

Lecture slides (CT4201/EC4215 – Computer Graphics)

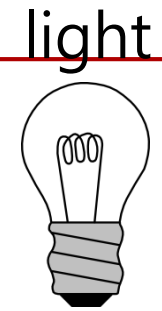
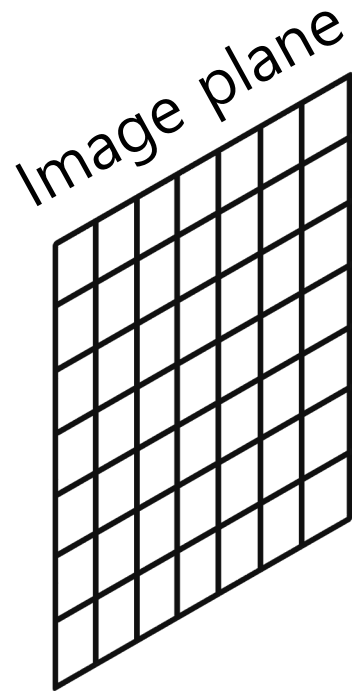
Ray Tracing

Lecturer: Bochang Moon

Ray Tracing

- A rendering technique:
 - Produce a 2D image from a scene (models)
- Image-order rendering:
 - Loop over pixels to decide pixel colors
- Object-order rendering:
 - Iterate objects and compute some pixel colors related to each object

