Lecture slides (CT4201/EC4215 – Computer Graphics)

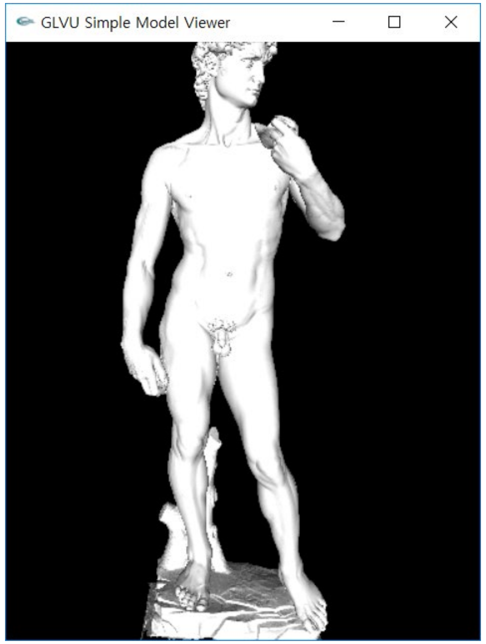
Transformation

Lecturer: Bochang Moon

Computer Graphics Laboratory

Transformation

- Fundamental operation to arrange objects in a 3D scene

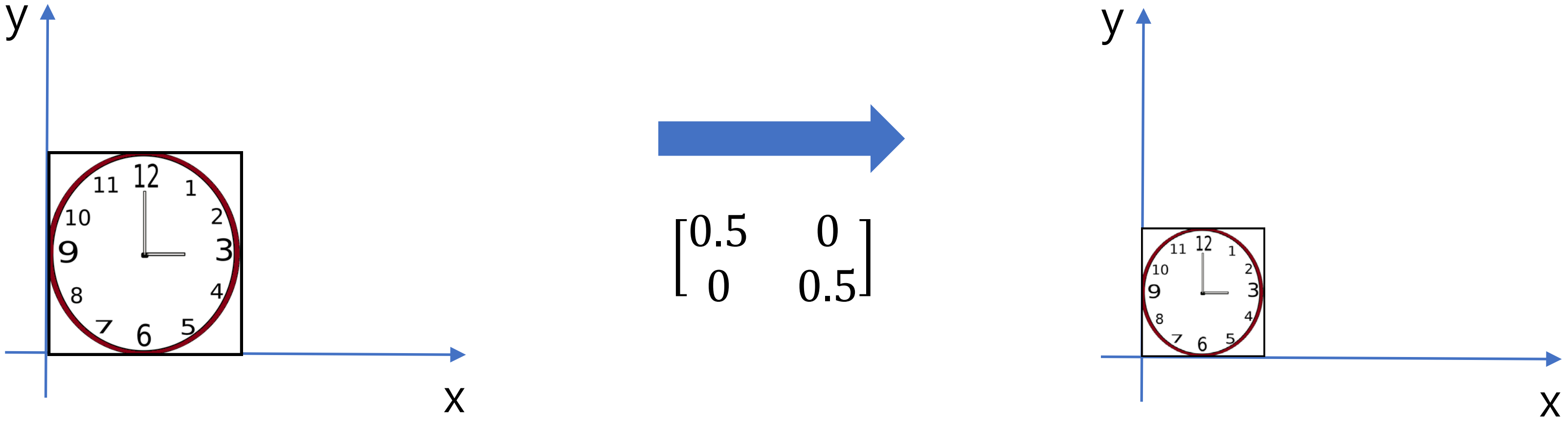


Object

3D scene

Transformation: Scaling

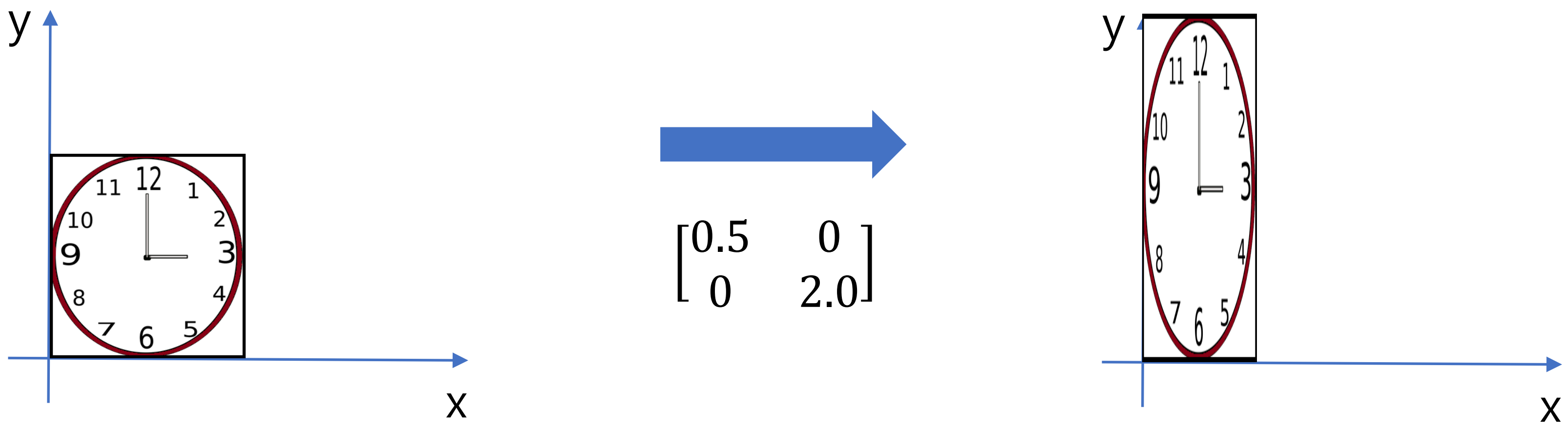
- Change length along the coordinate axes
  - $scale(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$
  - $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$



