

CT4201/EC4215: Computer Graphics

Acceleration Data Structures

BOCHANG MOON



Ray Tracing

- Procedure for Ray Tracing:
- For each pixel
 - Generate a primary ray (with depth 0)
 - While (depth < d) {
 - Find the closest intersection point between the ray and objects
 - If (there is a hit) then
 - Generate a shadow ray
 - If (there is no hit between the shadow ray and a light) then
 - Perform a shading
 - Generate a secondary ray (reflection or refraction ray) // increase the ray depth +1
 - Else
 - Perform a shading with background color }
 - Return background color

Naïve Ray Tracing

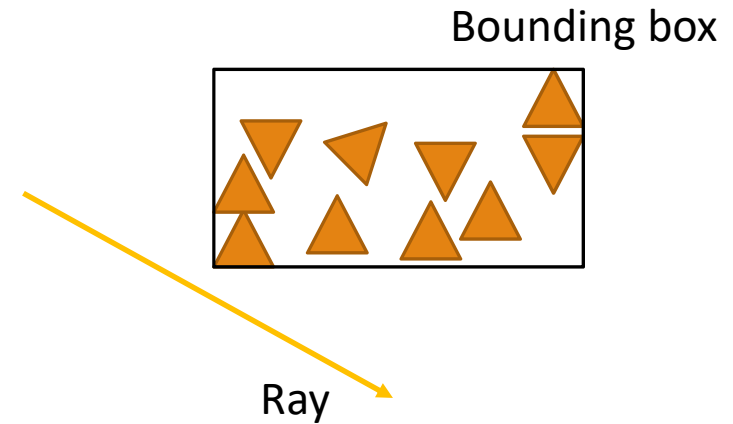
- Problem: find the closest intersection point between the ray $p(t) = e + td$
- For each triangle
 - Compute the intersection point (i.e., t) between a ray and triangle
 - If (there is a hit and $t < \text{stored } t$)
 - Store shading information and the ray parameter t
 - Return the shading information
- The complexity of this naïve algorithm is $O(N)$, where N is the number of triangles in the scene

Spatial Data Structures

- Group objects together into a hierarchy to accelerate the geometry processing
- The complexity using the acceleration data structures can be a sub-linear time (e.g., $O(\log N)$)
- Object partitioning:
 - Bounding Volume Hierarchy (BVH)
- Space partitioning:
 - Uniform Grids
 - Octree (3D) or QuadTree (2D)
 - Binary space partition tree (BSP)
 - kD-Trees

Bounding Boxes

- The key operation is to perform an intersection test between a ray and bounding box
 - Need to know only whether a ray hits the box or not



- Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$
- 2D version
 - $(x, y) \in [x_{min}, x_{max}] \times [y_{min}, y_{max}]$

Bounding Boxes

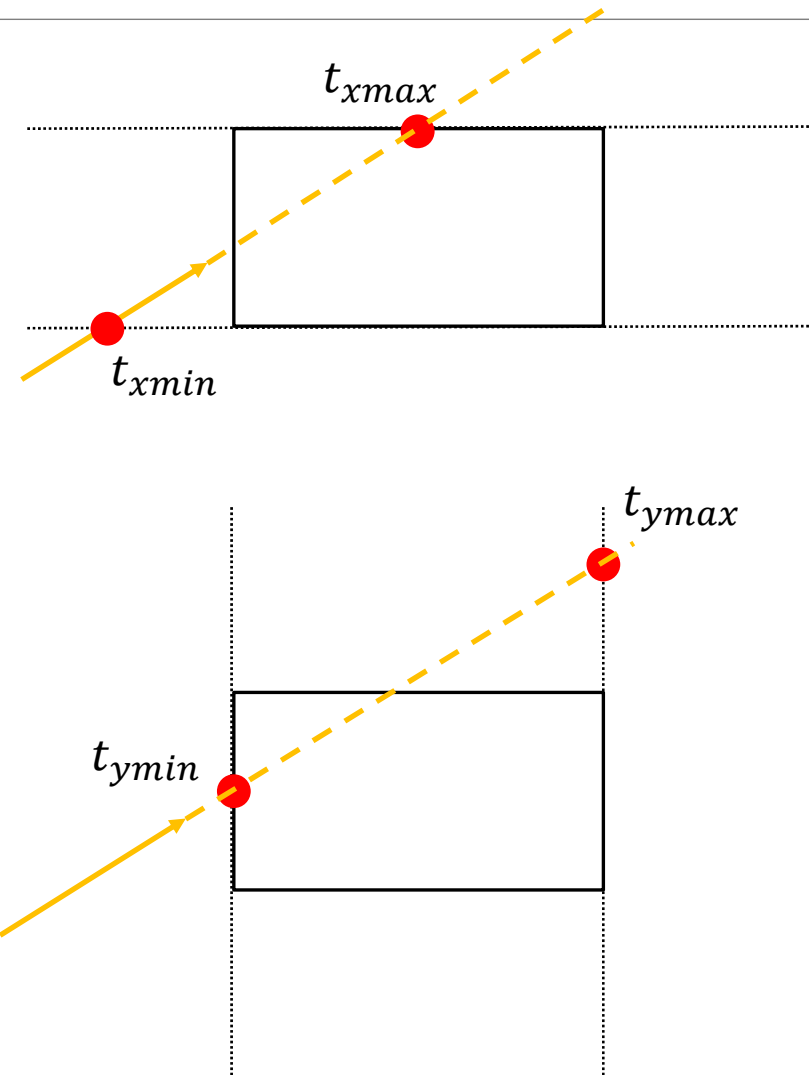
- Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$
- 2D version
 - $(x, y) \in [x_{min}, x_{max}] \times [y_{min}, y_{max}]$

- $t_{xmin} = \frac{x_{min} - x_e}{x_d}$

- $t_{xmax} = \frac{x_{max} - x_e}{x_d}$

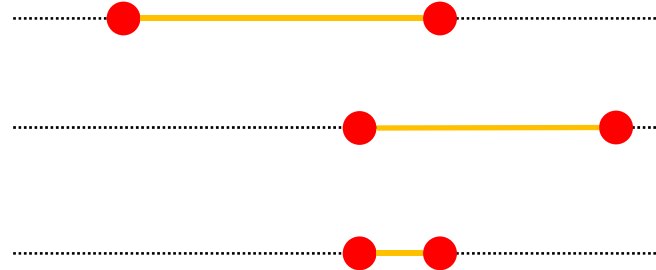
- $t_{ymin} = \frac{y_{min} - y_e}{y_d}$

- $t_{ymax} = \frac{y_{max} - y_e}{y_d}$



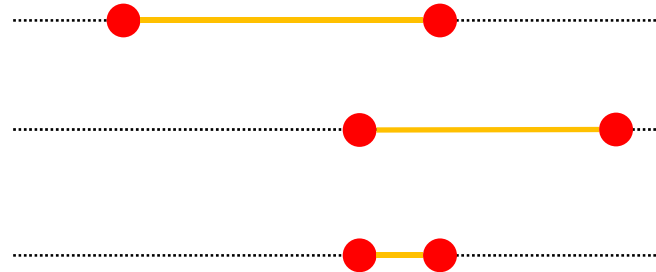
Bounding Boxes

- Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$
- $t_{xmin} = \frac{x_{min} - x_e}{x_d}$, $t_{xmax} = \frac{x_{max} - x_e}{x_d}$
- $t_{ymin} = \frac{y_{min} - y_e}{y_d}$, $t_{ymax} = \frac{y_{max} - y_e}{y_d}$
- $t \in [t_{xmin}, t_{xmax}]$
- $t \in [t_{ymin}, t_{ymax}]$
- $t \in [t_{xmin}, t_{xmax}] \cap [t_{ymin}, t_{ymax}]$
- A ray hits the box if and only if the two intervals overlap.