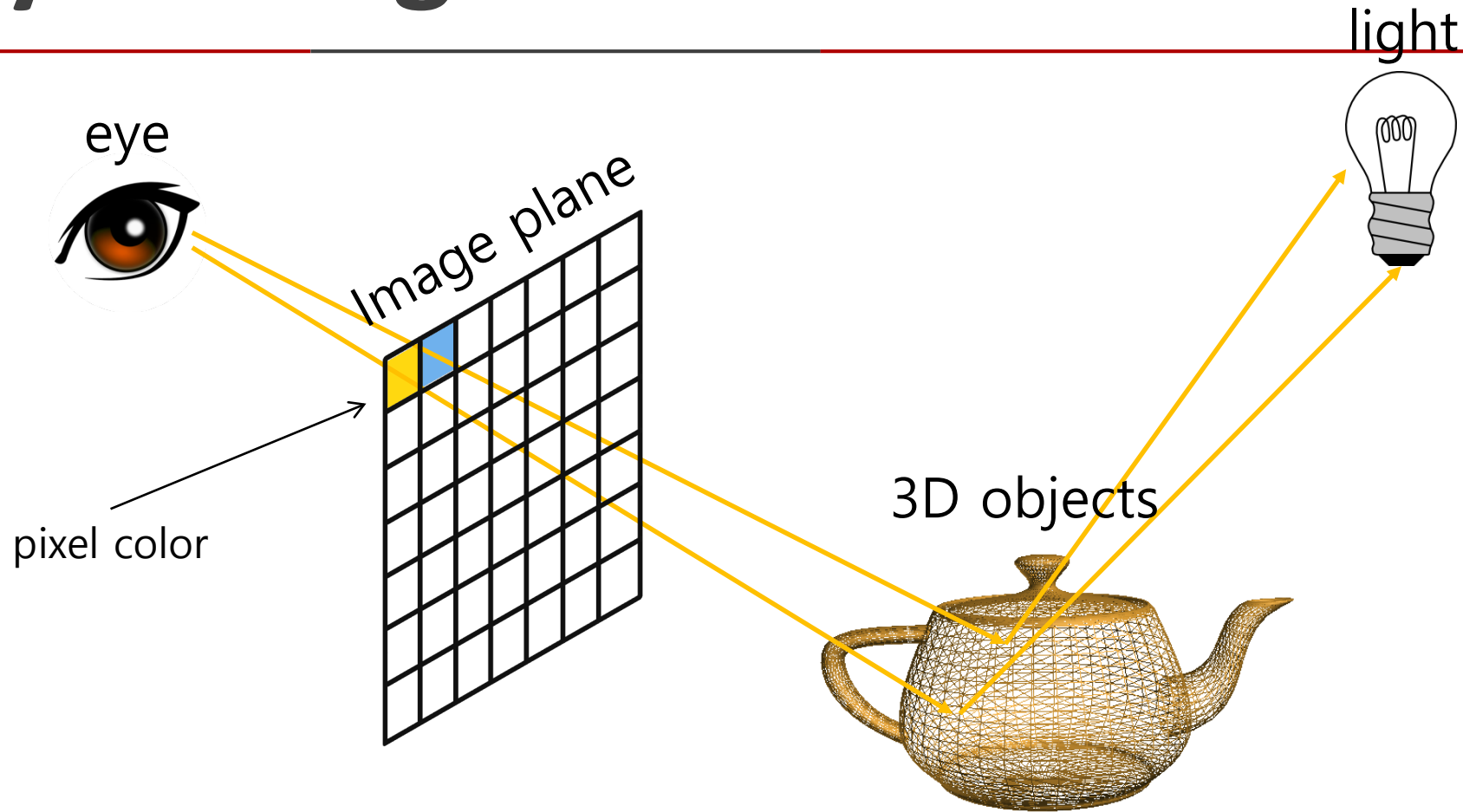
Rendering



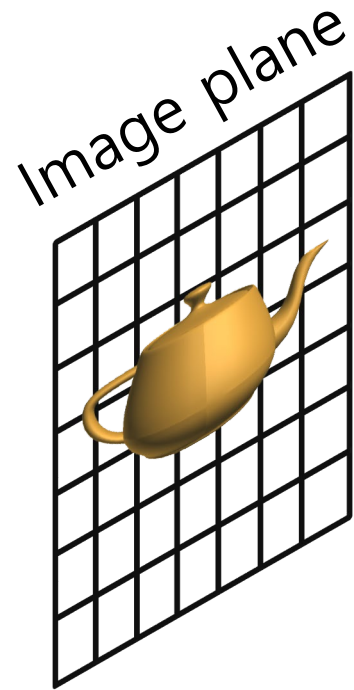
3D objects



Ray Tracing



Ray Tracing

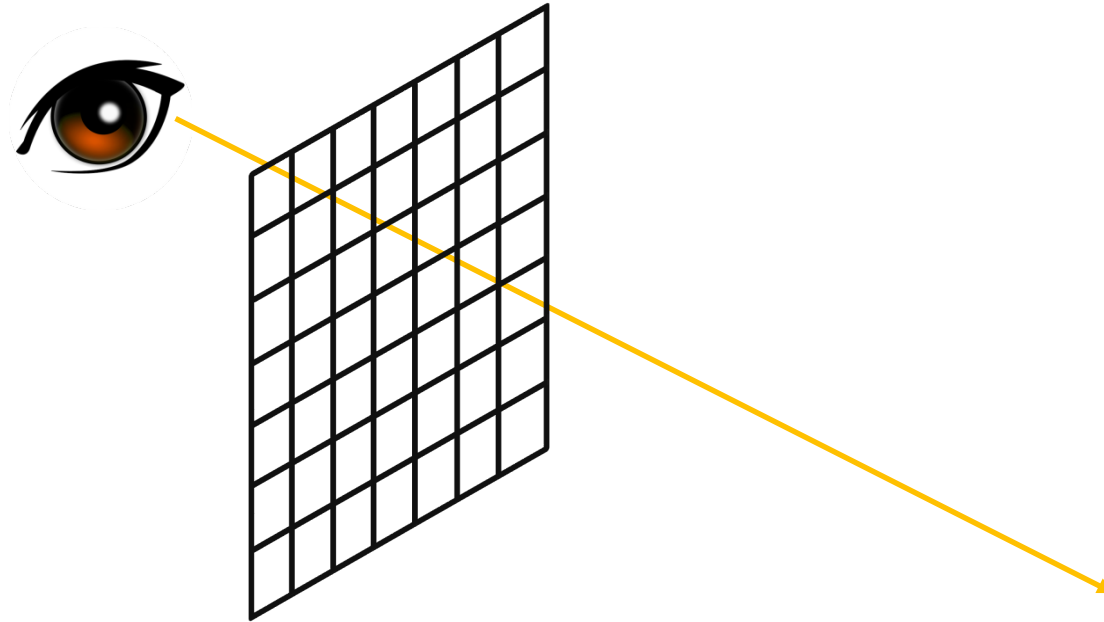


3D objects



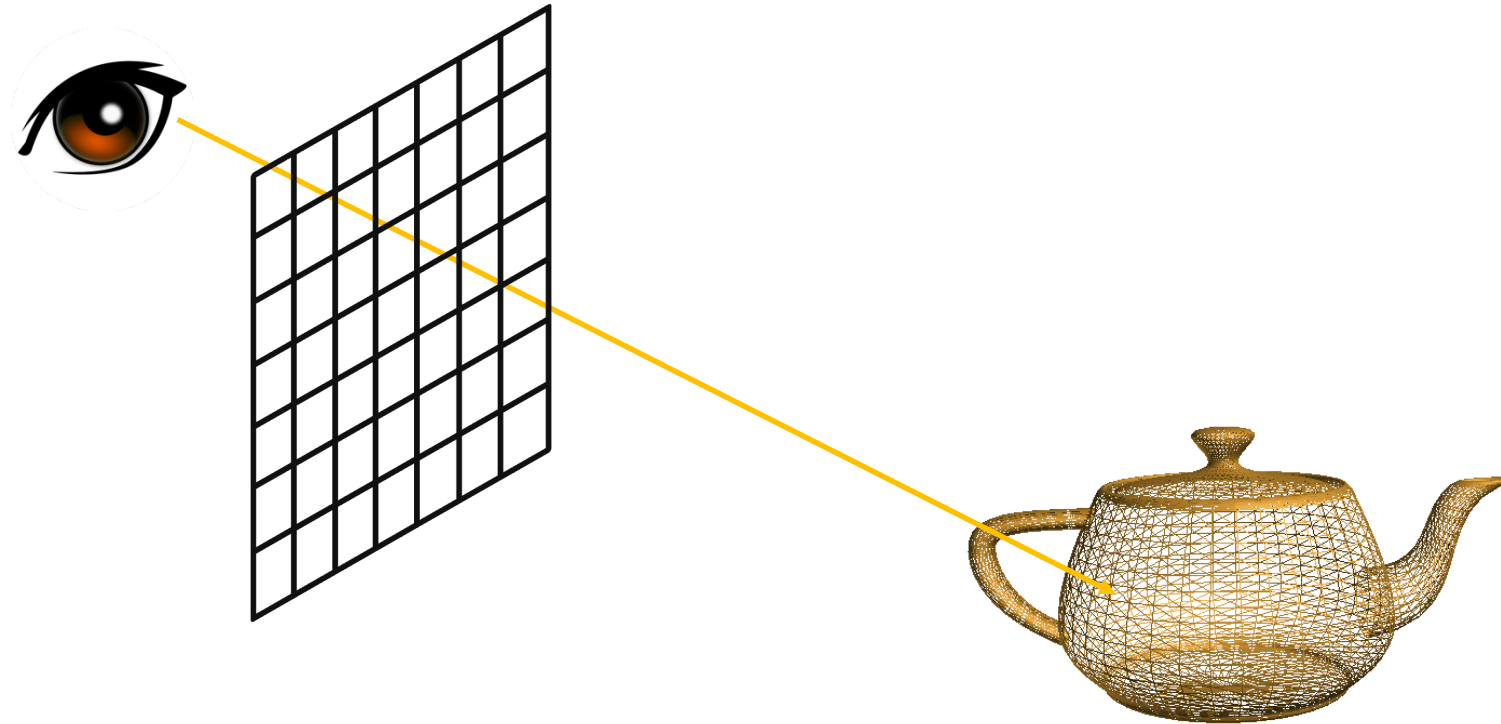
Basic Ray Tracer

