

CT4201/EC4215: Computer Graphics

# Whitted Ray Tracing

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BOCHANG MOON



# Whitted Ray Tracing

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- Recursively perform the ray casting to simulate refraction and reflection



Whitted (1980)



# An Improved Illumination Model [Whitted 80]


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- Phong illumination model

- $$I = \sum_{i=1}^{\# \text{ of lights}} \{L_a^i k_a + L_d^i k_d \max(0, \mathbf{n} \cdot \mathbf{l}^i) + L_s^i k_s \max(0, \mathbf{r}^i \cdot \mathbf{v})^s\}$$

- Whitted illumination model

- $$I = \sum_{i=1}^{\# \text{ of lights}} \{L_a^i k_a + L_d^i k_d \max(0, \mathbf{n} \cdot \mathbf{l}^i)\} + k_s S + k_t T$$

- S: intensity of light coming from a reflection ray
  - T: intensity of light that comes from a transmission ray
  - $k_s, k_t$ : coefficients for specular and transmission
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- S: intensity of light coming from a reflection ray
- T: intensity of light that comes from a transmission ray
- $k_s, k_t$ : coefficients for specular and transmission
  
- Q. how we can compute the rays?
  - Note that we need to generate the secondary rays to find S and T

# Specular Reflection

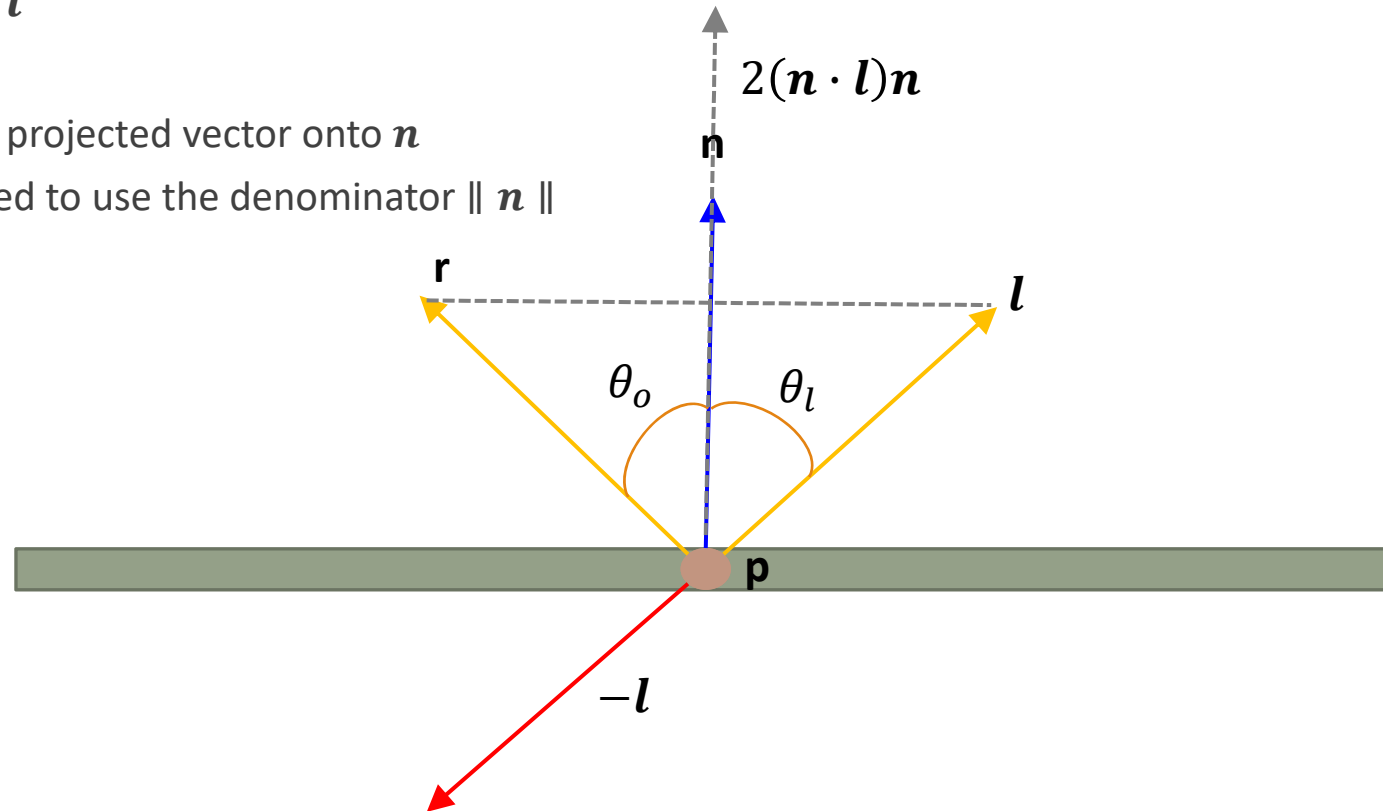
- The reflection vector,  $r$ , can be computed as the following:

- $r = 2(n \cdot l)n - l$

- $n \cdot l$ : length of the projected vector onto  $n$

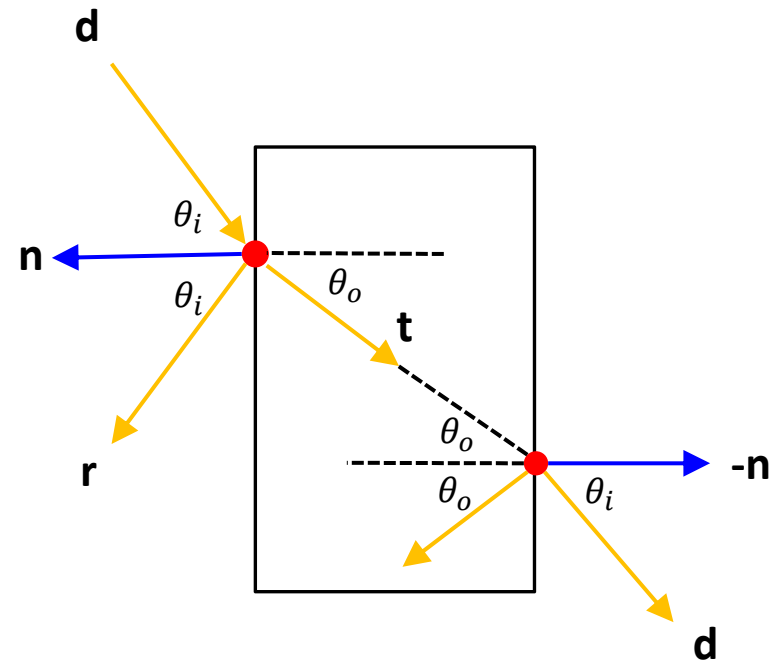
- Note: we don't need to use the denominator  $\|n\|$

- e.g., mirror



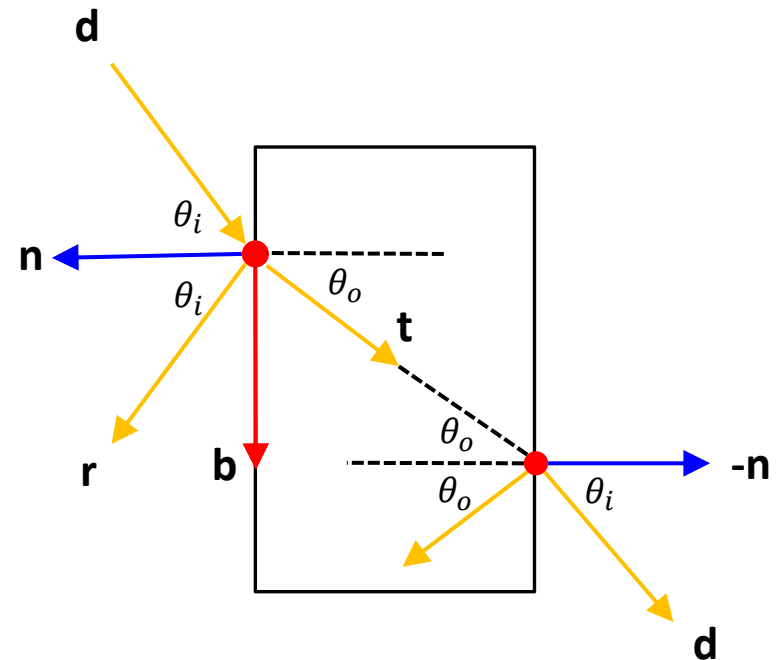
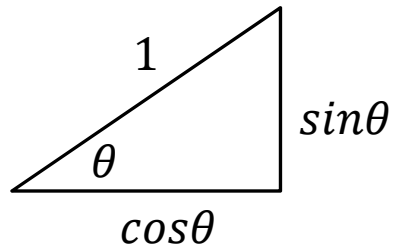
# Specular Refraction