Bounding Boxes

- Procedure for testing the intersection
 - Compute $t_{xmin}, t_{xmax}, t_{ymin}, t_{ymax}$
 - If ($t_{xmin} > t_{ymax}$ or $t_{xmax} < t_{ymin}$)
 - No hit
 - else
 - Hit



Bounding Boxes

- Negative x_d or y_d :
 - A ray will hit x_{max} (or y_{max}) before it hits x_{min} (or y_{min})
 - If ($x_d \geq 0$) then
 - $t_{min} = (x_{min} - x_e)/x_d$
 - $t_{max} = (x_{max} - x_e)/x_d$
 - else
 - $t_{min} = (x_{max} - x_e)/x_d$
 - $t_{max} = (x_{min} - x_e)/x_d$
 - If ($y_d \geq 0$) then
 - $t_{min} = (y_{min} - y_e)/y_d$
 - $t_{max} = (y_{max} - y_e)/y_d$
 - else
 - $t_{min} = (y_{max} - y_e)/y_d$
 - $t_{max} = (y_{min} - y_e)/y_d$

Bounding Boxes

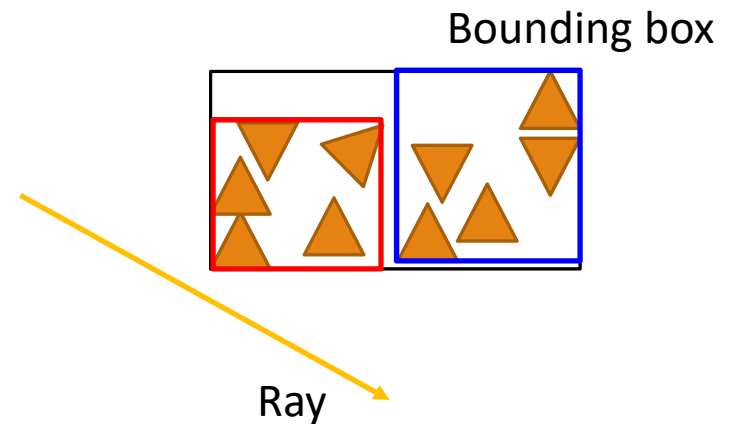
- Zero x_d or y_d :
 - Divide-by-zero issue
- Given a number $a \in \mathbb{R}^+$, IEEE floating point rules provide the following:
 - $\frac{+a}{+0} = \infty$
 - $\frac{-a}{+0} = -\infty$
 - $[t_{xmin}, t_{xmax}] = [-\infty, -\infty], [\infty, \infty]$: no hit
 - $[t_{xmin}, t_{xmax}] = [-\infty, \infty]$: hit
 - The previous code works for +0 denominator
- How about -0 denominator?
 - We can test a reciprocal of the ray direction (e.g., $1/x_d$)

Bounding Boxes

- -0 denominator?
 - If ($x_d \geq 0$) then
 - $t_{min} = (x_{min} - x_e)/x_d$
 - $t_{max} = (x_{max} - x_e)/x_d$
 - else
 - $t_{min} = (x_{max} - x_e)/x_d$
 - $t_{max} = (x_{min} - x_e)/x_d$
- Problem: the first if statements will be true because $-0 == 0$ is true (IEEE floating point standard), so we can miss valid hits.
 - A remedy is test a reciprocal of the ray direction (e.g., $1/x_d$) instead of x_d
 - More detail:
 - An Efficient and Robust Ray–Box Intersection Algorithm, Williams et al. 2005

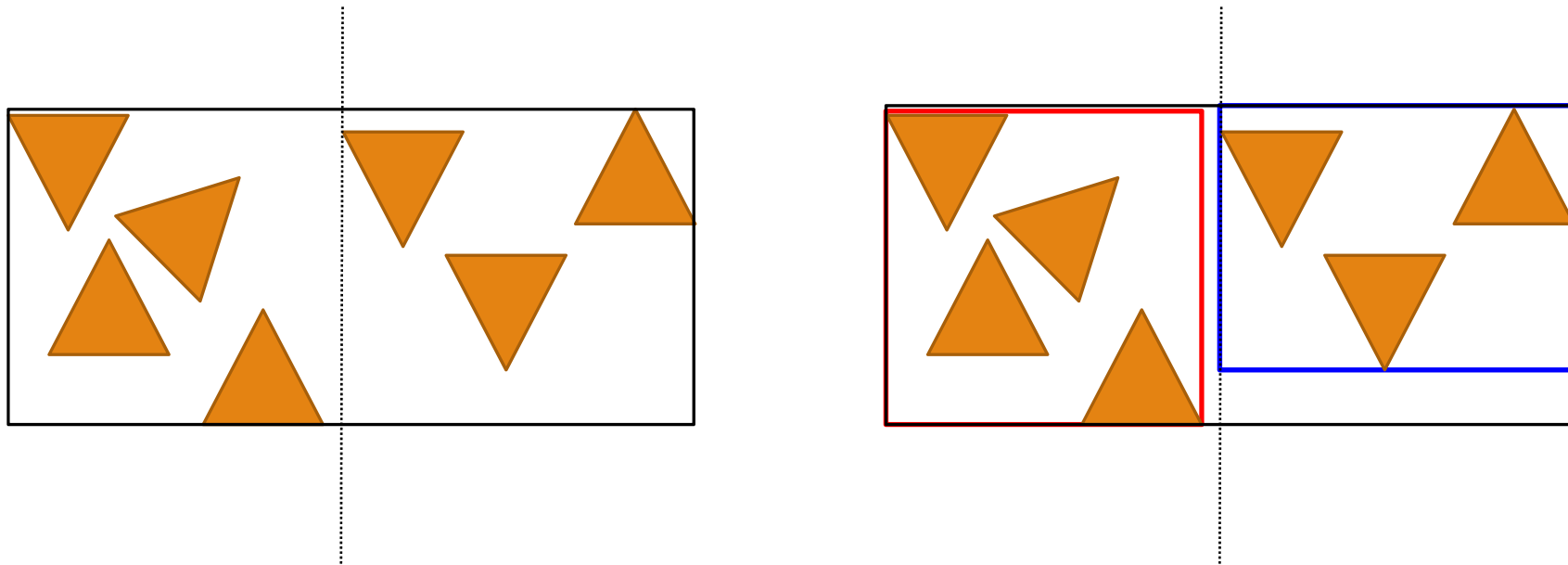
Hierarchical Bounding Boxes

- Motivation: expensive as we need to test all primitives within a bounding box that a ray hits
- Solution: the bounding boxes can be built in a hierarchical way
- Two popular hierarchical methods:
 - Bounding volume hierarchy (BVH)
 - Kd-tree



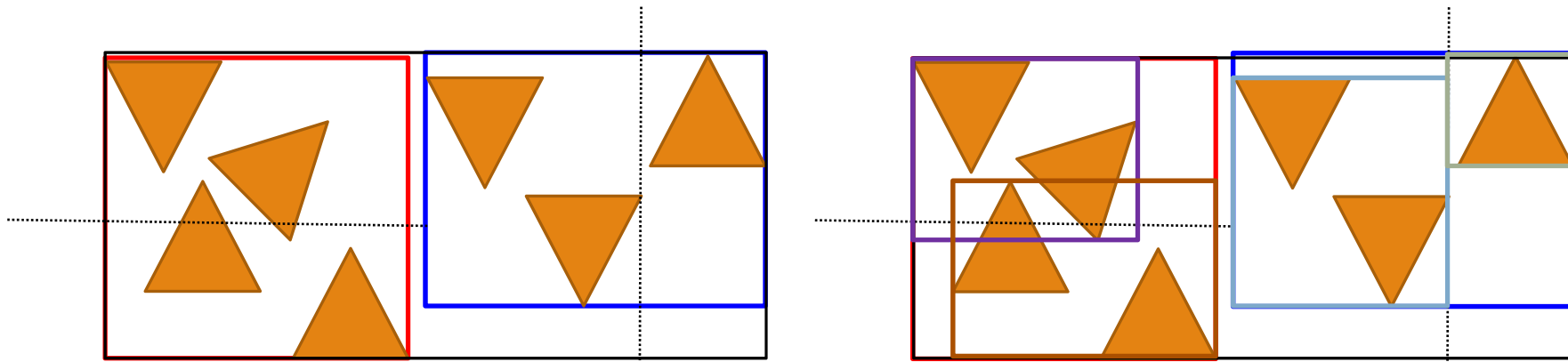
Bounding Volume Hierarchy

- Step 1. Compute a bounding box of primitives
 - e.g., Axis-Aligned Bounding Box (AABB) $[x_{min}, y_{min}, z_{min}] \times [x_{max}, y_{max}, z_{max}]$
- Step 2. Split the primitives into two groups and compute the child BVs
- Step 3. Go to Step 1 until the number of primitives $< k$