- Ray generation
 - Compute the origin and direction of a ray per pixel, by considering the camera and image plane



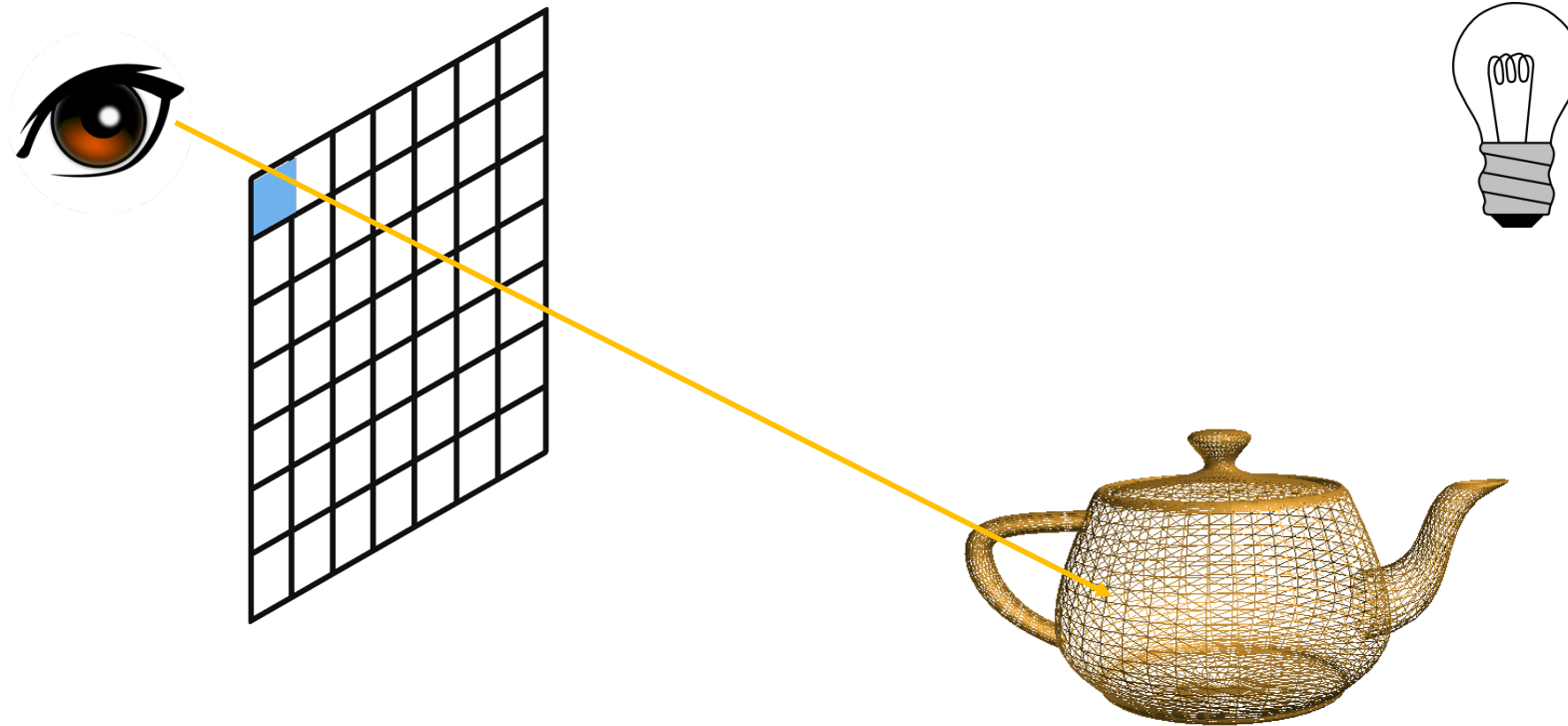
Basic Ray Tracer

- Ray intersection
 - Find the closest intersection point between the ray and objects