- Dielectric materials (e.g., diamonds, glass, water, and air)
  - Transparent objects that refract light
- Snell's law
  - $n_i \sin \theta_i = n_o \sin \theta_o$ 
    - $n_i, n_o$ : indices of refraction
  - $n_i^2 (1 - \cos^2 \theta_i) = n_o^2 (1 - \cos^2 \theta_o)$ 
    - $\because$  trigonometric identity ( $\sin^2 \theta + \cos^2 \theta = 1$ )
  - $\cos^2 \theta_o = 1 - \frac{n_i^2 (1 - \cos^2 \theta_i)}{n_o^2}$



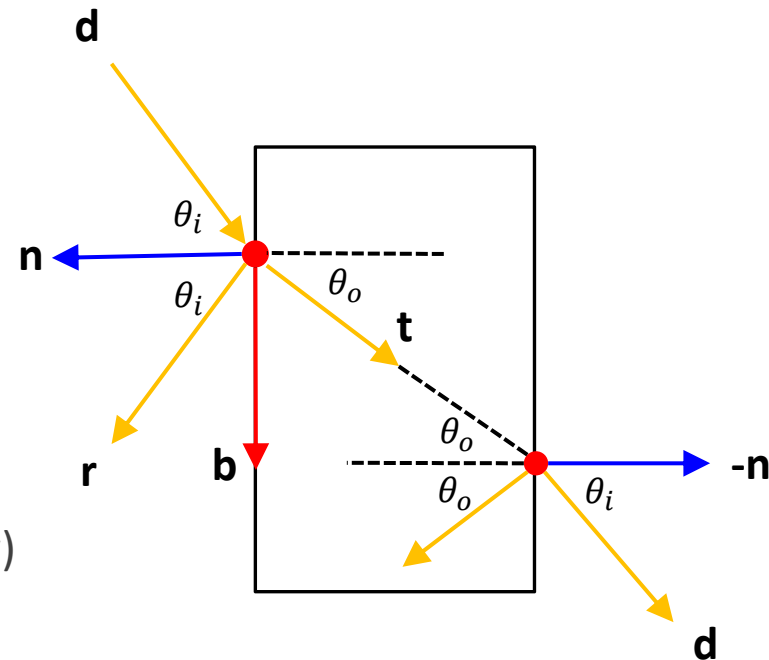
# Specular Refraction

- Vectors  $\mathbf{n}$  and  $\mathbf{b}$  forms an orthonormal basis
- $\mathbf{t} = \sin\theta_o \mathbf{b} - \cos\theta_o \mathbf{n}$ 
  - Unknowns:  $\mathbf{t}, \mathbf{b}$
- $\mathbf{d} = \sin\theta_i \mathbf{b} - \cos\theta_i \mathbf{n},$
- $\mathbf{b} = \frac{\mathbf{d} + \mathbf{n} \cos\theta_i}{\sin\theta_i}$
- Trigonometric functions