$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

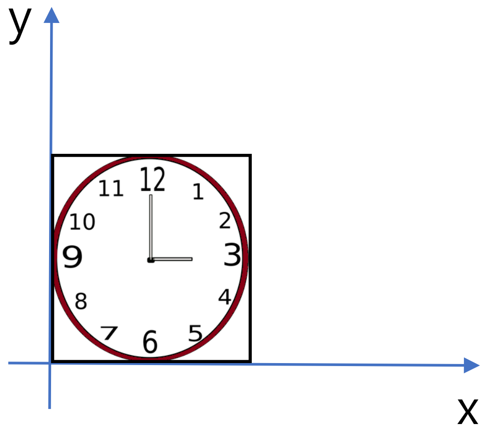
Transformation: Scaling

- Change length along the coordinate axes
  - $scale(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$
  - $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$

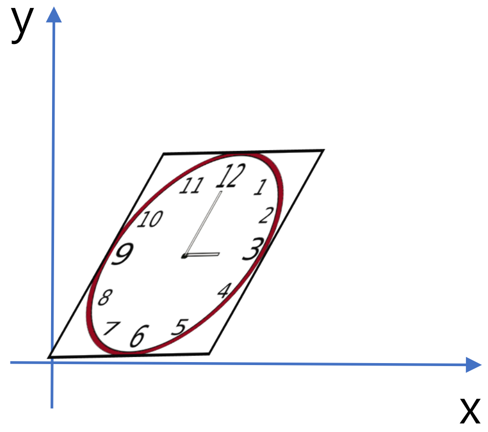


Transformation: Shearing

- Push objects sideways along horizontal or vertical direction
  - $shear_x(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$
  - $shear_y(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$

