Generate a secondary ray (reflection or refraction ray) // increase the ray depth +1
    - Else
      - $c = c + \text{background color}$
  - Set the pixel color with c

# Whitted Ray Tracing

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- Missing effects?
  - Soft shadow
  - Glossy reflection
  - Diffuse reflection
  - Depth-of-field effect
  - Motion blur
  - ...
- Advanced ray tracing
  - Distributed ray tracing
  - Path tracing and photon mapping



Whitted (1980)

# Further Readings

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- Chapter 4, 13
- Papers
  - T. Whitted. An improved illumination model for shading display. *Communications of the ACM*, 23(6):343–349, 1980