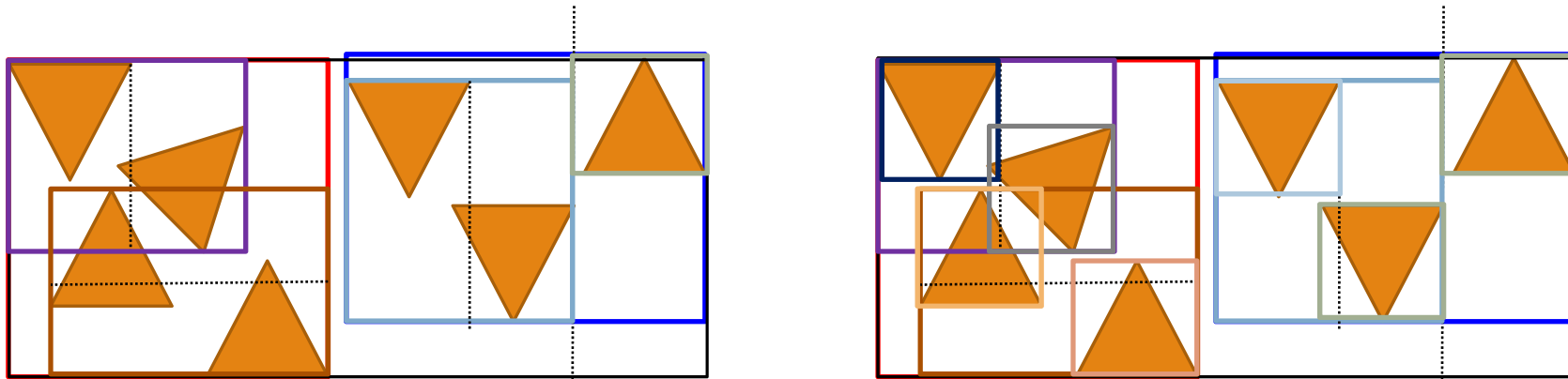
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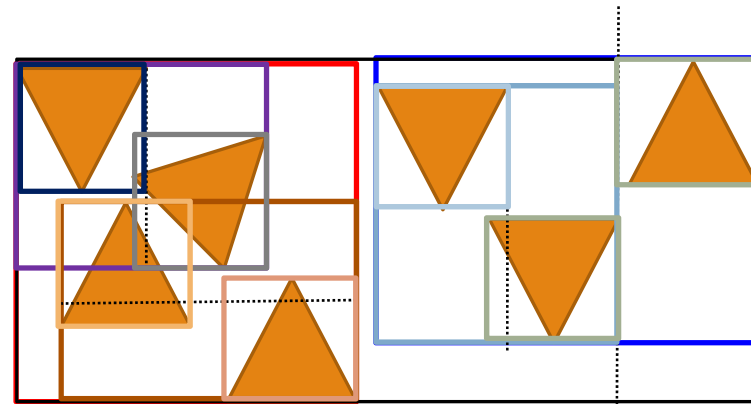
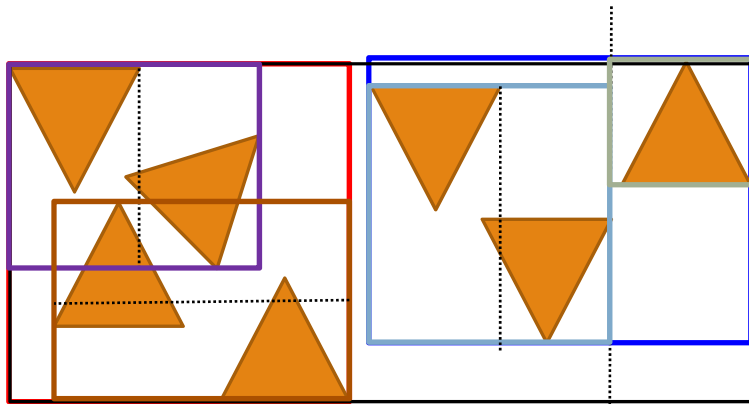
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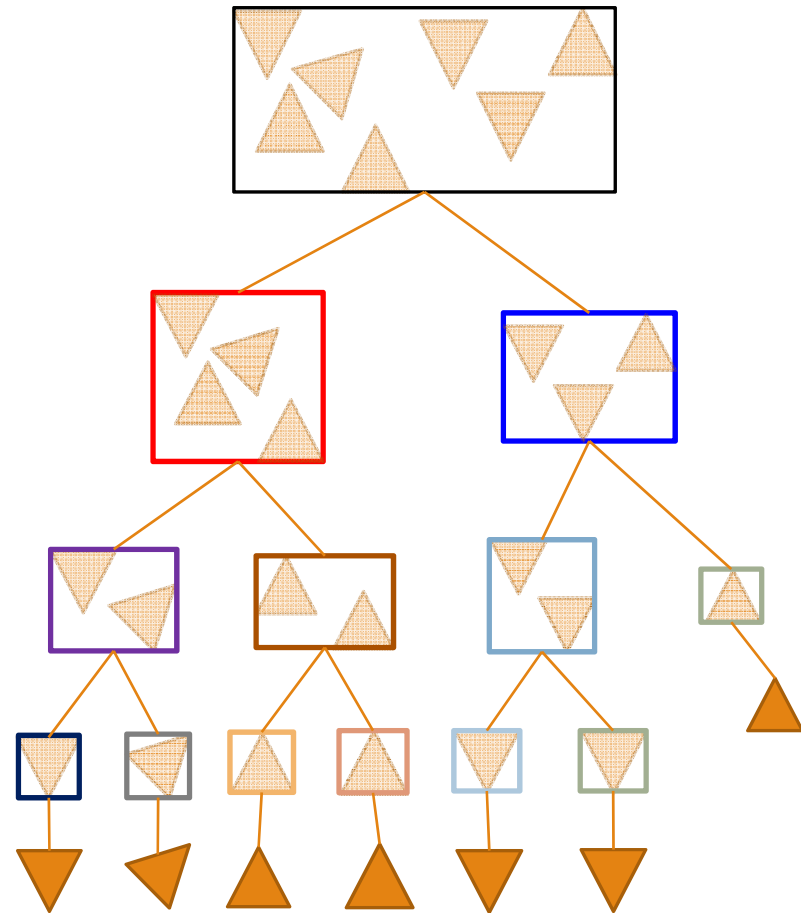
Bounding Volume Hierarchy

- Where should we split the primitives?
 - Midpoint of a volume
 - Sort the primitives, and select the median
 - Other approaches?
 - Surface Area Heuristic (SAH)



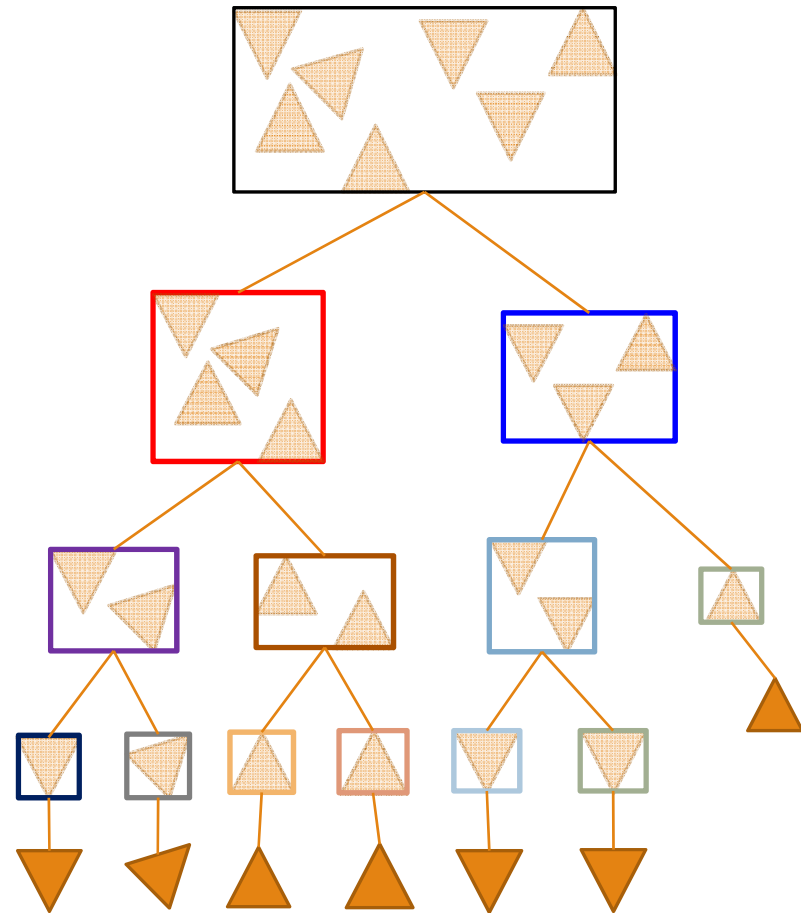
Bounding Volume Hierarchy

- Traversal procedure:
 - Check whether the intersection occurs
 - If (hit and $t < \text{ray.t}$) then
 - If (the BV is a leaf node)
 - Find the closest intersection point between the ray and triangle
 - If (the ray hits triangles) then
 - $\text{ray.t} = t$ (from the closest intersection)
 - Store some shading info.
 - else
 - Check an intersection using its child BVs