# Specular Refraction

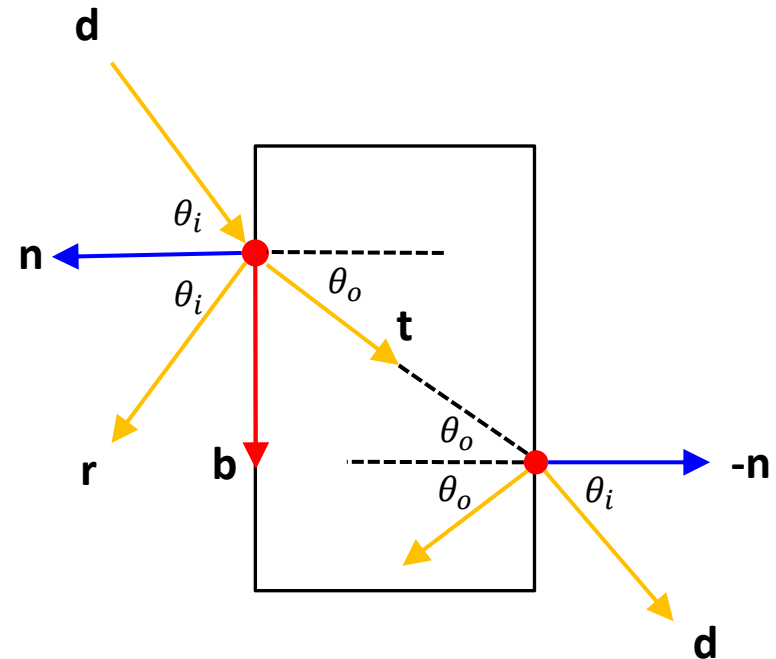
- $\mathbf{t} = \sin\theta_o \mathbf{b} - \cos\theta_o \mathbf{n}$ 
  - Unknowns:  $\mathbf{t}, \mathbf{b}$
- $\mathbf{b} = \frac{\mathbf{d} + n \cos\theta_i}{\sin\theta_i}$
- By plug-in  $\mathbf{b}$  into the equation for  $\mathbf{t}$ :
- $\mathbf{t} = \sin\theta_o \frac{\mathbf{d} + n \cos\theta_i}{\sin\theta_i} - \cos\theta_o \mathbf{n}$
- $= \frac{n_i(\mathbf{d} + n \cos\theta_i)}{n_o} - \cos\theta_o \mathbf{n}$  (by Snell's law)
- $= \frac{n_i(\mathbf{d} - n(\mathbf{d} \cdot \mathbf{n}))}{n_o} - \mathbf{n} \sqrt{1 - \frac{n_i^2(1 - (\mathbf{d} \cdot \mathbf{n})^2)}{n_o^2}}$
- $\cos^2 \theta_o = 1 - \frac{n_i^2(1 - \cos^2 \theta_i)}{n_o^2}$





# Specular Refraction

- The transmitting ray's direction  $\mathbf{t}$
- $$\mathbf{t} = \frac{n_i(\mathbf{d} - \mathbf{n}(\mathbf{d} \cdot \mathbf{n}))}{n_o} - \mathbf{n} \sqrt{1 - \frac{n_i^2(1 - (\mathbf{d} \cdot \mathbf{n})^2)}{n_o^2}}$$
- $1 - \frac{n_i^2(1 - (\mathbf{d} \cdot \mathbf{n})^2)}{n_o^2} < 0$ 
  - No refracted ray (i.e., all energy is reflected)
- Q. what are the amount of reflected and refracted energy (i.e., reflectivity)?