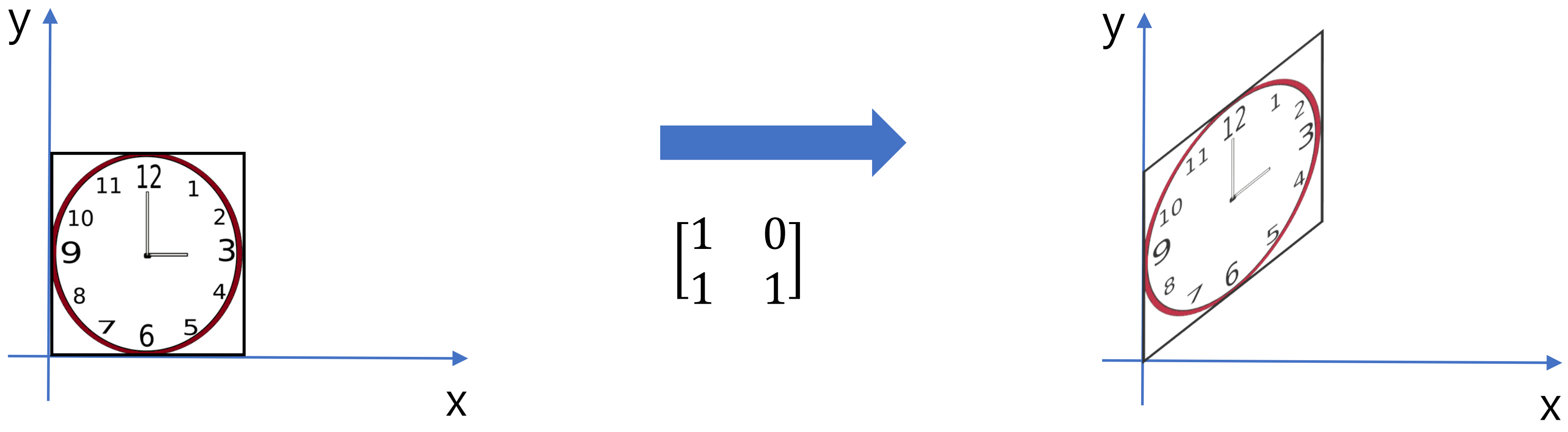


$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$



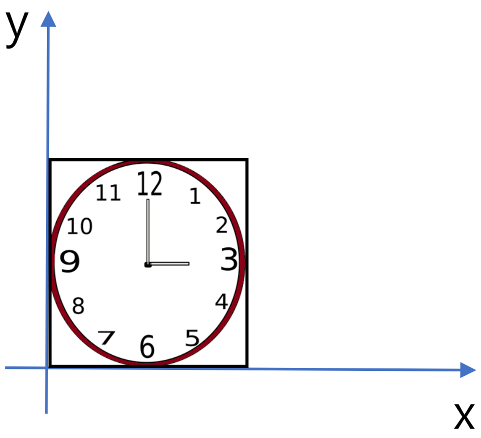
Transformation: Shearing

- Push objects sideways along horizontal or vertical direction
  - $shear_x(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$
  - $shear_y(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$



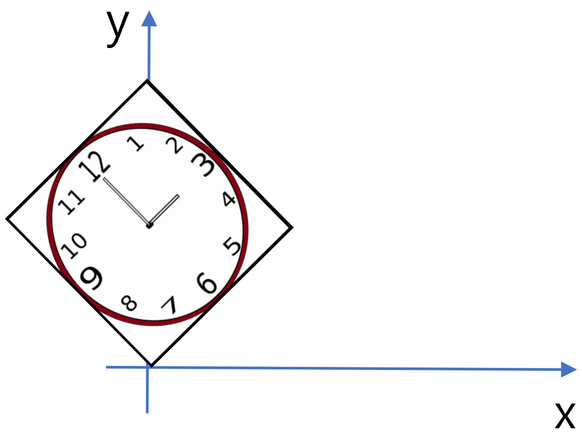
Transformation: Rotation

- Rotate objects counterclockwise
  - $rotate(\theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$