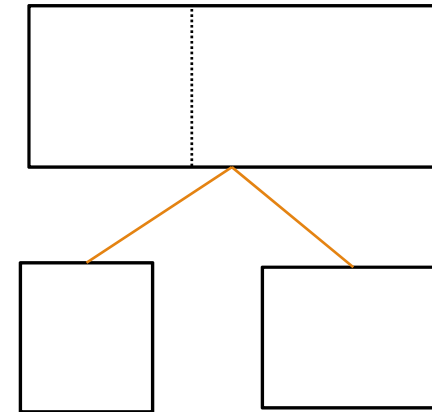
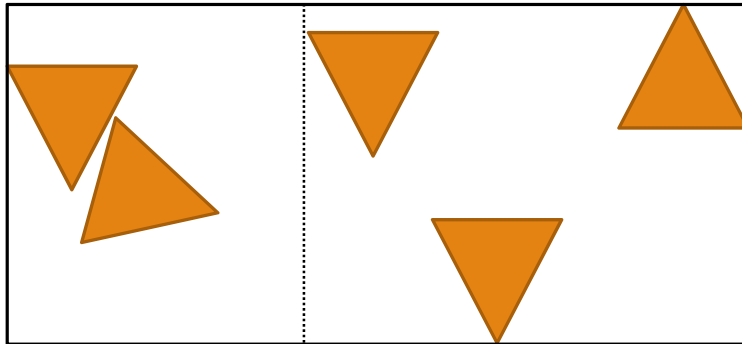
Bounding Volume Hierarchy

- Properties of BVH
 - Object partitioning: split primitives
 - Some BVs can overlap each other



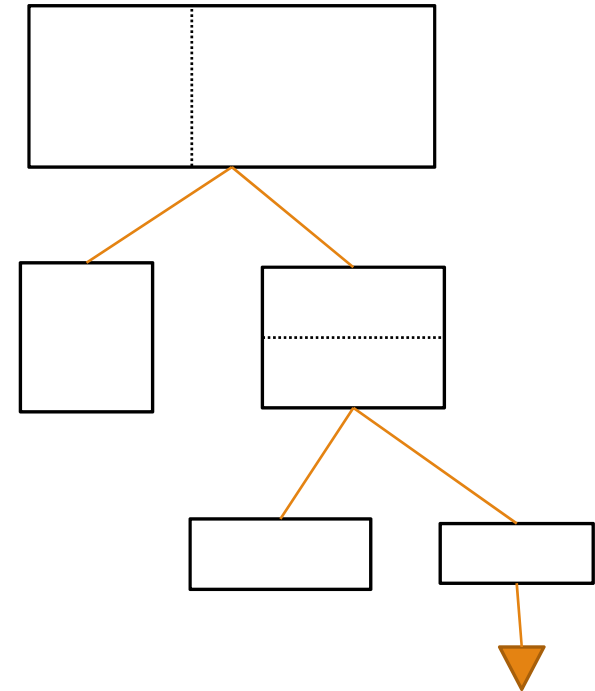
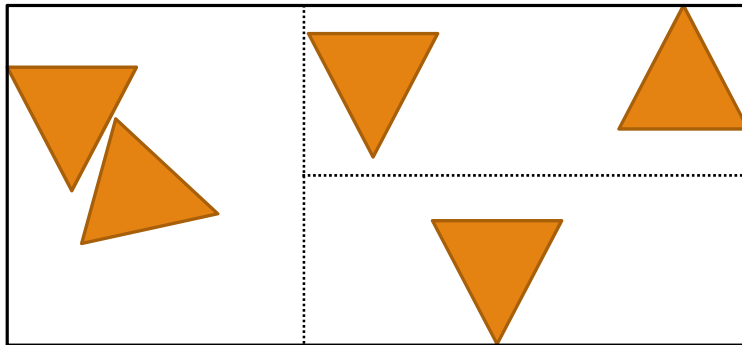
Kd-trees

- Recursively split space with axis-aligned planes



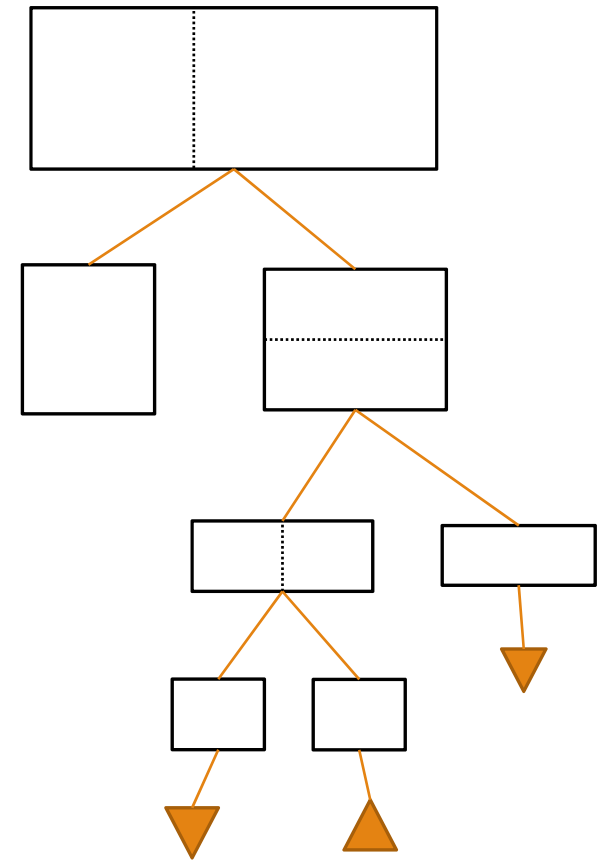
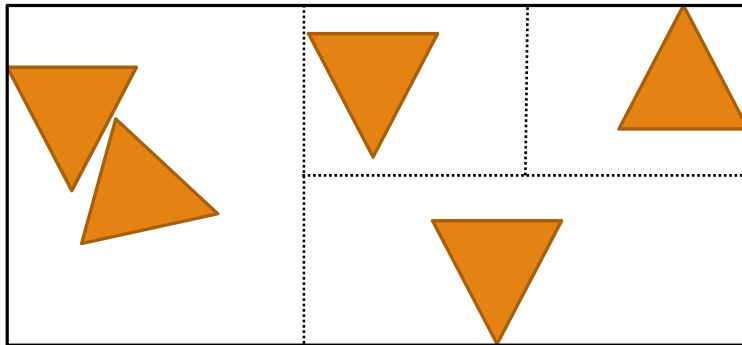
Kd-trees

- Recursively split space with axis-aligned planes



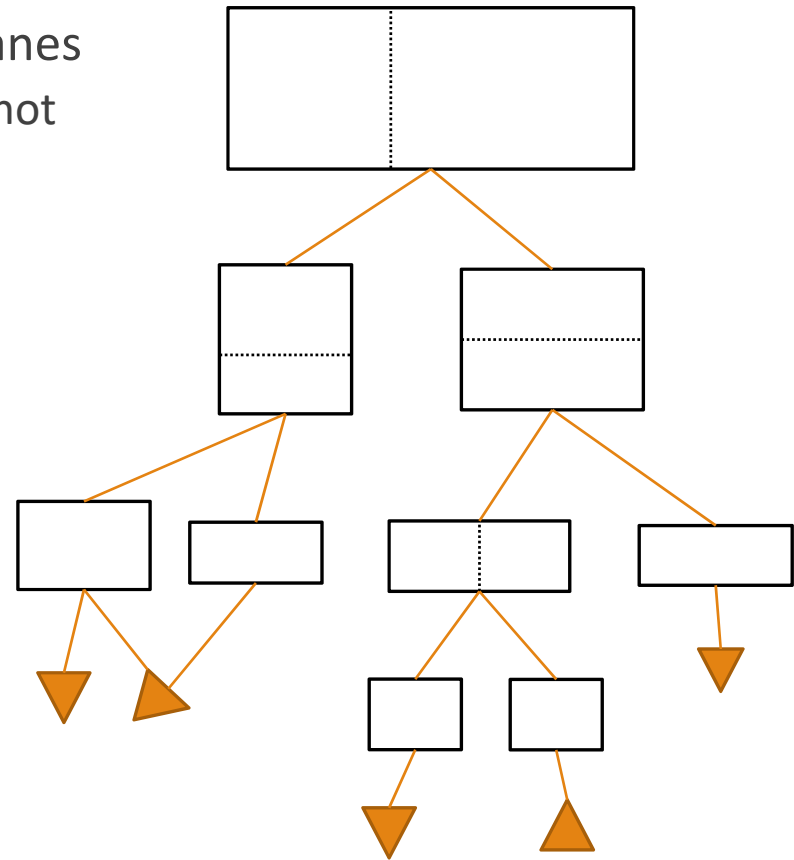
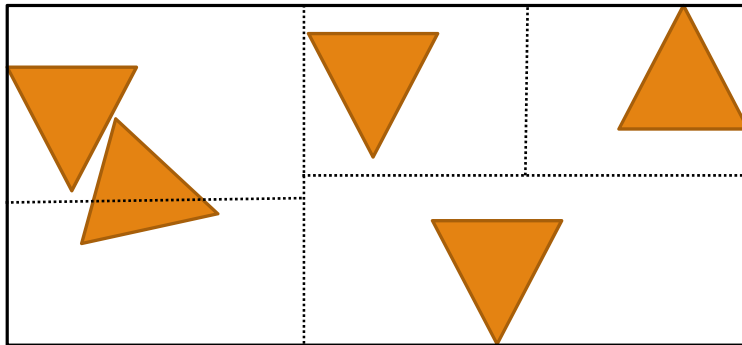
Kd-trees

