

CT4201/EC4215: Computer Graphics

# Transformation

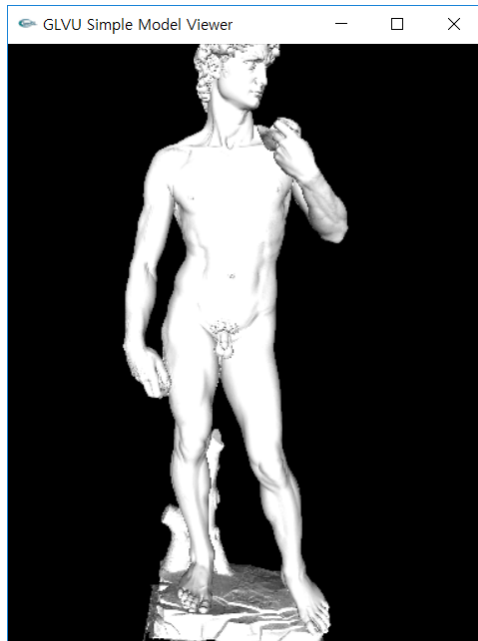
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BOCHANG MOON



# Transformation

- Fundamental operation to arrange objects in a 3D scene



Object



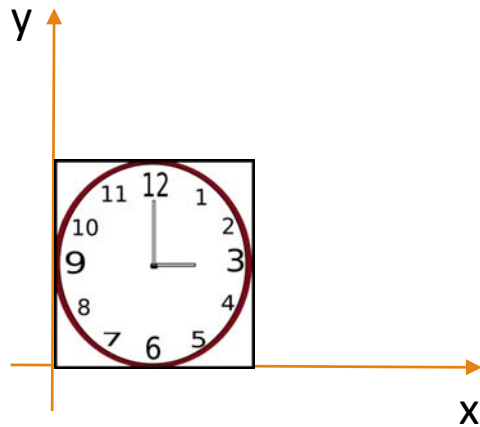
3D scene

# Transformation: Scaling

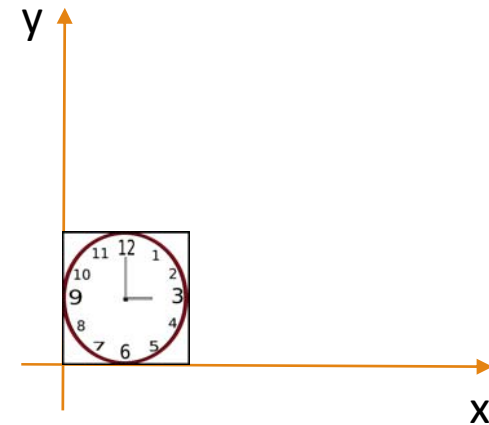
- Change length along the coordinate axes

- $scale(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$

- $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$



$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

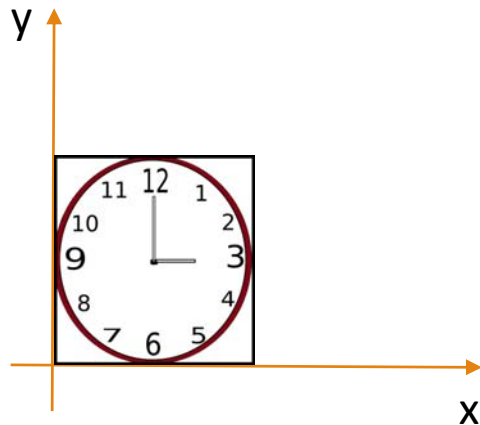


# Transformation: Scaling

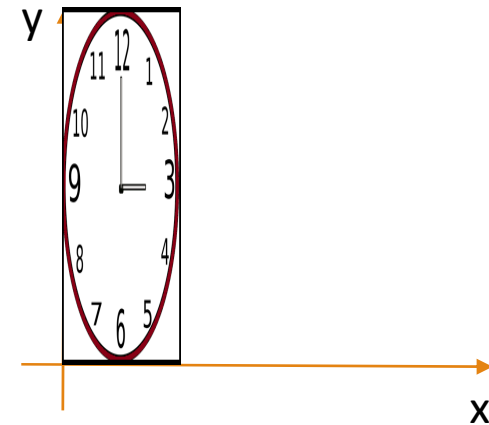
- Change length along the coordinate axes