Basic Ray Tracer

- Shading
 - Compute the pixel color using the geometry, material, and lights at the intersection point



Basic Ray Tracer

- For each pixel do
 - Compute a primary ray (viewing ray)
 - Find the closest intersection point between the ray and a scene
 - Determine a pixel color

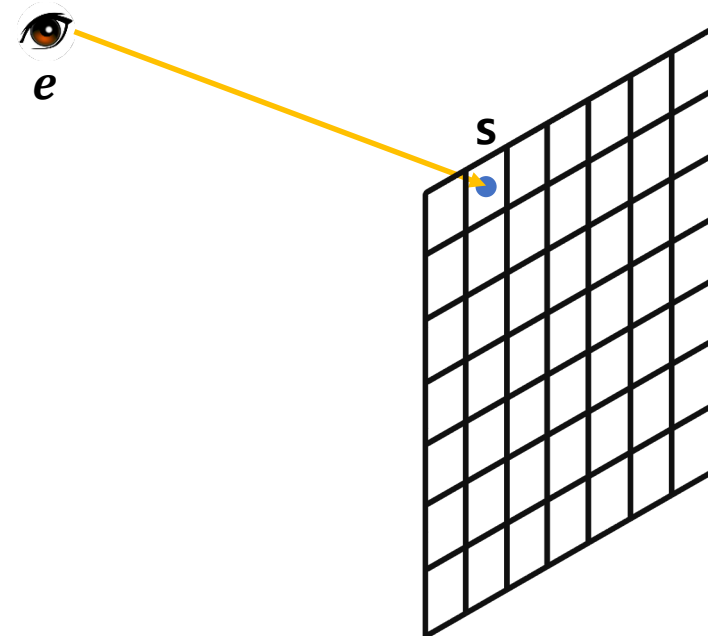
Primary Ray Generation

- Mathematical representation for a ray
 - 3D parametric line: $\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$

- Properties

- $\mathbf{p}(0) = \mathbf{e}$, $\mathbf{p}(1) = \mathbf{s}$
- $\mathbf{p}(t_1)$ is closer to the eye than $\mathbf{p}(t_2)$ when $0 < t_1 < t_2$
- When $t < 0$, $\mathbf{p}(t)$ is behind the eye
- \mathbf{e} is a given value

- Q. How can we compute \mathbf{s} ?

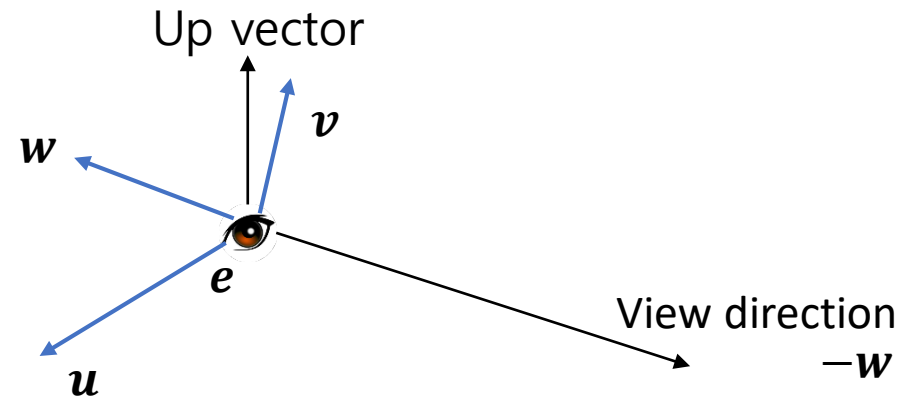


Primary Ray Generation

- Mathematical representation for a ray
 - 3D parametric line: $p(t) = e + t(s - e)$

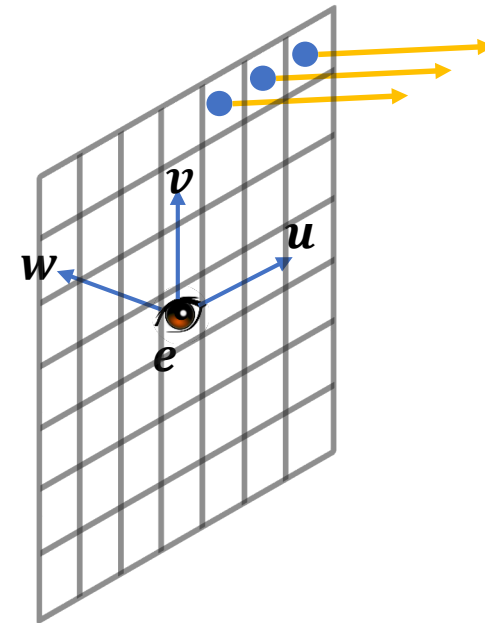
- u, v, w forms a right-handed coordinate system