- Recursively split space with axis-aligned planes



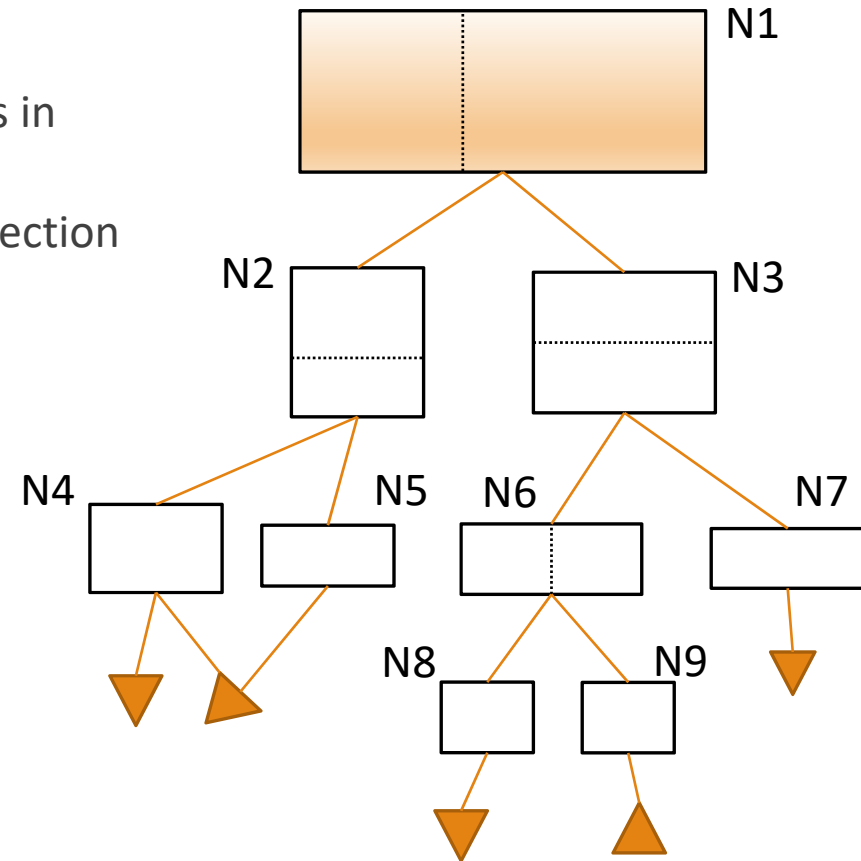
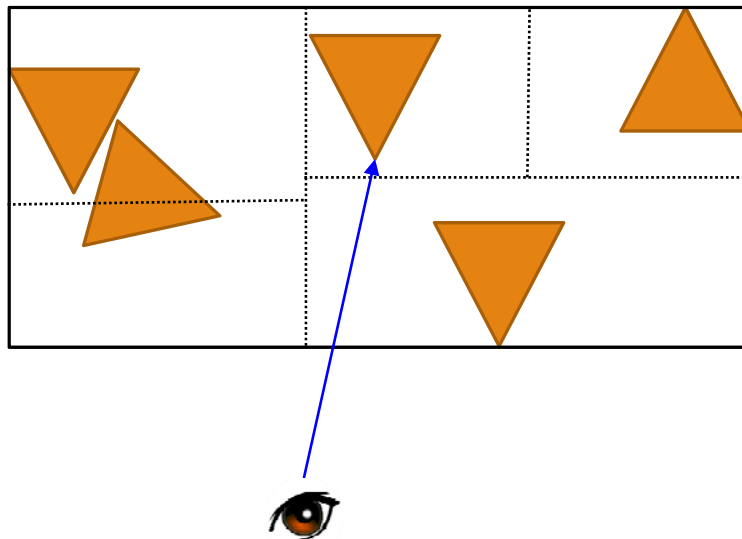
Kd-trees

- Recursively split space with axis-aligned planes
 - Some nodes can point same triangles if we cannot split them



Kd-trees

- Traversal
 - Front-to-back traversal: traverse child nodes in order along a ray
 - Can terminate traversal as soon as an intersection between a ray and triangle is found



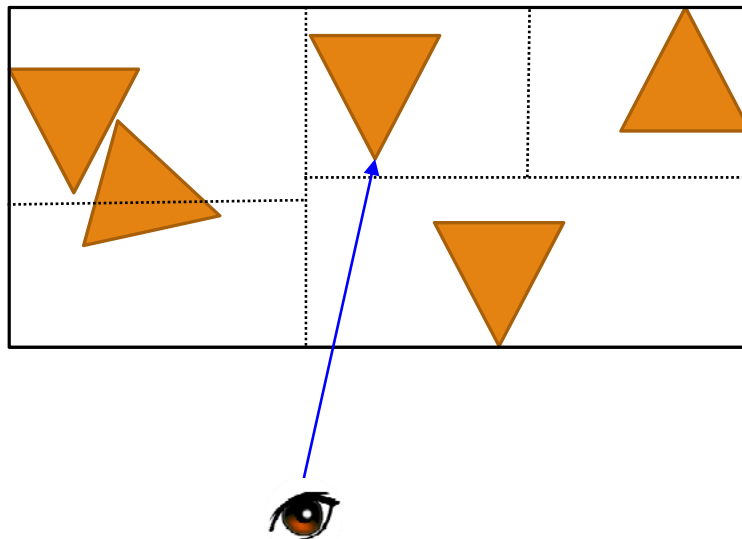
Current node: N1

Stack:



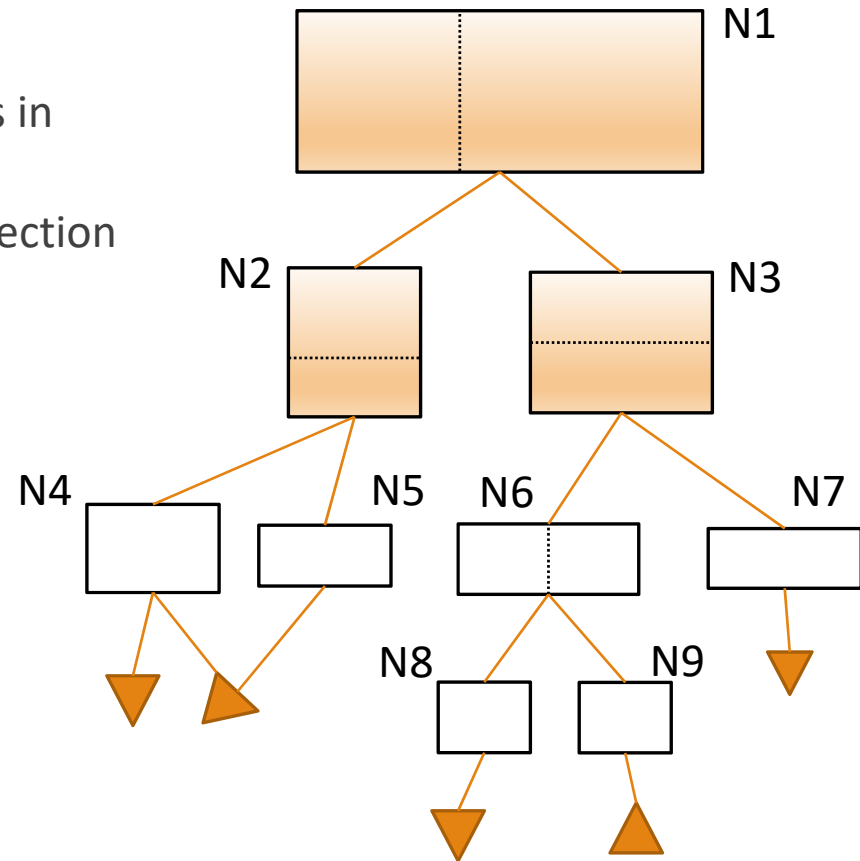
Kd-trees

- Traversal
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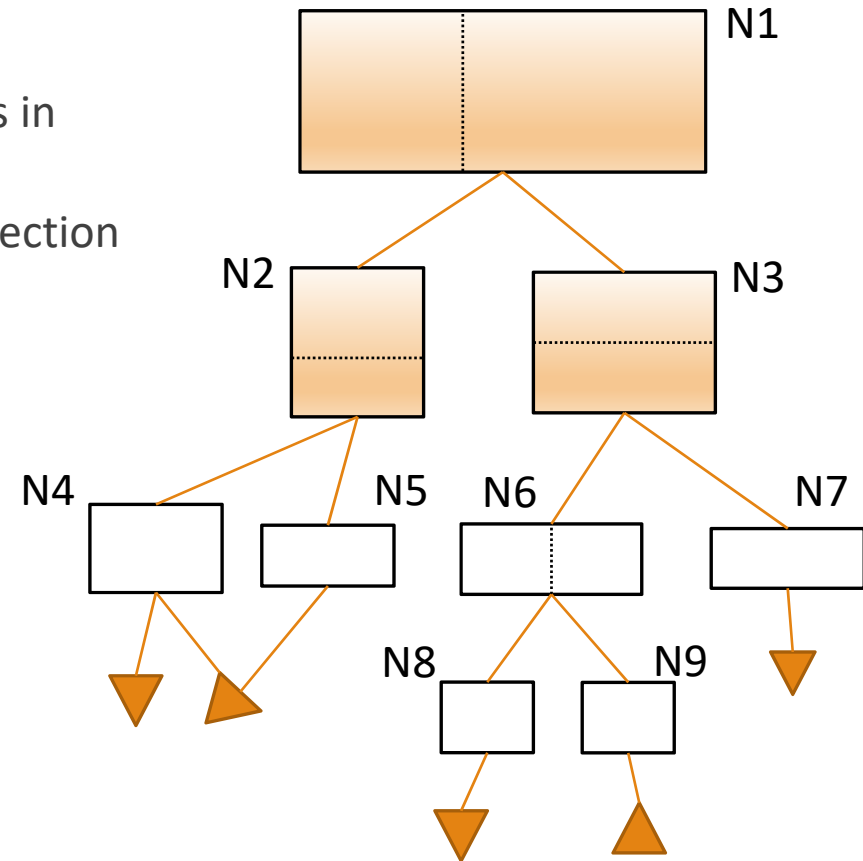
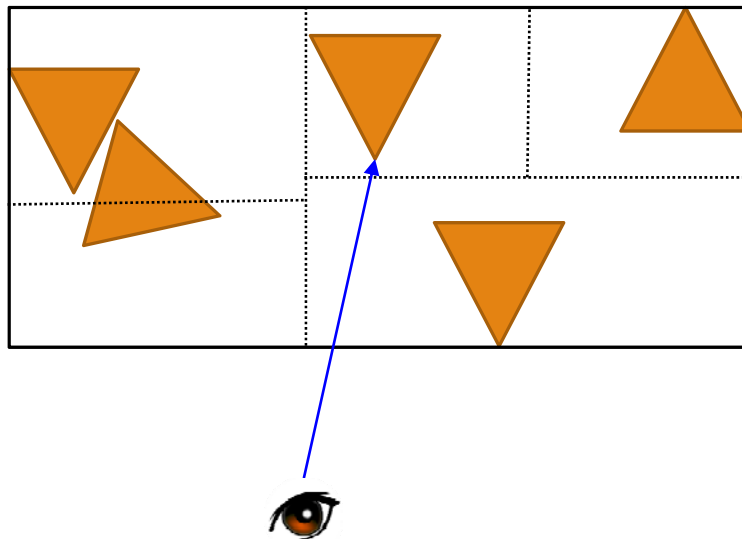
Current node: N2

Stack: N3



Kd-trees

- Traversal
 - Front-to-back traversal: traverse child nodes in order along a ray
 - Can terminate traversal as soon as an intersection between a ray and triangle is found



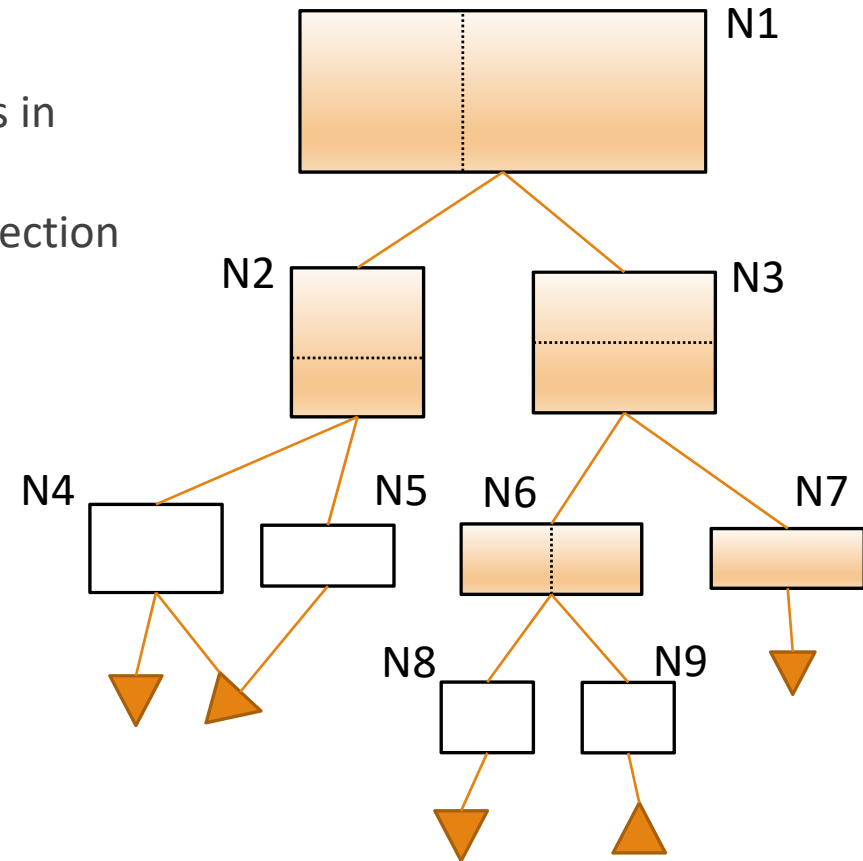
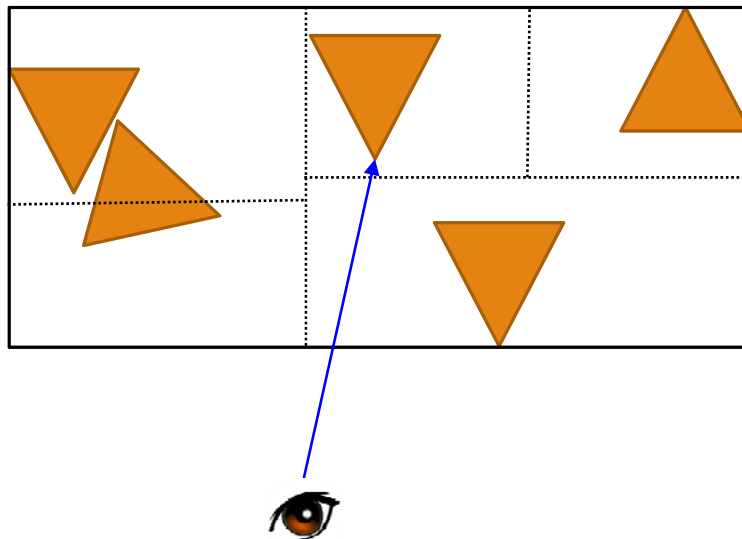
Current node: N3

Stack:



Kd-trees

- Traversal
 - Front-to-back traversal: traverse child nodes in order along a ray
 - Can terminate traversal as soon as an intersection between a ray and triangle is found



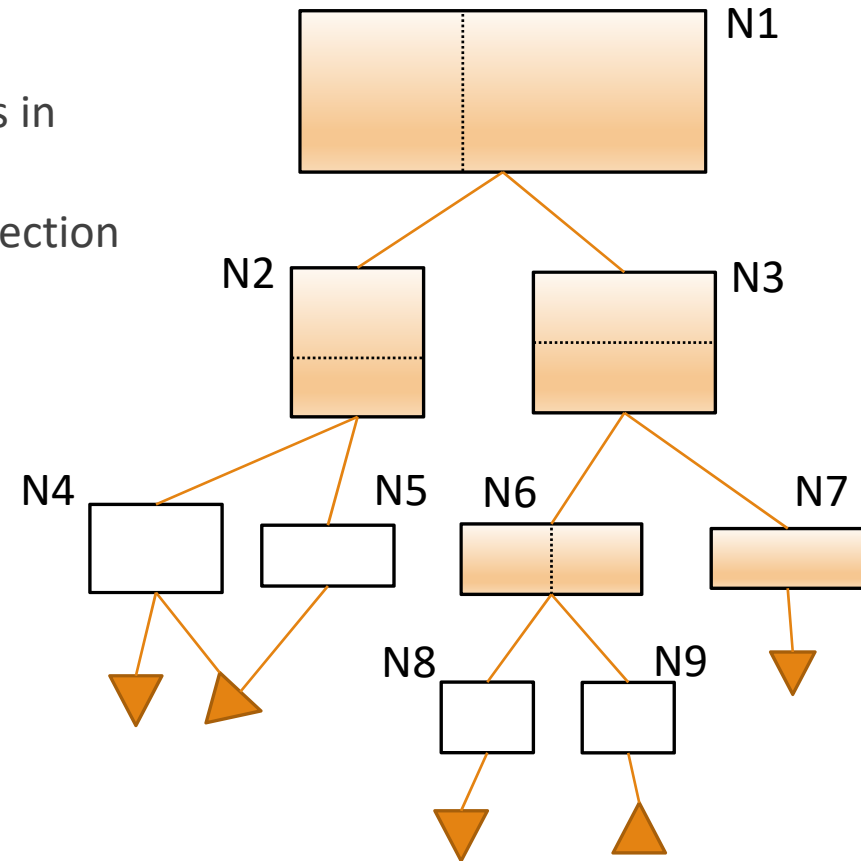
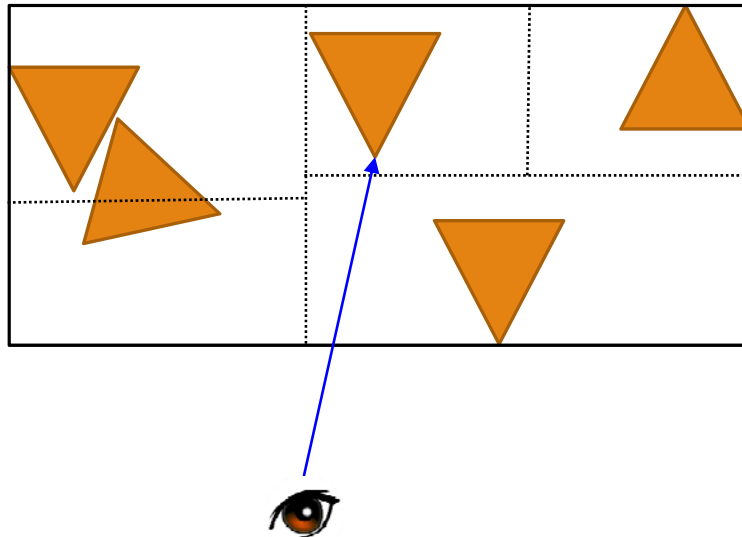
Current node: N7