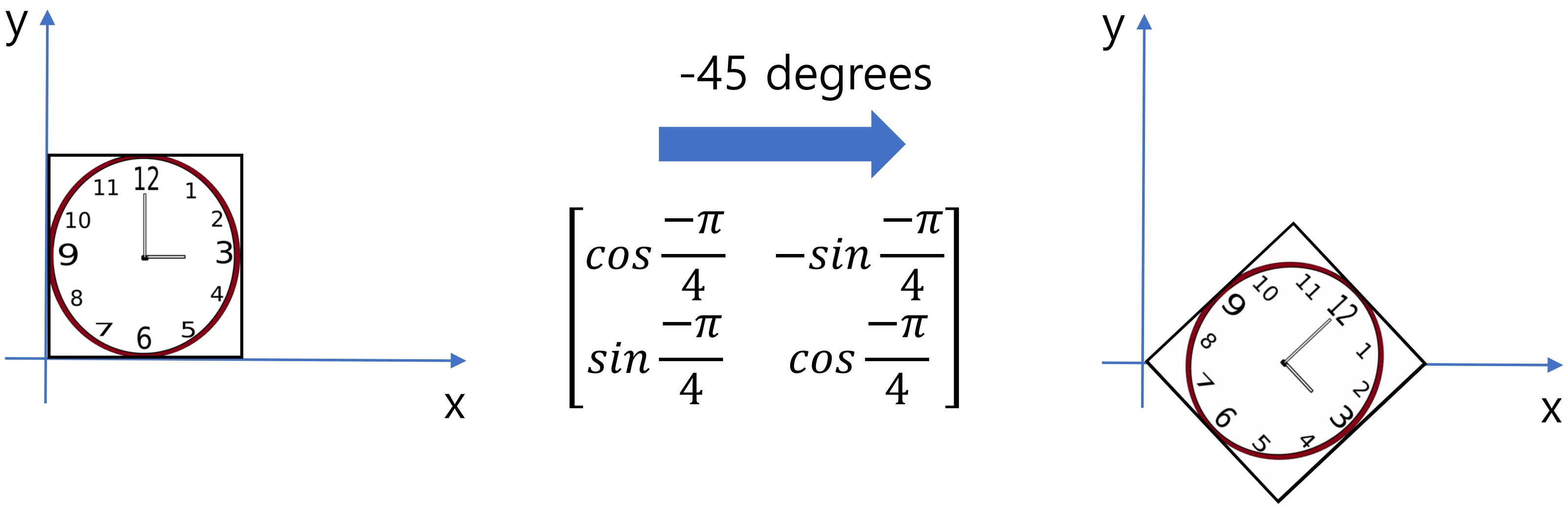
45 degrees

$$\begin{bmatrix} cos\frac{\pi}{4} & -sin\frac{\pi}{4} \\ sin\frac{\pi}{4} & cos\frac{\pi}{4} \end{bmatrix}$$



Transformation: Rotation

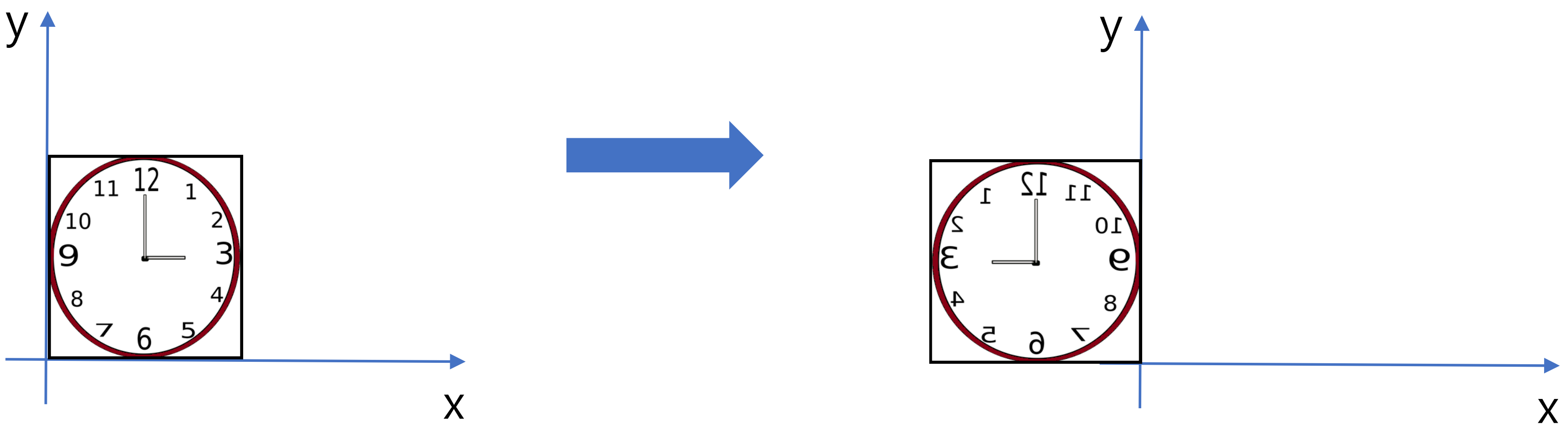
- Rotate objects counterclockwise
  - $rotate(\theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$



$$\begin{bmatrix} cos\frac{-\pi}{4} & -sin\frac{-\pi}{4} \\ sin\frac{-\pi}{4} & cos\frac{-\pi}{4} \end{bmatrix}$$