- Two kinds of views
 - Orthographic view
 - Perspective view



Orthographic Views

- All primary rays have the same direction, $-w$
- The primary ray starts on the image plane defined by e, u, v
- The image plane is defined with four numbers:
 - l, r : positions of left and right edges of the image plane
 - b, t : positions of bottom and top edges
- To make an image with $n_x \times n_y$
 - Pixels are spaced as the following:
 - $\frac{r-l}{n_x}$ horizontally, $\frac{t-b}{n_y}$ vertically
- Position (α, β) in the image plane is corresponding to a pixel (i, j) in the raster image:
 - $\alpha = l + \frac{(r-l)(i+0.5)}{n_x}$
 - $\beta = b + \frac{(t-b)(j+0.5)}{n_y}$
 - (α, β) are the coordinates of the pixel's position on the image plane



Orthographic Views

- Procedure to generate orthographic viewing rays

- Compute α and β

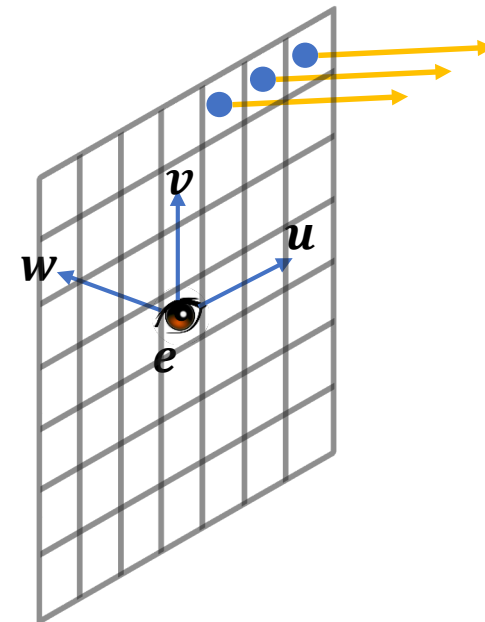
- $\alpha = l + \frac{(r-l)(i+0.5)}{n_x}$
- $\beta = b + \frac{(t-b)(j+0.5)}{n_y}$

- $ray.direction := -w$

- $ray.origin := e + \alpha u + \beta v$

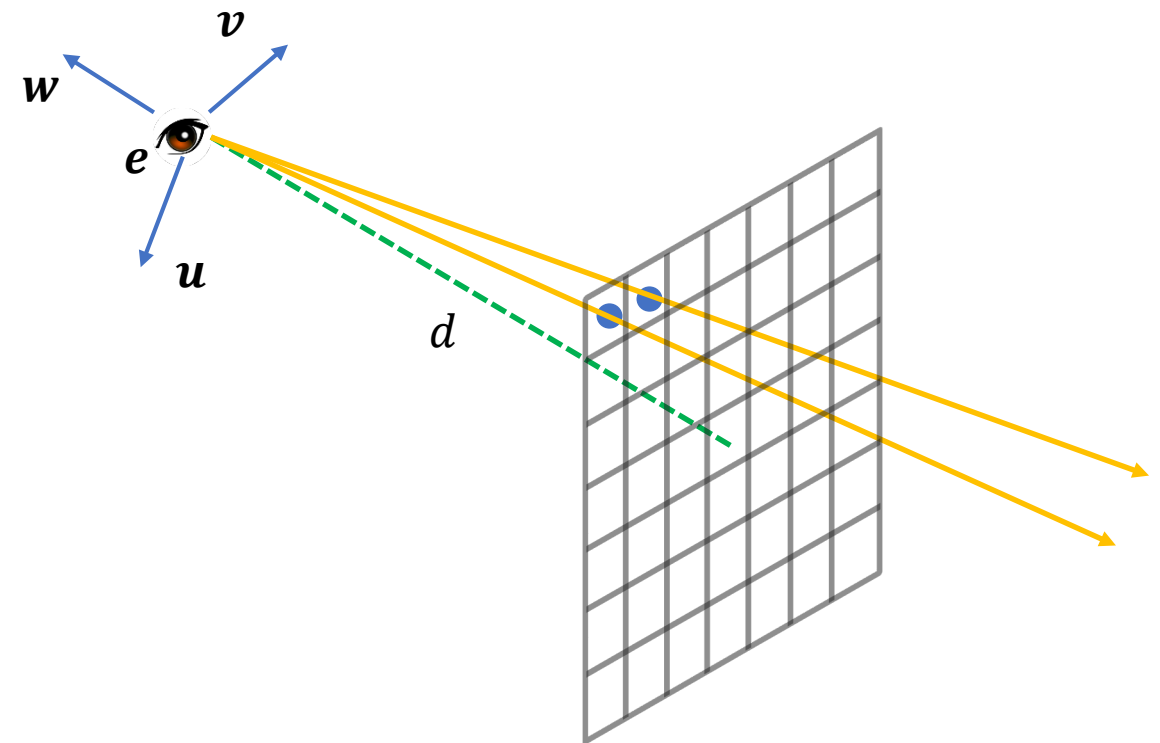
- Properties

- Same direction for all rays
- Different origins for rays



Perspective Views

- All rays have the same origin, e , but have different directions
- The image plane is placed with a distance, d , in front of e
 - d : image plane distance (called the focal length)
- Procedure to generate perspective viewing rays
 - Compute α and β
 - $\alpha = l + \frac{(r-l)(i+0.5)}{n_x}$
 - $\beta = b + \frac{(t-b)(j+0.5)}{n_y}$
- $ray.direction := -dw + \alpha u + \beta v$
- $ray.origin := e$