- $scale(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$

- $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$



$$\begin{bmatrix} 0.5 & 0 \\ 0 & 2.0 \end{bmatrix}$$

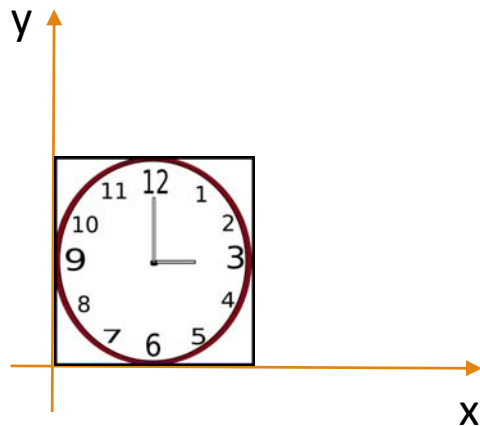


# Transformation: Shearing

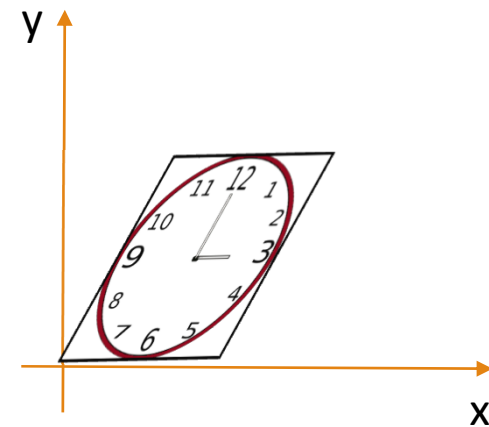
- Push objects sideways along horizontal or vertical direction

- $shear_x(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$

- $shear_y(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$



$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

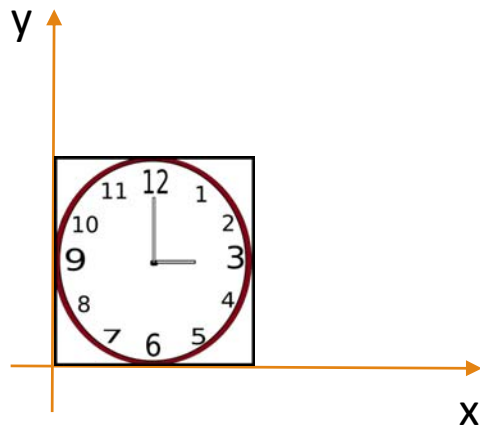


# Transformation: Shearing

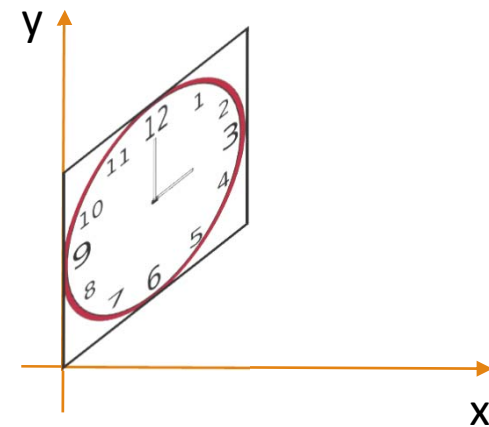
- Push objects sideways along horizontal or vertical direction

