

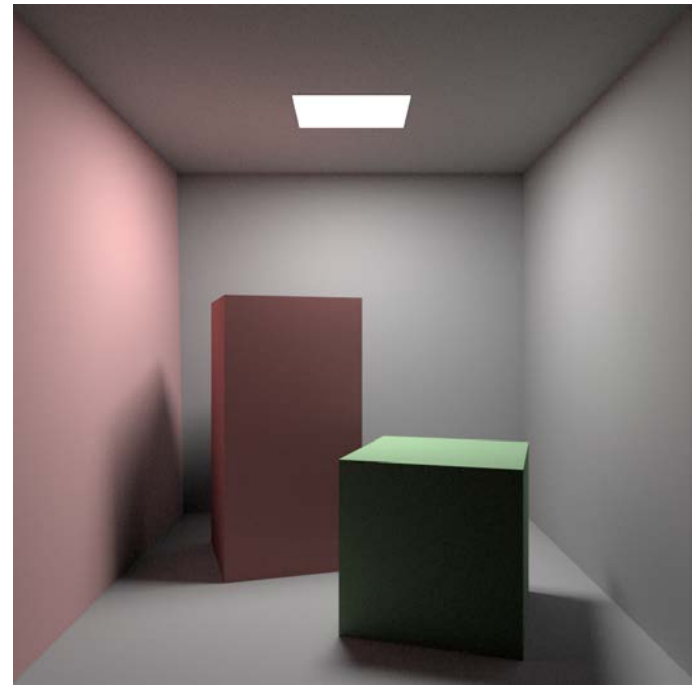
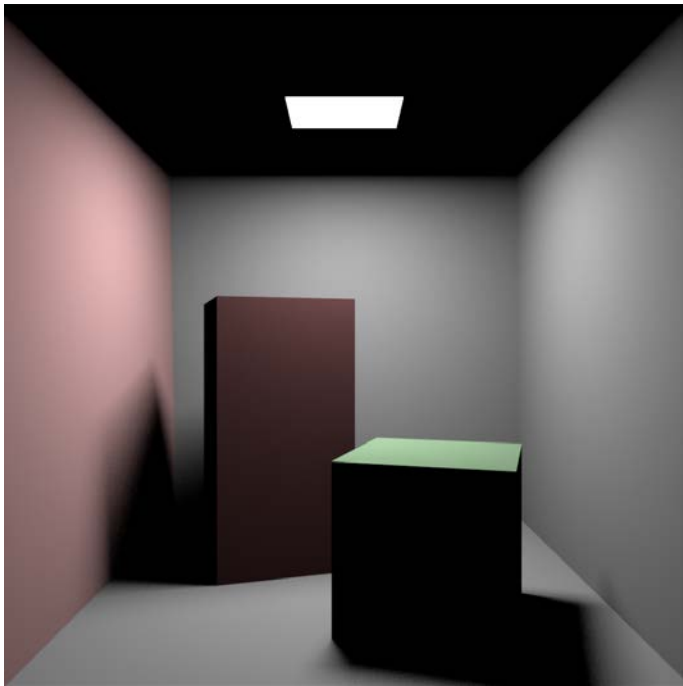
CT4510: Computer Graphics