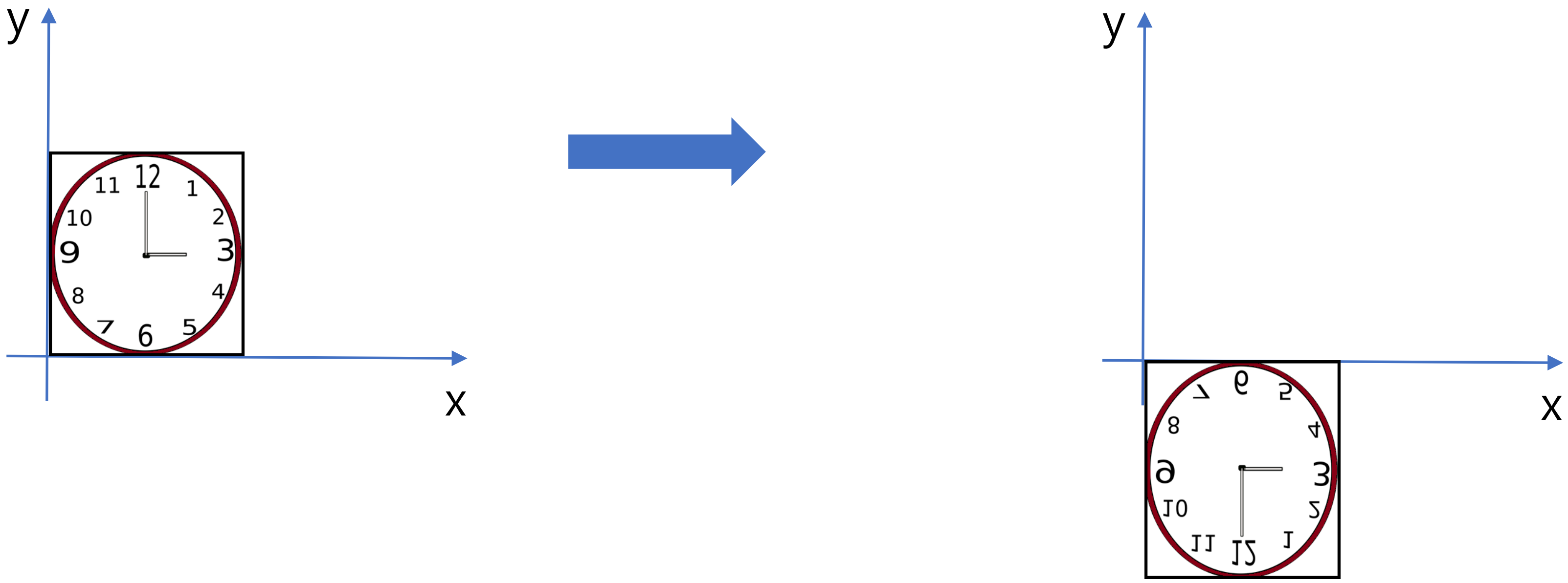
Transformation: Reflection

- Reflect objects across either of the coordinate axes
  - $reflect\_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$



Transformation: Reflection

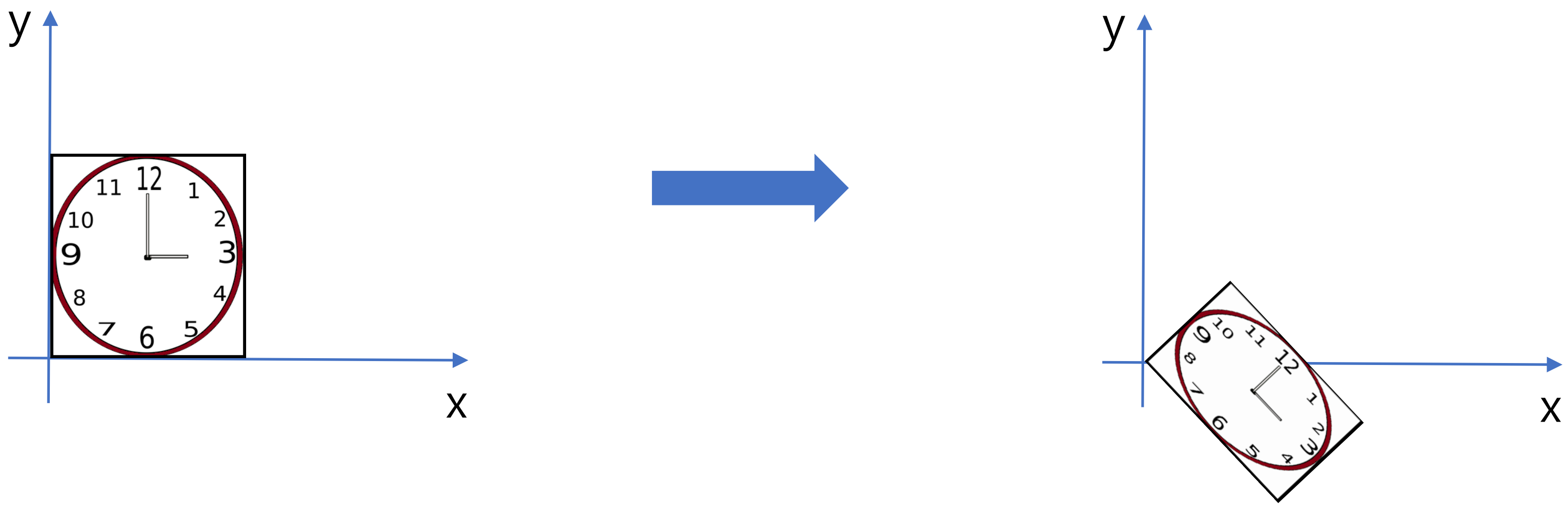
- Reflect objects across either of the coordinate axes
  - $reflect\_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$