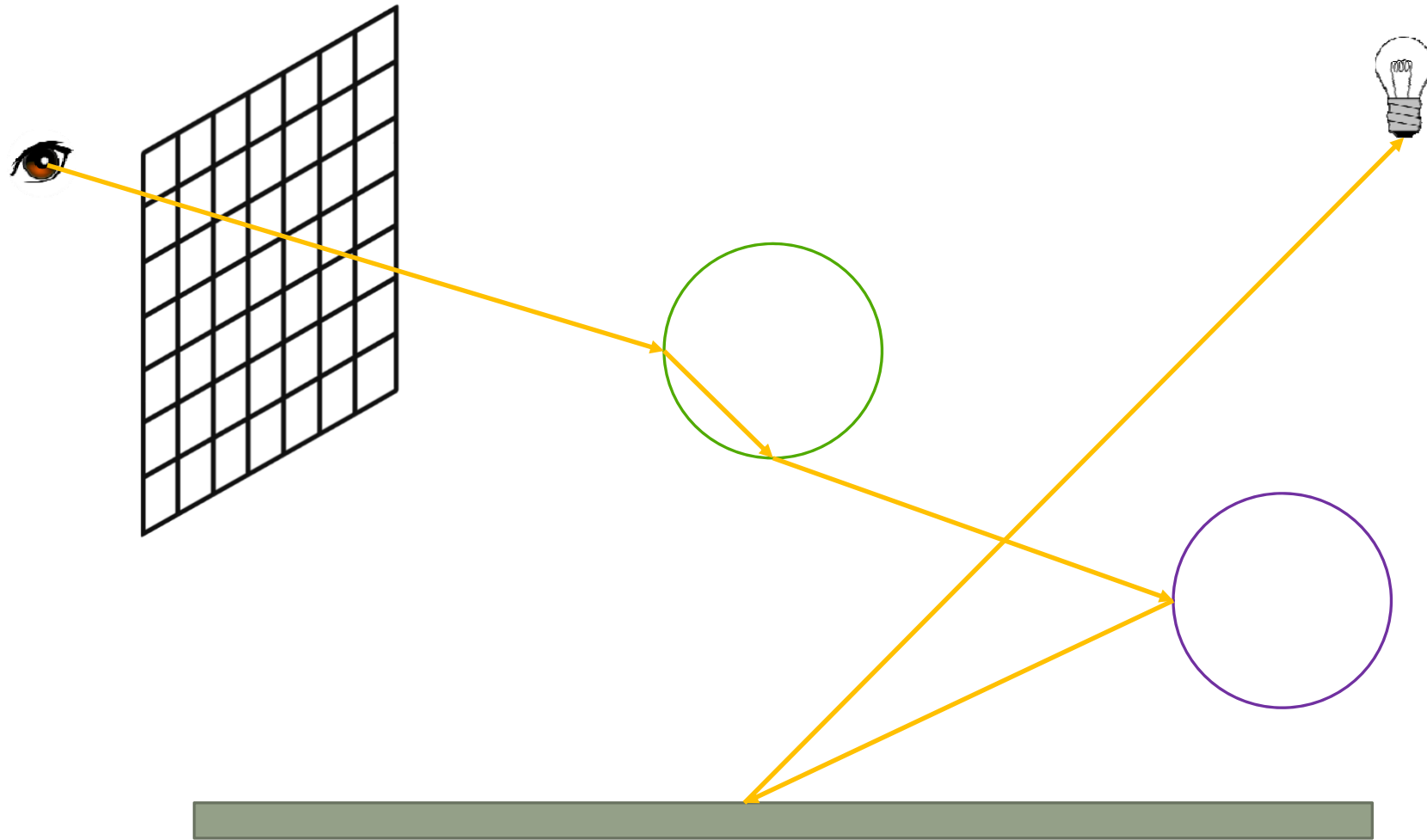


# Specular Refraction

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- The reflectivity varies with the incident direction of light and it is determined by *Fresnel equations* (from physics).
- Schlick approximation (1994) of the Fresnel equations
  - $R(\theta_i) = R_0 + (1 - R_0)(1 - \cos\theta_i)^5$
  - $R_0 = \left(\frac{n_o - 1}{n_o + 1}\right)^2$

# Whitted Ray Tracing



# Whitted Ray Tracing

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- For each pixel
  - Color  $c = (0, 0, 0)$
  - Generate a primary ray (camera ray)
  - Find the closest intersection point between the ray and objects
  - If (there is a hit) then
    - Generate a shadow ray
    - If (there is no hit between the shadow ray and a light) then
      - $c = c + \text{shading}()$
    - Generate a secondary ray (reflection or refraction ray)
    - Go to the third line
  - Else
    - $c = c + \text{background color}$
  - Set the pixel color with  $c$
- Issue: cannot trace a ray path that has infinite depth

# Whitted Ray Tracing

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- For each pixel
  - Color  $c = (0, 0, 0)$
  - Generate a primary ray (with depth 0)
  - While (depth < d)
    - Find the closest intersection point between the ray and objects
    - If (there is a hit) then
      - Generate a shadow ray
      - If (there is no hit between the shadow ray and a light) then
        - $c = c + \text{shading}()$
      - Generate a secondary ray (reflection or refraction ray) // increase the ray depth +1
    - Else
      - $c = c + \text{background color}$
  - Set the pixel color with  $c$

# Whitted Ray Tracing

- Missing effects?
  - Soft shadow
  - Glossy reflection
  - Diffuse reflection
  - Depth-of-field effect
  - Motion blur
  - ...
- Advanced ray tracing
  - Distributed ray tracing
  - Path tracing and photon mapping



Whitted (1980)

# Further Readings

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- Chapter 4, 13
- Papers
  - T. Whitted. An improved illumination model for shading display. Communications of the ACM, 23(6):343–349, 1980