- $shear_x(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$

- $shear_y(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$



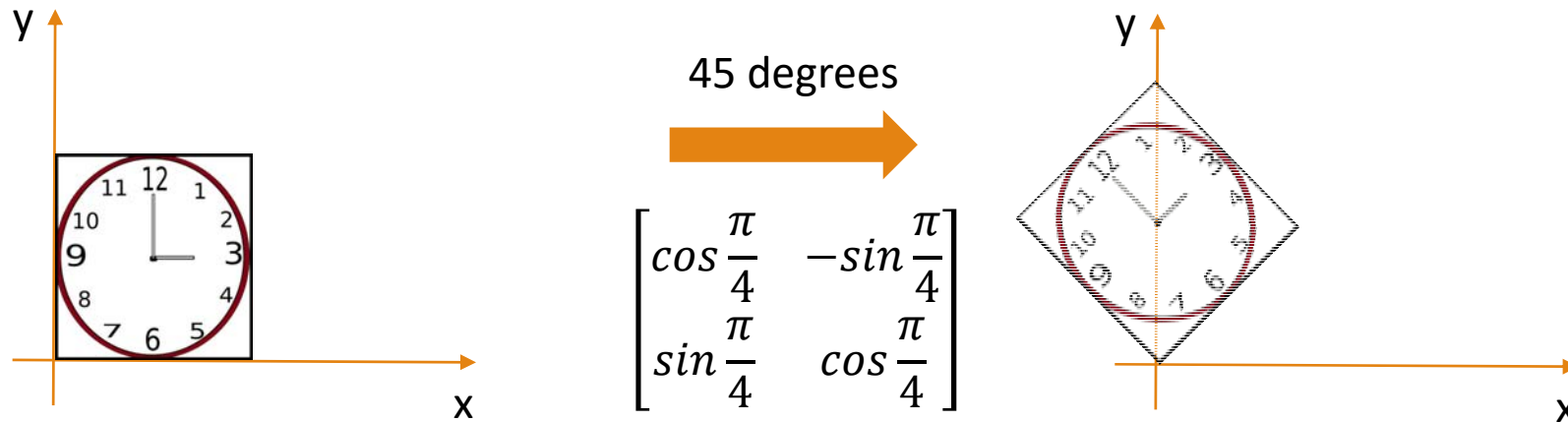
$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$



# Transformation: Rotation

- Rotate objects counterclockwise

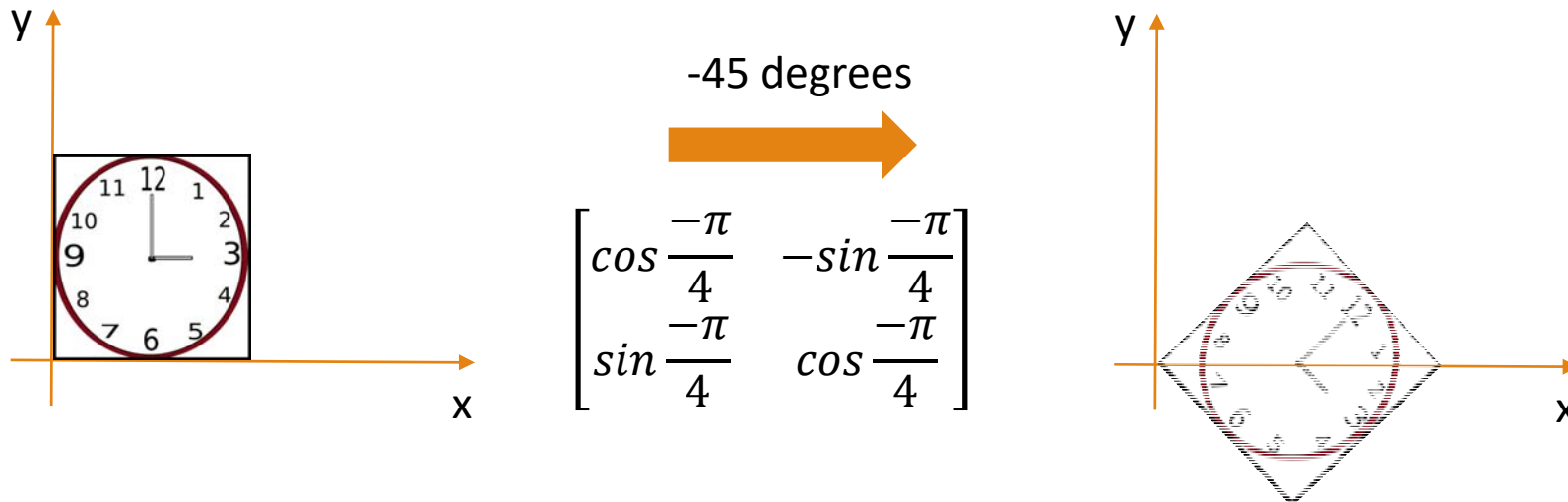
- $rotate(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$