Transformation: Composition

- Apply more than one transformation to an object

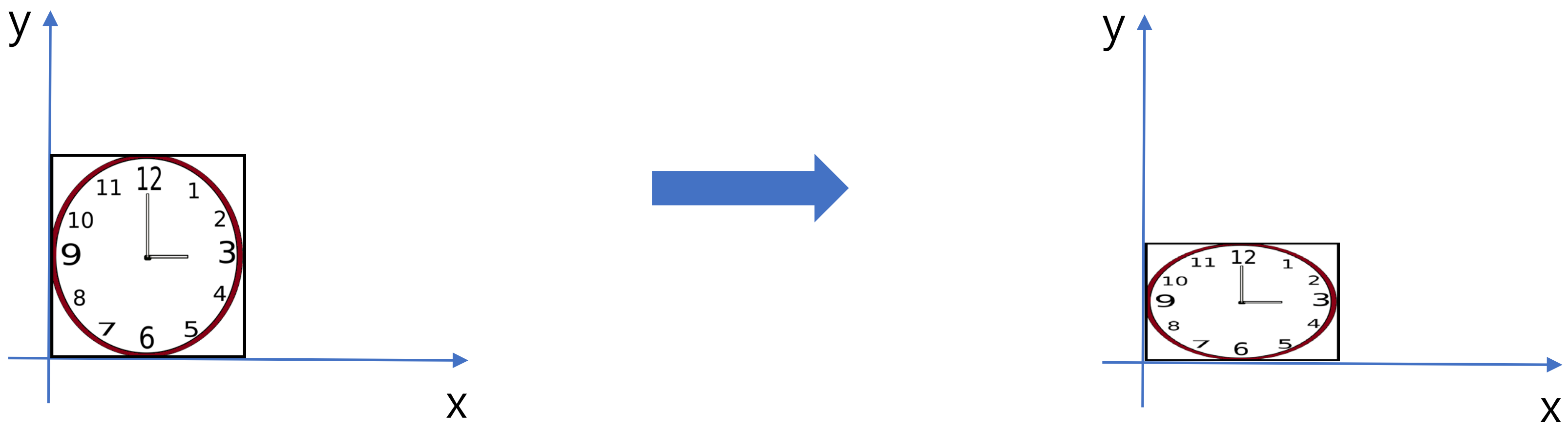
- e.g. apply a scale S, and then a rotation R
  - $\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = RS \begin{bmatrix} x_{old} \\ y_{old} \end{bmatrix}$



Transformation: Composition

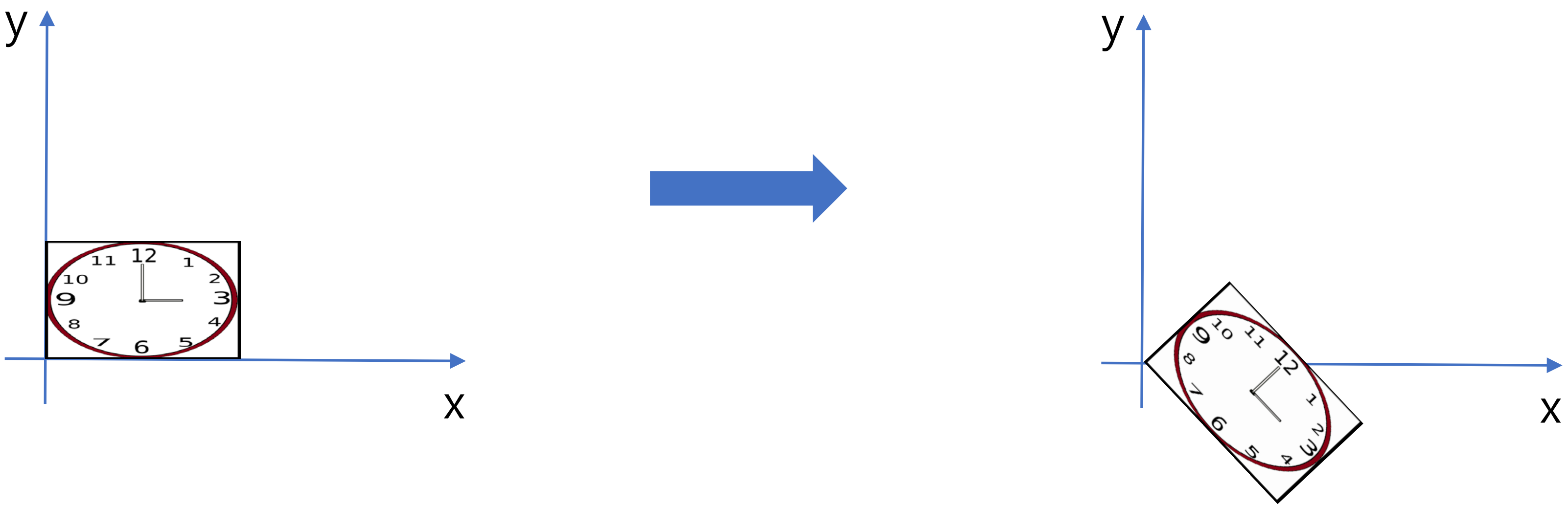
- Apply more than one transformation to an object