Intersection between Ray and Object

- Generated ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$
- The next task is to find the closest intersection point between a ray and objects
 - i.e., need to find a t in the interval $[t_0, t_1]$ (e.g., $[0, +\infty)$)
- Objects
 - Sphere
 - Triangle
 - Multiple objects

Intersection between Ray and Sphere

- Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$
- Implicit surface: $f(\mathbf{p}) = 0$
- Intersection points should satisfy both equations
 - $f(\mathbf{p}(t)) = f(\mathbf{e} + t\mathbf{d}) = 0$
- Let's define a sphere with center $\mathbf{c} = (x_c, y_c, z_c)$ and radius r
 - $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - r^2 = 0$
 - $(\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - r^2 = 0$ (vector form)
 - A point \mathbf{p} that satisfies this equation is on the sphere
- By plug-in the parametric ray equation,
 - $(\mathbf{e} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{e} + t\mathbf{d} - \mathbf{c}) - r^2 = 0$
 - By rearranging terms with respect to t (unknown value):
 - $(\mathbf{d} \cdot \mathbf{d})t^2 + 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$

Intersection between Ray and Sphere

- A quadratic equation in t

- $(\mathbf{d} \cdot \mathbf{d})t^2 + 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$

- The solutions for $at^2 + bt + c = 0$

- $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

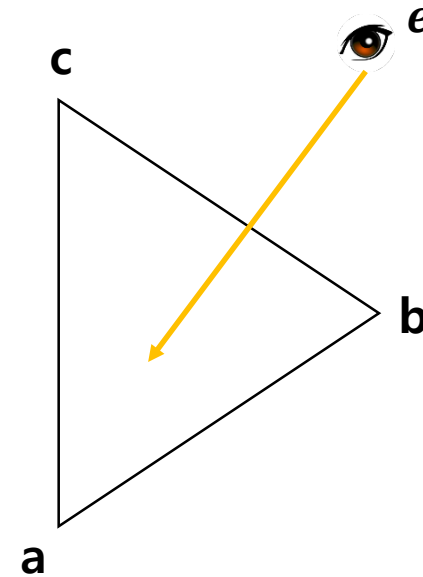
- $b^2 - 4ac$ (called discriminant)

- When $b^2 - 4ac < 0$, there is no solution (the ray does not intersect with the sphere)
 - When $b^2 - 4ac = 0$, a solution exists (the ray touches the sphere)
 - When $b^2 - 4ac > 0$, two solutions exist (the ray enters and leaves the sphere)

- $t = \frac{-\mathbf{d} \cdot (\mathbf{e} - \mathbf{c}) \pm \sqrt{(\mathbf{d} \cdot (\mathbf{e} - \mathbf{c}))^2 - (\mathbf{d} \cdot \mathbf{d})((\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2)}}{(\mathbf{d} \cdot \mathbf{d})}$

Intersection between Ray and Triangle

- Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$
- Intersection point:
 - $\mathbf{e} + t\mathbf{d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$
- Solving the equation for t, β, γ :
 - $x_e + tx_d = x_a + \beta(x_b - x_a) + \gamma(x_c - x_a)$
 - $y_e + ty_d = y_a + \beta(y_b - y_a) + \gamma(y_c - y_a)$
 - $z_e + tz_d = z_a + \beta(z_b - z_a) + \gamma(z_c - z_a)$
 - Can be rewritten:
 - $$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$



Intersection between Ray and Triangle

- Cramer's rule can be utilized to solve the 3 x 3 linear system

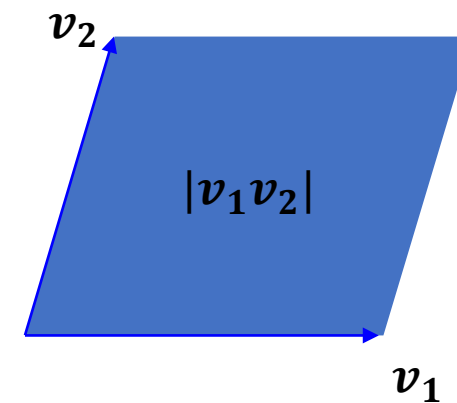
$$\circ \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\circ x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{|A|}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{|A|}, \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{|A|}$$

$$\circ \text{ where } |A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \text{ } | \cdot | \text{ is the determinant}$$

$$\circ |A| = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$\circ \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} = b_2 c_3 - c_2 b_3$$



Intersection between Ray and Triangle

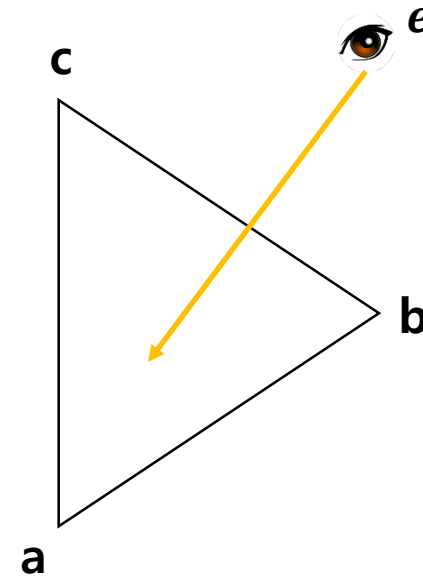
$$\circ \begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

$$\circ \beta = \frac{\begin{vmatrix} x_a - x_e & x_a - x_c & x_d \\ y_a - y_e & y_a - y_c & y_d \\ z_a - z_e & z_a - z_c & z_d \end{vmatrix}}{|A|}$$

$$\circ \gamma = \frac{\begin{vmatrix} x_a - x_b & x_a - x_e & x_d \\ y_a - y_b & y_a - y_e & y_d \\ z_a - z_b & z_a - z_e & z_d \end{vmatrix}}{|A|}$$