Stack: N6



Kd-trees

- Traversal
 - Front-to-back traversal: traverse child nodes in order along a ray
 - Can terminate traversal as soon as an intersection between a ray and triangle is found



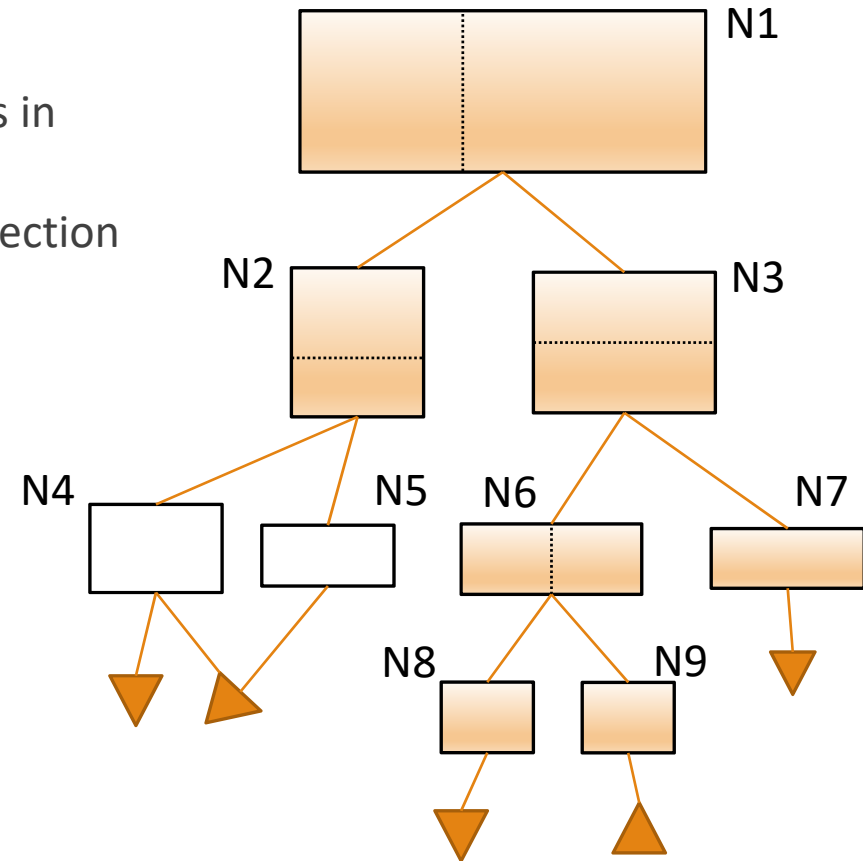
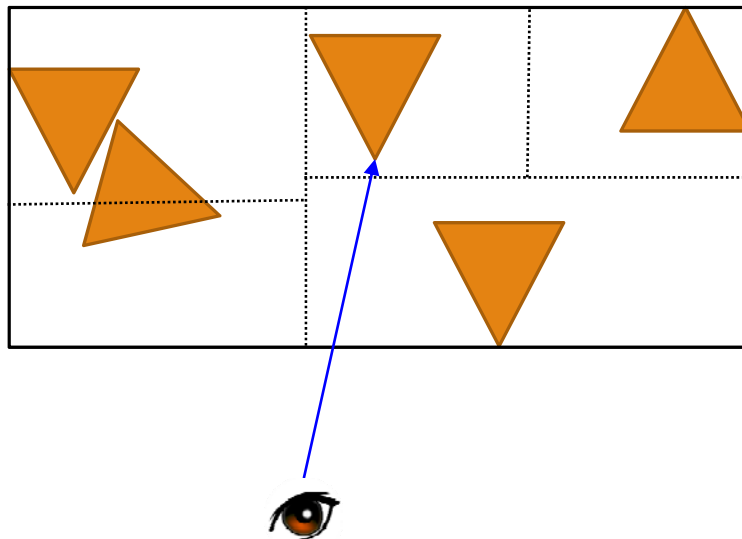
Current node: ▼

Stack: N6



Kd-trees

- Traversal
 - Front-to-back traversal: traverse child nodes in order along a ray
 - Can terminate traversal as soon as an intersection between a ray and triangle is found



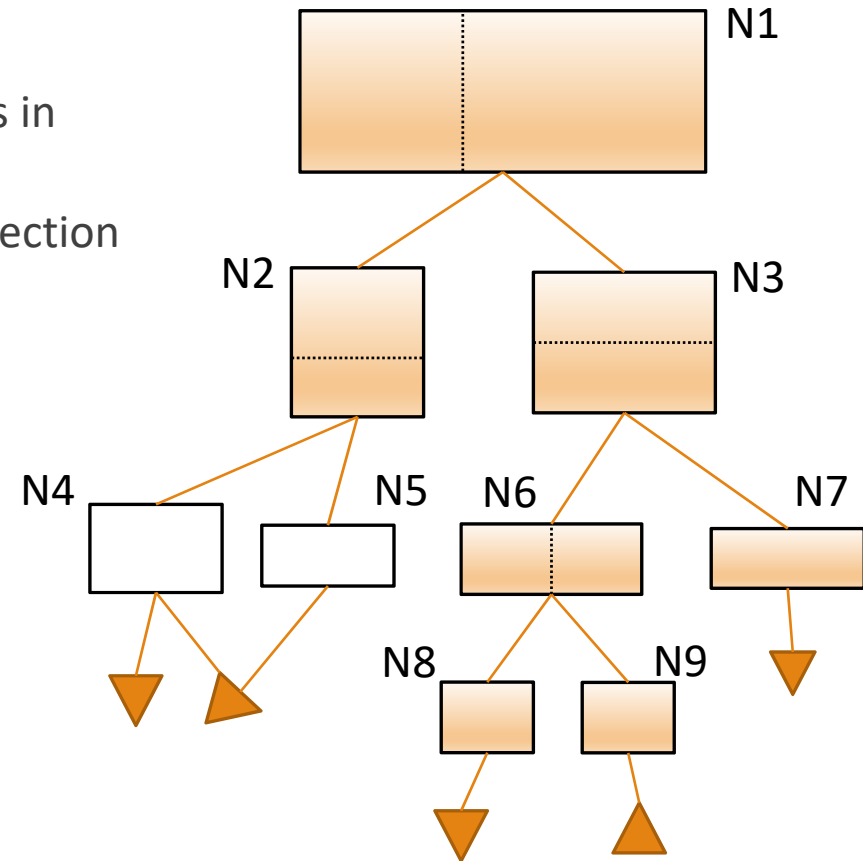
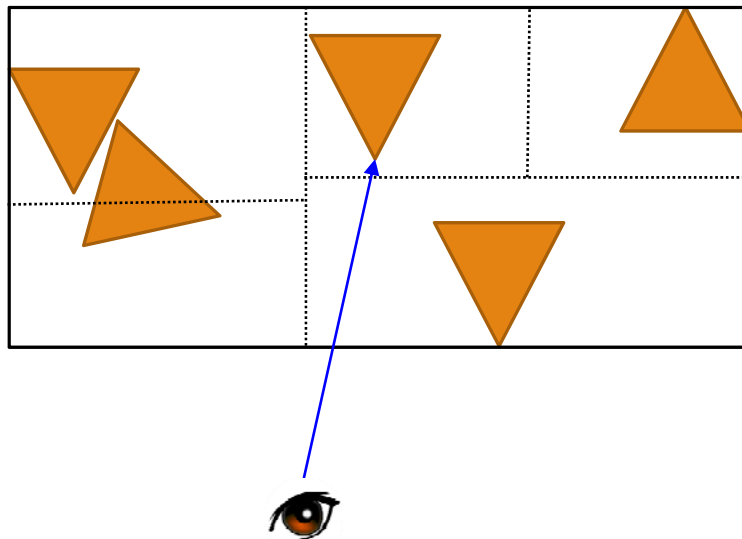
Current node: N6