Global Illumination Methods

BOCHANG MOON

Global Illumination

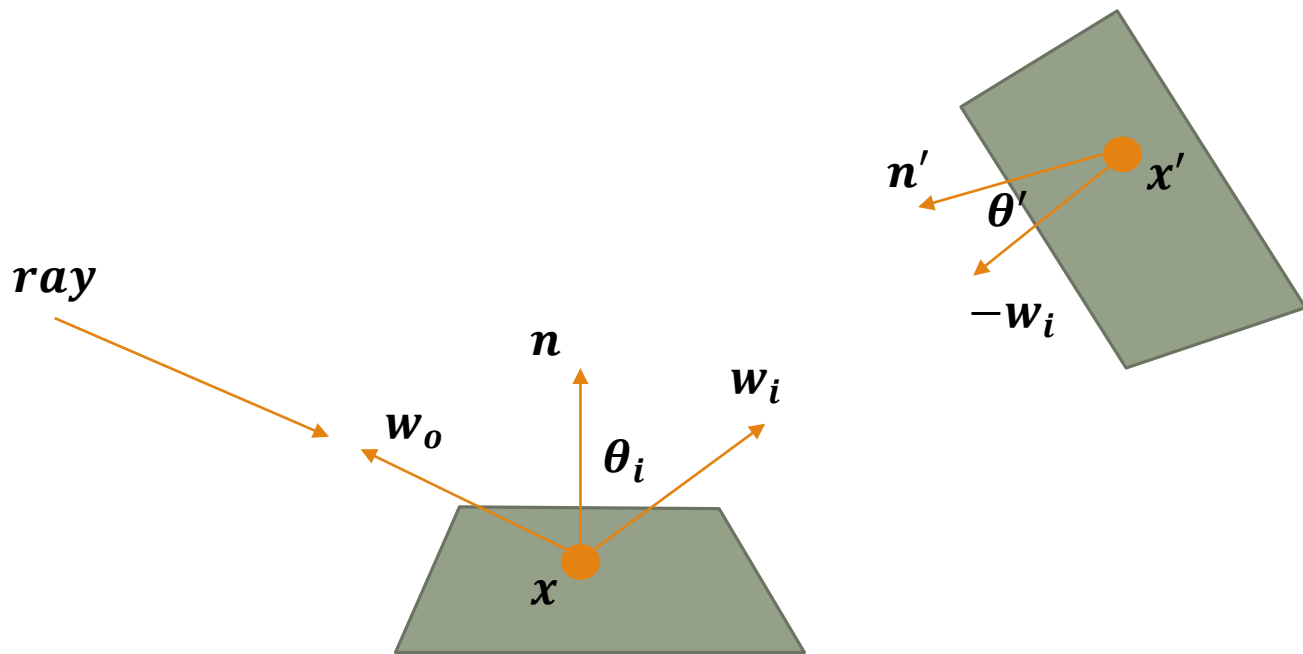
- Distributed ray tracing introduces a noticeable improvement on rendering quality, but it still misses some rendering effects



Global Illumination

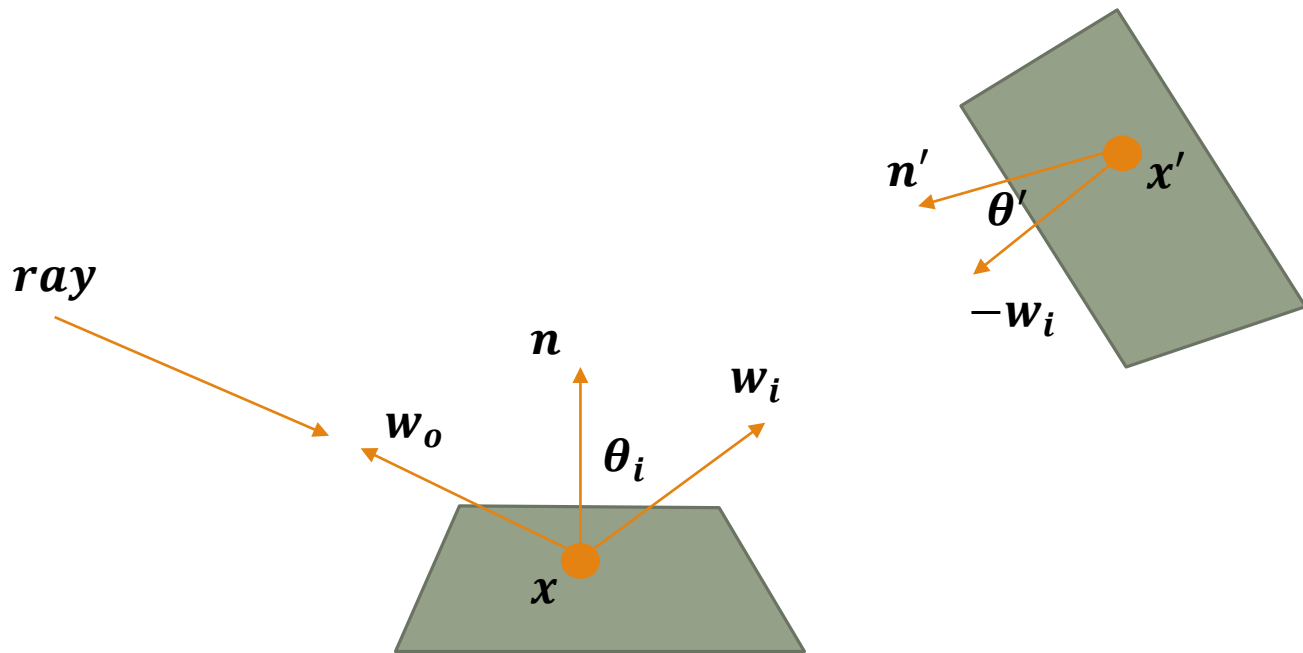
- Rendering equation [Kajiya 86] (another form: [Immel and Cohen 86])