- e.g. apply a scale S, and then a rotation R
  - $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S \begin{bmatrix} x_{old} \\ y_{old} \end{bmatrix}$



Transformation: Composition

- Apply more than one transformation to an object
- e.g. apply a scale S, and then a rotation R
  - $\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = R \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

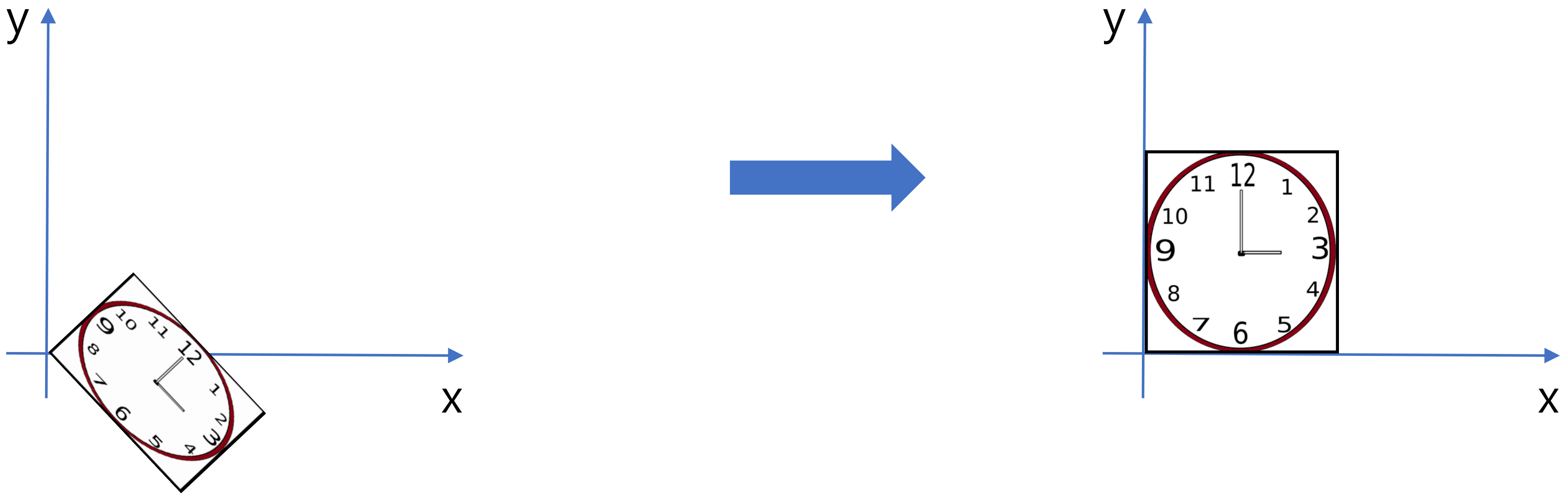


Inverse Transformation

- Your transformation
  - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
  - $x_{old} = T^{-1}x_{new}$