# Transformation: Rotation

- Rotate objects counterclockwise

- $rotate(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

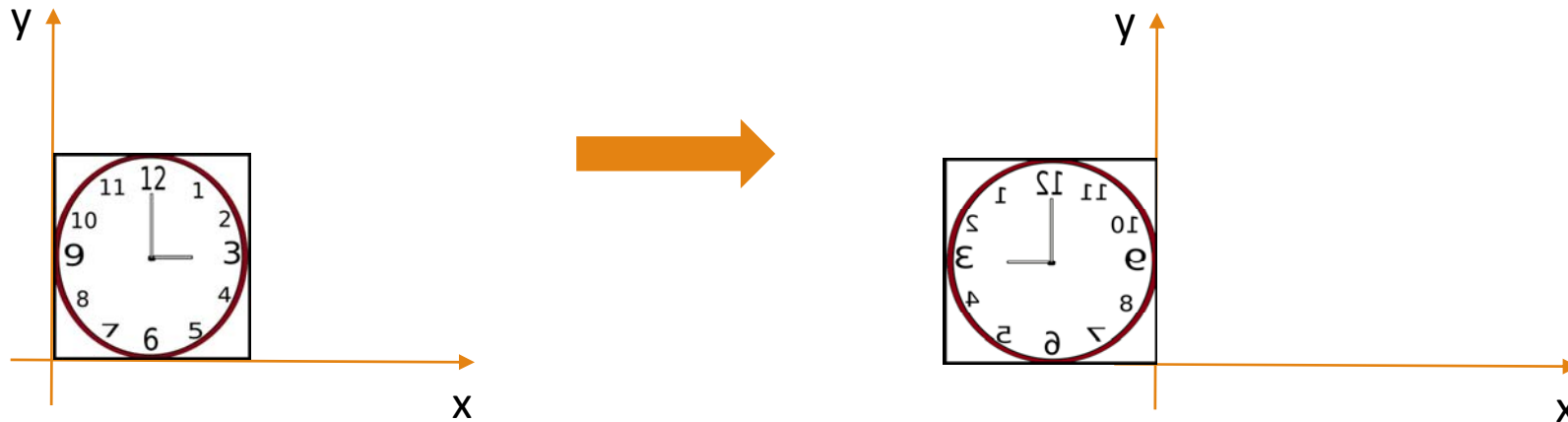




# Transformation: Reflection

- Reflect objects across either of the coordinate axes

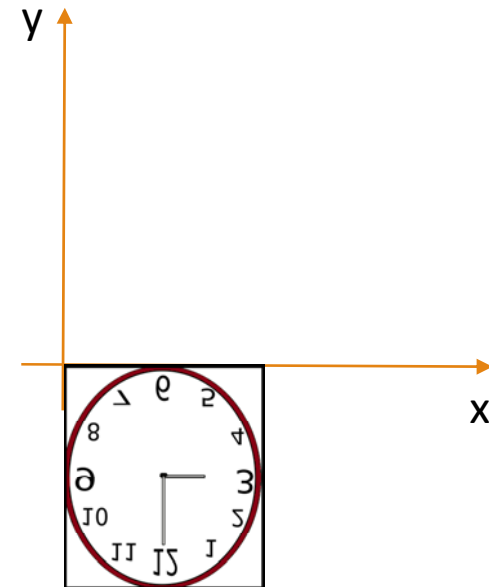
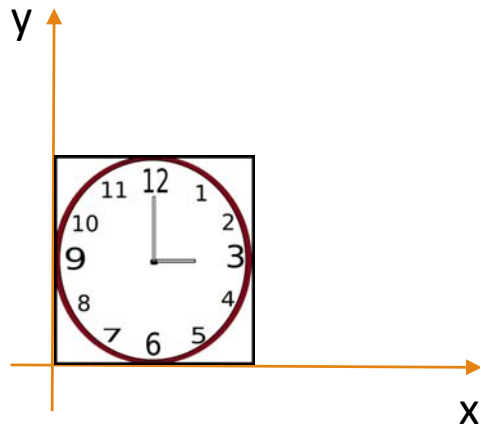
- $reflect\_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$