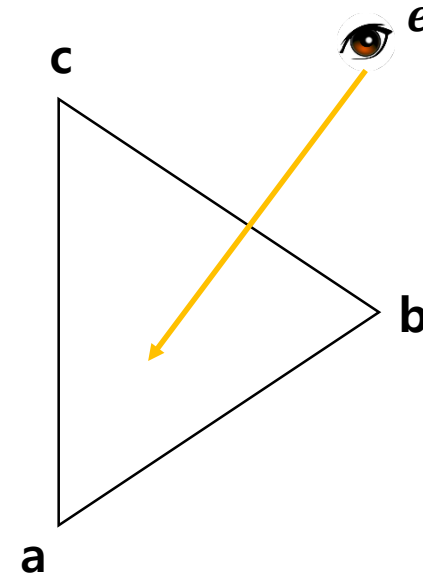
$$\circ t = \frac{\begin{vmatrix} x_a - x_b & x_a - x_c & x_a - x_e \\ y_a - y_b & y_a - y_c & y_a - y_e \\ z_a - z_b & z_a - z_c & z_a - z_e \end{vmatrix}}{|A|}$$

$$\circ \text{ where } |A| = \begin{vmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{vmatrix}$$



Intersection between Ray and Triangle

- Procedure (with early termination) for finding the intersection:
 - Input: a ray, vertex a, b, c , interval $[t_0, t_1]$
 - Compute t
 - If $(t < t_0)$ or $(t > t_1)$ then
 - return false
 - Compute γ
 - If $(\gamma < 0)$ or $(\gamma > 1)$ then
 - return false
 - Compute β
 - If $(\beta < 0)$ or $(\beta > 1 - \gamma)$ then
 - return false
 - return true



Intersection between Ray and Objects

- Procedure for finding the *closest* intersection:
 - hit = false
 - For each object o do
 - If (o is intersected with the ray at a parameter t and $t \in [t_0, t_1]$) then
 - hit = true
 - store some information (e.g., o , normal, etc.) for shading
 - $t_1 = t$
 - return hit

Basic Ray Tracer

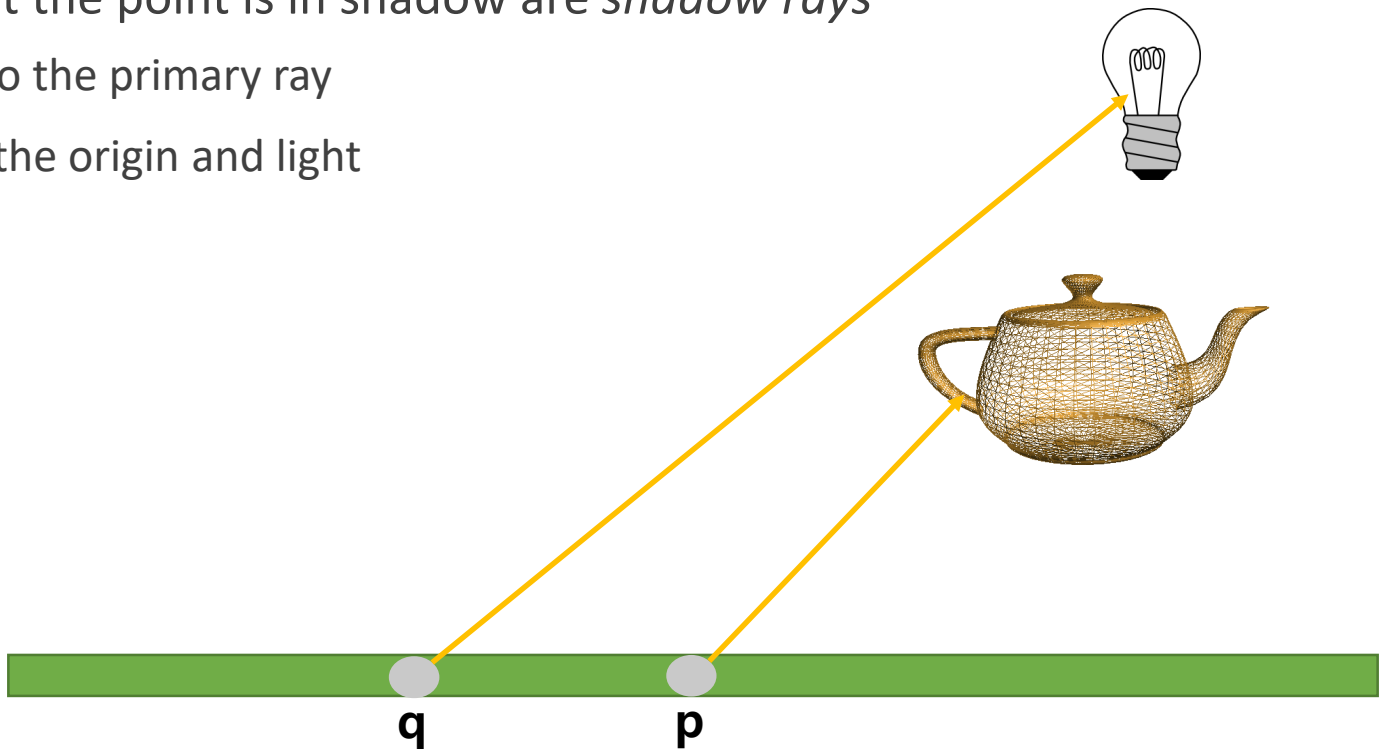
- For each pixel do
 - Compute a primary ray (viewing ray)
 - Find the closest intersection point between the ray and a scene
 - Determine a pixel color
 - e.g., we can apply the Phong illumination model here