Stack:



Kd-trees

- Traversal
 - Front-to-back traversal: traverse child nodes in order along a ray
 - Can terminate traversal as soon as an intersection between a ray and triangle is found



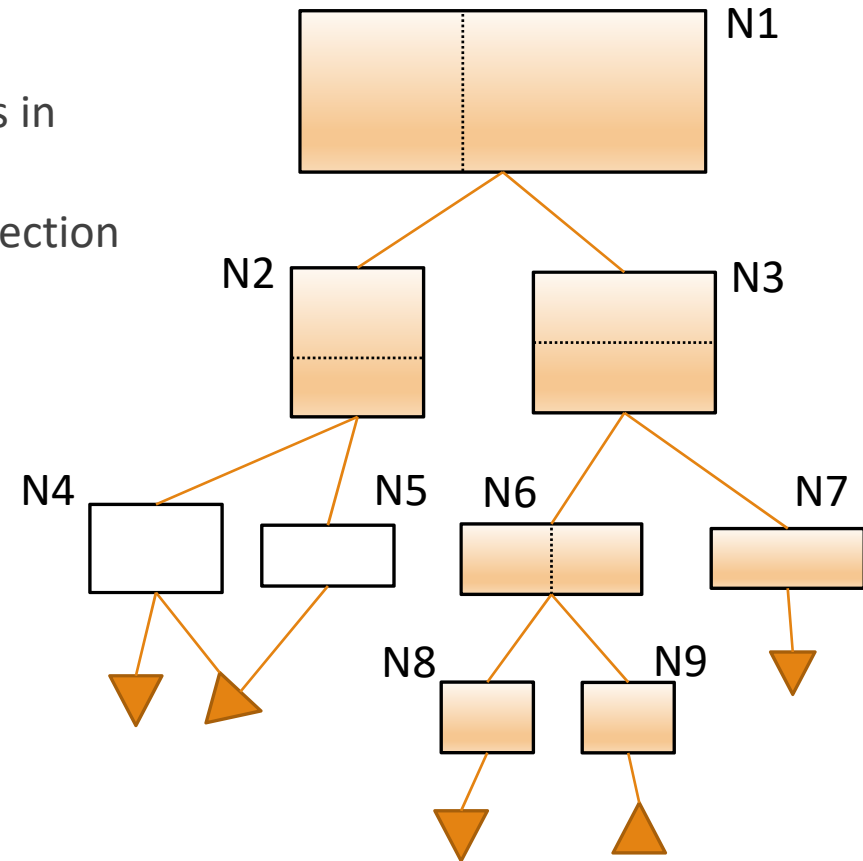
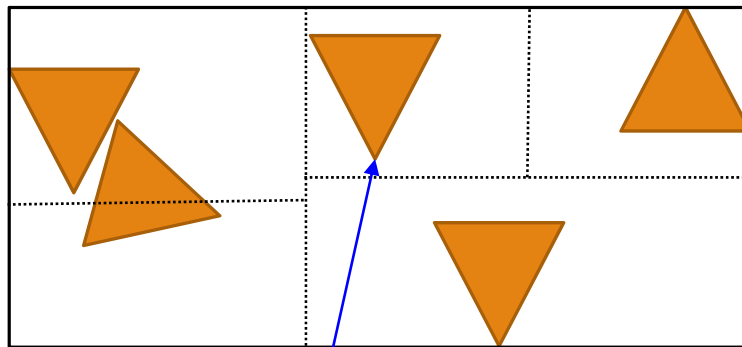
Current node: N8

Stack: N9



Kd-trees

- Traversal
 - Front-to-back traversal: traverse child nodes in order along a ray
 - Can terminate traversal as soon as an intersection between a ray and triangle is found



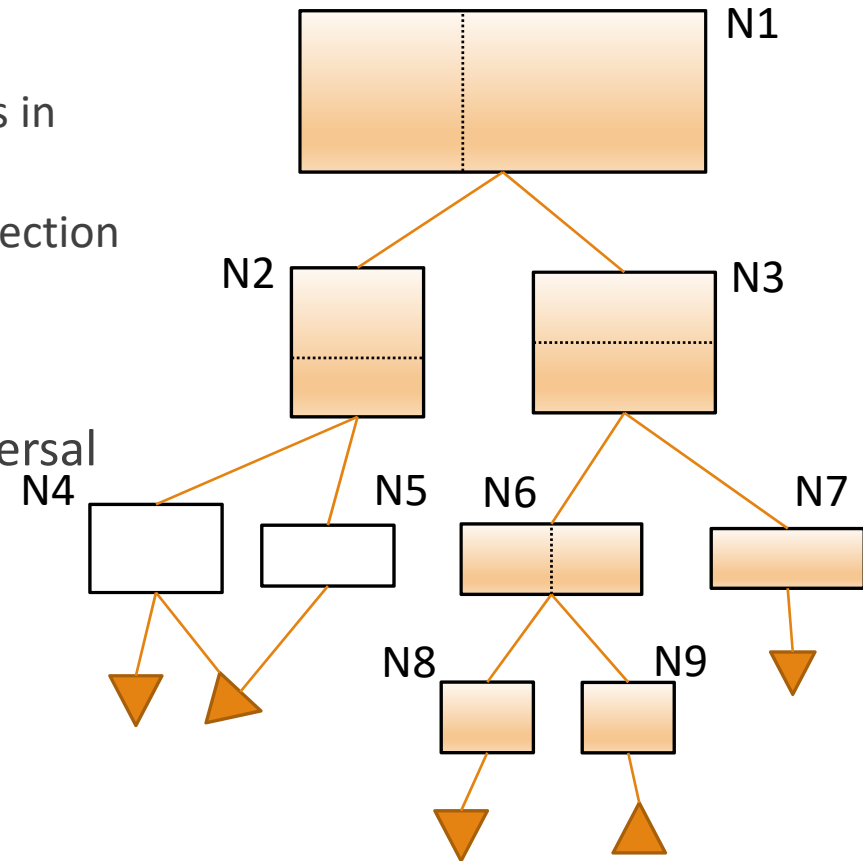
Current node: ▼ (hit and finish)

Stack: N9



Kd-trees

- Traversal
 - Front-to-back traversal: traverse child nodes in order along a ray
 - Can terminate traversal as soon as an intersection between a ray and triangle is found
- What's difference compared to the traversal on BVH?



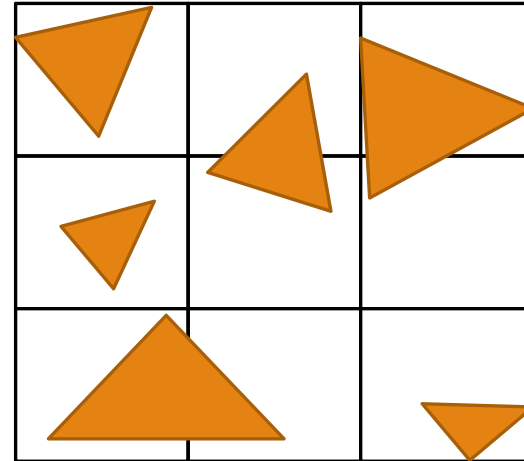
Current node: ▼ (hit and finish)

Stack: N9



Other Structures

- Uniform grids
 - Partition the whole space into equal-size cells
- Binary space partition (BSP) tree
 - Recursively split space with planes (arbitrary orientations)
 - Kd-tree is a special case of BSP tree: it uses an axis-aligned plane for partitioning
- Octree
 - Recursively split space but each inner node has 8 equal-size voxels



Discussion Points

- Axis-aligned bounding box (AABB)?
 - Cheap to compute the intersection
 - Bounding box may be too loose
 - Oriented bound box (OBB) can be better to fit objects, but this requires more complex computations
 - Other shapes (e.g., sphere) can be utilized
 - What's the ideal bounding volume?

Discussion Points

- What's the best hierarchy?
 - Usually need to consider the following:
 - Pre-processing time (construction)
 - Run-time (rendering)
 - Memory to save all the nodes
 - Deformable objects can require run-time constructions
 - Hybrid?
 - Maintain two-level hierarchy
 - e.g., top-level: grids, low-level: kd-tree



Further Readings

- Chapter 12