# Transformation: Reflection

- Reflect objects across either of the coordinate axes

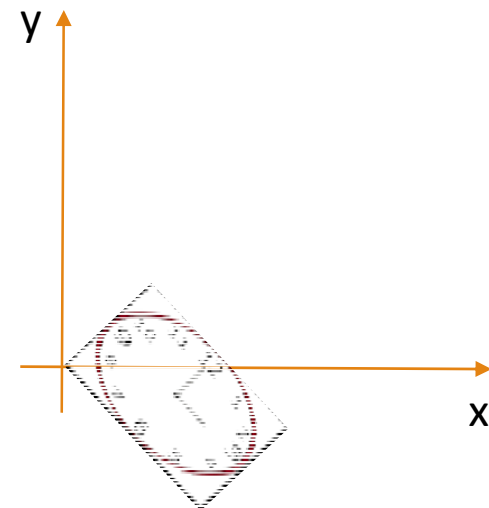
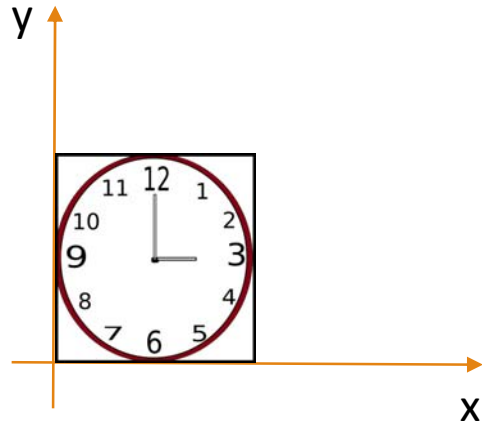
- $reflect_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$



# Transformation: Composition

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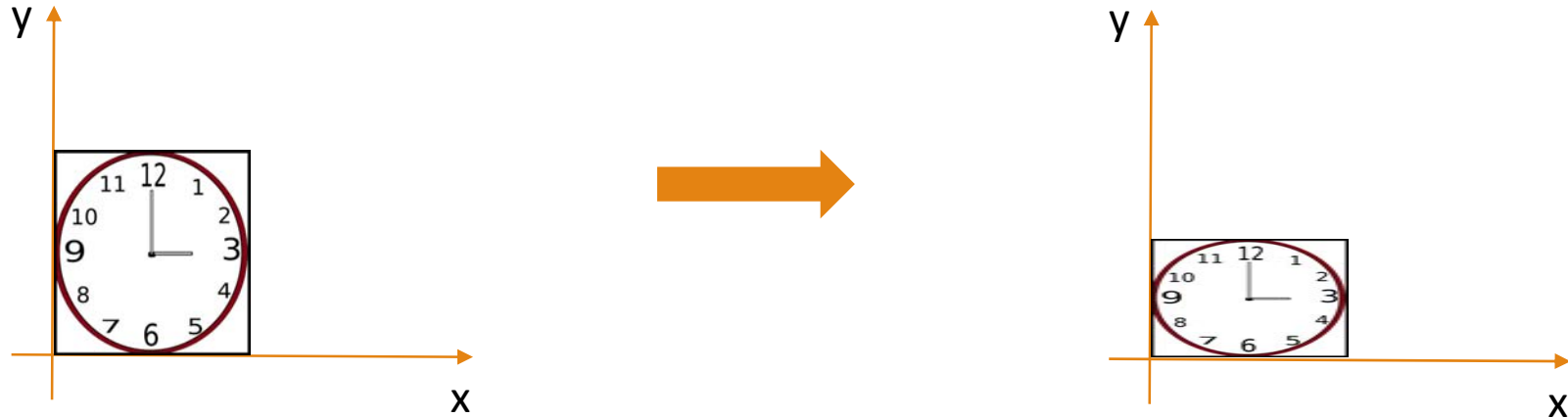
- Apply more than one transformation to an object
- e.g. apply a scale  $S$ , and then a rotation  $R$ 
  - $\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = RS \begin{bmatrix} x_{old} \\ y_{old} \end{bmatrix}$