Basic Ray Tracer

- For each pixel do
 - Compute a primary ray (viewing ray)
 - If (ray intersects an object with $t \in [0, \infty)$) then
 - Compute a hit record that contains some information (normal, materials, ...)
 - Evaluate an illumination model and set a pixel color
 - Else
 - Set a pixel color to background color

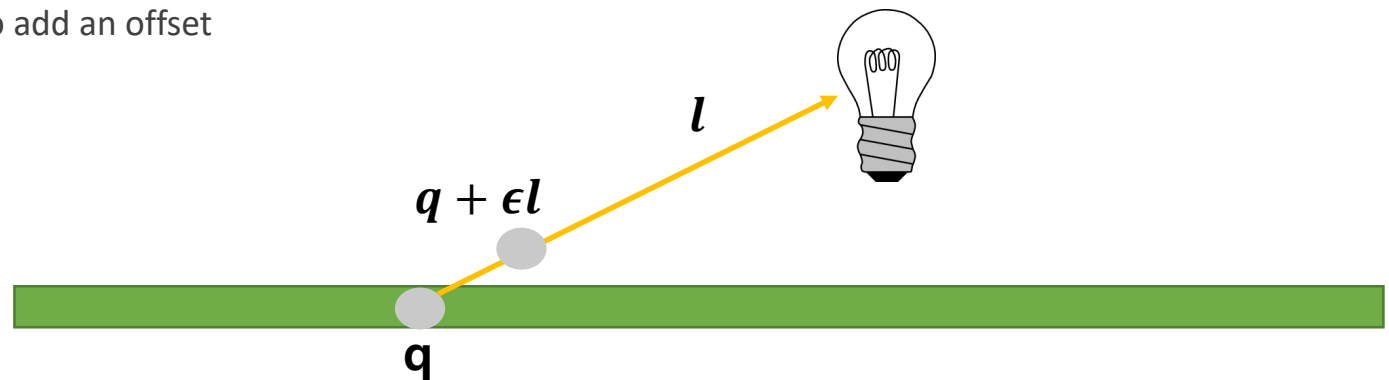
Shadows

- Assume there are two intersection points, p and q
 - p is in shadow, but q is not in shadow
- Rays to determine whether or not the point is in shadow are *shadow rays*
 - Generate a shadow ray similar to the primary ray
 - Check there is any hit between the origin and light
 - $t = [t_0, t_1]$
 - e.g., $t = [0, t_1]$



Shadows

- Assume there are two intersection points, p and q
 - p is in shadow, but q is not in shadow
- Rays to determine whether or not the point is in shadow are *shadow rays*
 - Generate a shadow ray similar to the primary ray
 - Check there is any hit between the origin and light
 - $t = [t_0, t_1]$
 - Due to numerical issues, the shadow ray can intersect the surface on which the point lies
 - A naïve but common approach is to add an offset
 - $t = [\epsilon, t_1]$



Shadows

- Pseudocode to implement shadows (based on the Phong illumination)
- Input: a ray $\mathbf{e} + t\mathbf{d}$, $[t_0 = 0, t_1 = \infty]$
- If (there is a hit between the ray and objects) then
 - $\mathbf{p} = \mathbf{e} + t\mathbf{d}$ // p is the closest intersection from e
 - **color** $\mathbf{c} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$
 - If (there is no hit between the shadow ray and a light) then
 - $\mathbf{c} = \mathbf{c} + k_a L_a + L_d k_d \max(0, \mathbf{n} \cdot \mathbf{l}) + L_s k_s \max(0, \mathbf{r} \cdot \mathbf{v})^s$
 - return c
- Else
 - return background color

Some History of Ray Tracing

- Rene Descartes (1637) used ray tracing to explain the phenomena of rainbow
- In rendering, the ray casting was presented by Arthur Appel (1968)
 - Ray casting (discussed so far) tends to be interchangeable to ray tracing
 - Ray tracing generates additional rays (e.g., secondary rays) to simulate global illumination effects
 - Ray tracing becomes popular due to the Whitted's paper (1980)
 - T. Whitted. An improved illumination model for shading display. Communications of the ACM, 23(6):343–349, 1980

Further Readings

- Chapter 4