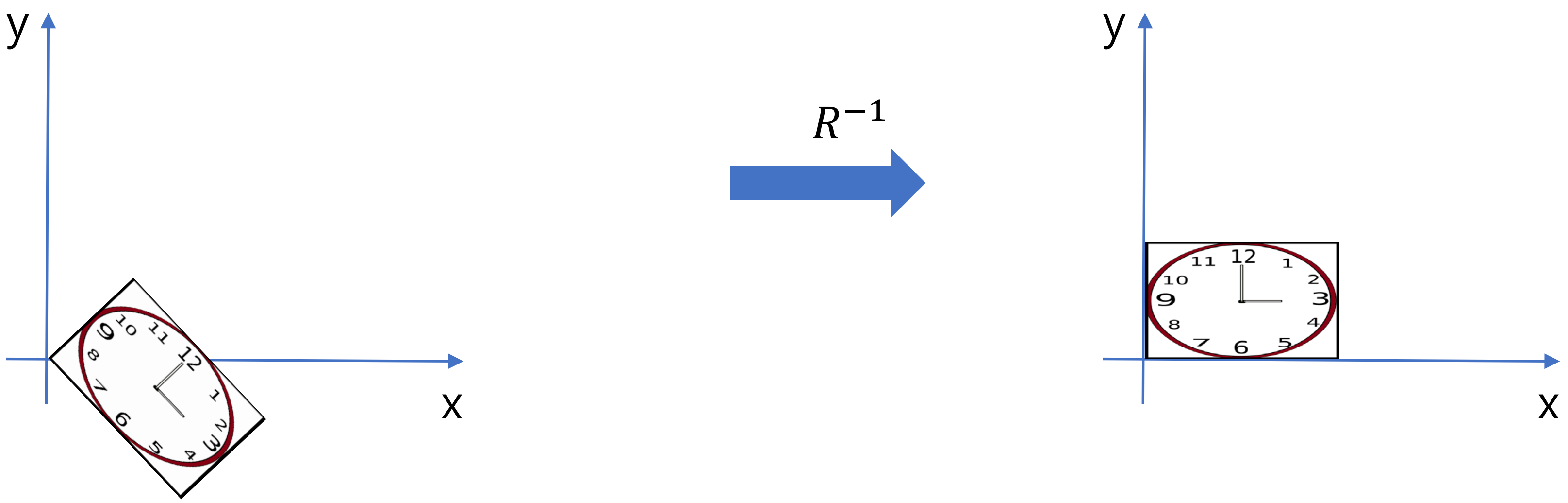
Inverse Transformation

- Your transformation
  - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
  - $x_{old} = T^{-1}x_{new} = (RS)^{-1}x_{new} = S^{-1}R^{-1}x_{new}$



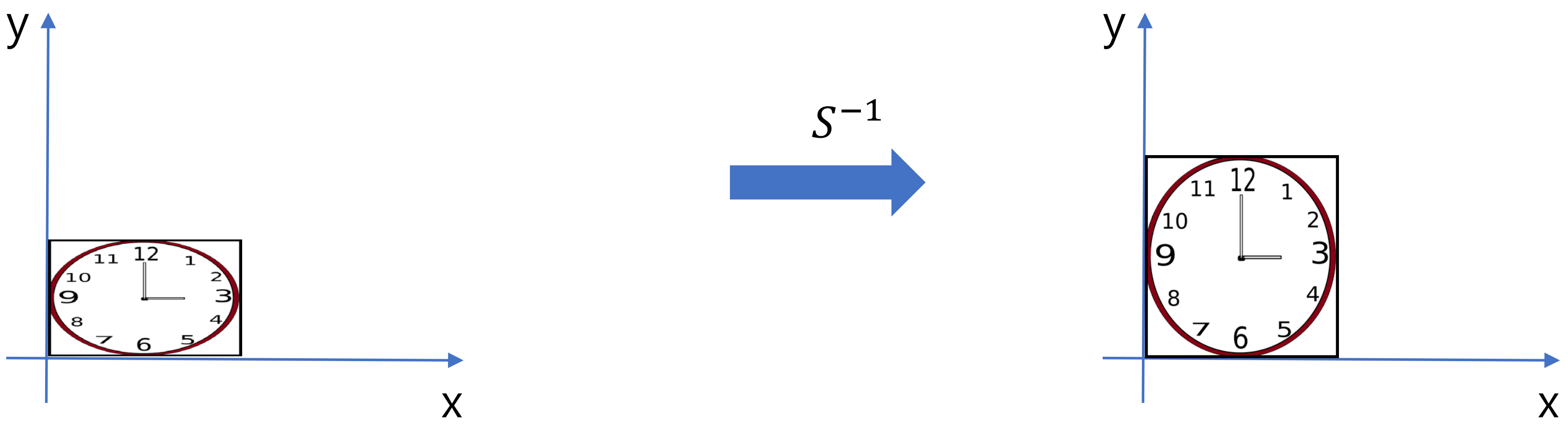
Inverse Transformation

- Your transformation
  - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
  - $x_{old} = T^{-1}x_{new} = (RS)^{-1}x_{new} = S^{-1}R^{-1}x_{new}$



Inverse Transformation

- Your transformation
  - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
  - $x_{old} = T^{-1}x_{new} = (RS)^{-1}x_{new} = S^{-1}R^{-1}x_{new}$



Inverse Matrix

- M = 2 x 2 matrix
  - $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$
  - $M^{-1} = \frac{1}{M_{11} \times M_{22} - M_{12} \times M_{21}} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix}$