# Transformation: Composition

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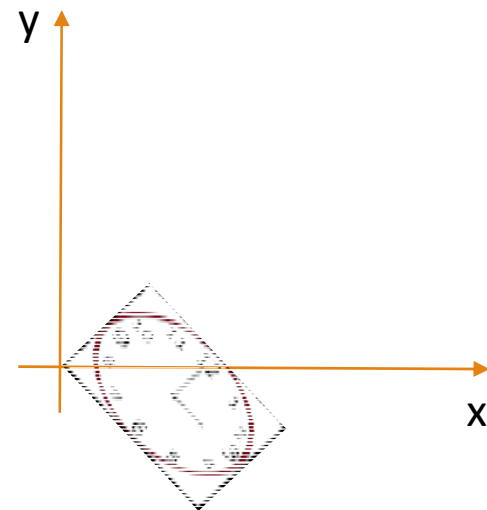
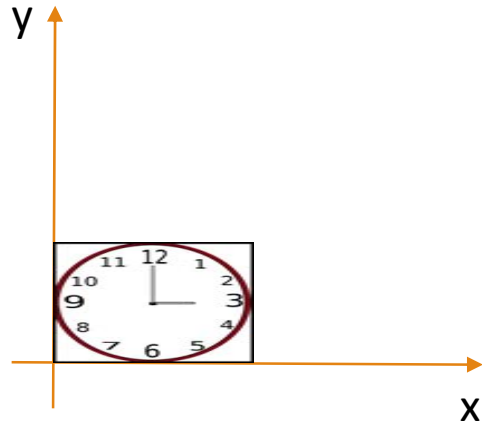
- Apply more than one transformation to an object
- e.g. apply a scale  $S$ , and then a rotation  $R$ 
  - $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S \begin{bmatrix} x_{old} \\ y_{old} \end{bmatrix}$



# Transformation: Composition

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- Apply more than one transformation to an object
- e.g. apply a scale  $S$ , and then a rotation  $R$ 
  - $\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = R \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$



# Inverse Transformation

- Your transformation
  - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
  - $x_{old} = T^{-1}x_{new}$



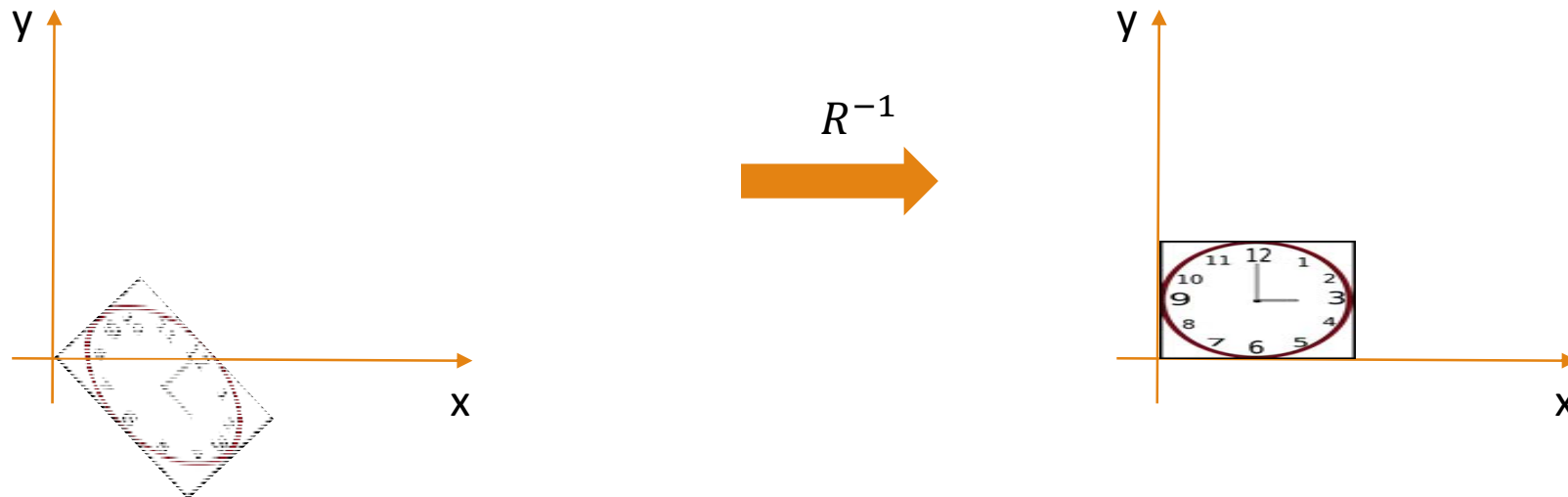
# Inverse Transformation

- Your transformation
  - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
  - $x_{old} = T^{-1}x_{new} = (RS)^{-1}x_{new} = S^{-1}R^{-1}x_{new}$



# Inverse Transformation

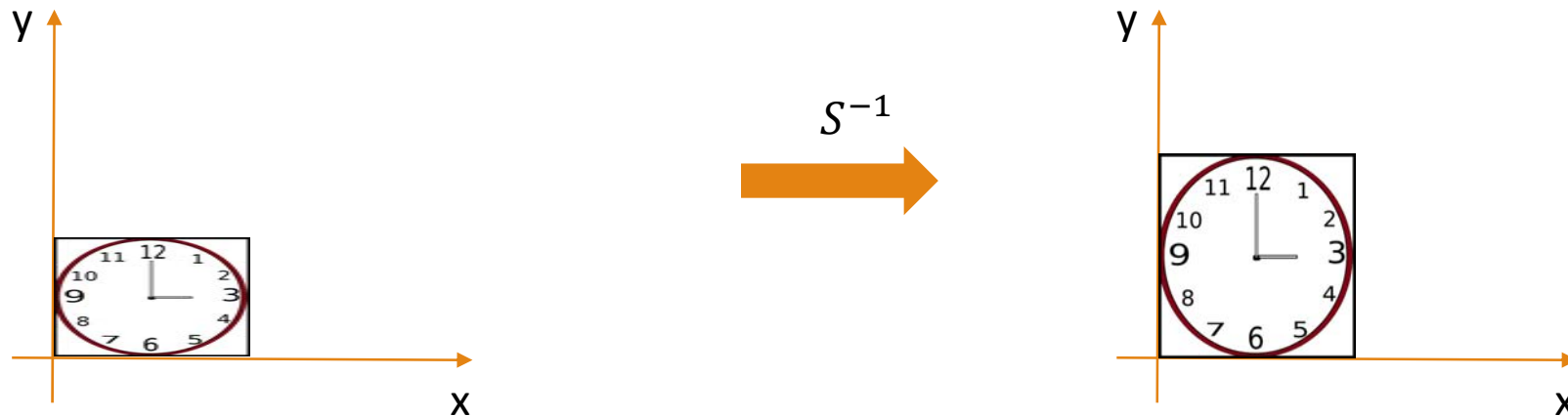
- Your transformation
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- Undo your transformation
  - $x_{old} = T^{-1}x_{new} = (RS)^{-1}x_{new} = S^{-1}R^{-1}x_{new}$





# Inverse Transformation

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  - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
  - $x_{old} = T^{-1}x_{new} = (RS)^{-1}x_{new} = S^{-1}R^{-1}x_{new}$



# Inverse Matrix

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- $M = 2 \times 2$  matrix
  - $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$
  - $M^{-1} = \frac{1}{M_{11} \times M_{22} - M_{12} \times M_{21}} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix}$