- $$L_S(\mathbf{x}, \mathbf{w}_o) = \int_{\text{all } \mathbf{x}'} \frac{\rho(\mathbf{w}_i, \mathbf{w}_o) L_S(\mathbf{x}', \mathbf{x} - \mathbf{x}') v(\mathbf{x}, \mathbf{x}') \cos\theta_i \cos\theta'}{\|\mathbf{x} - \mathbf{x}'\|^2} dA$$



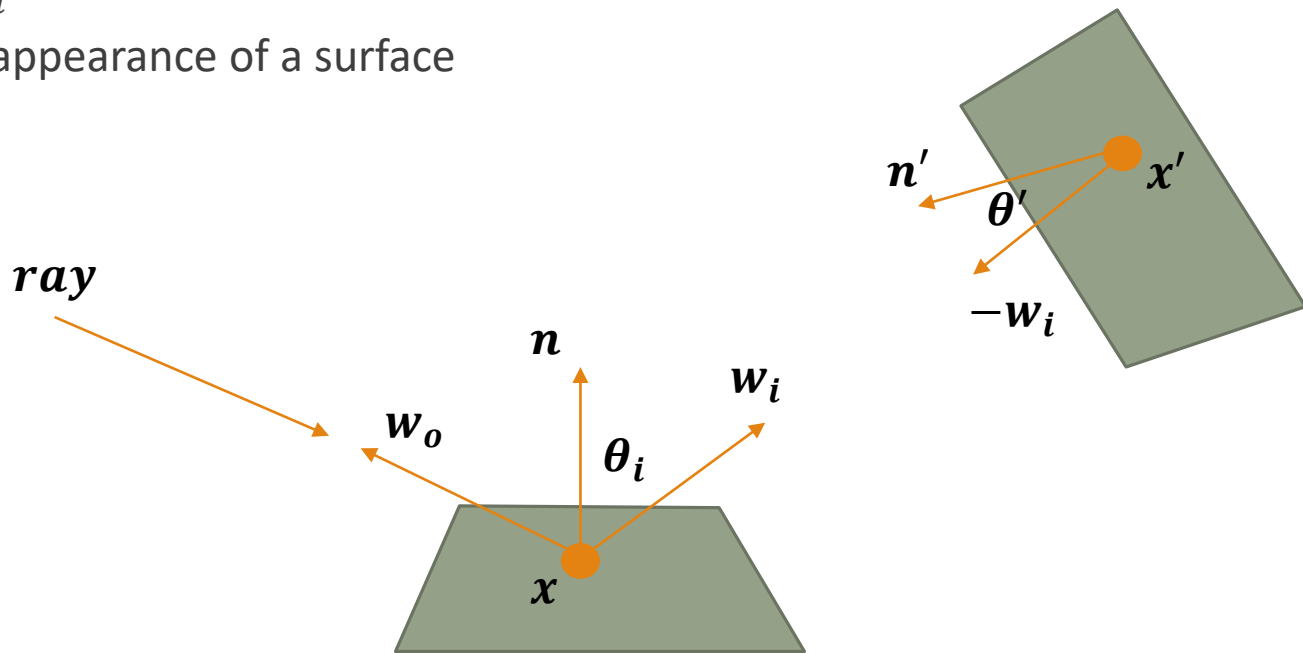
Global Illumination

- $L_S(\mathbf{x}, \mathbf{w}_o) = \int_{\text{all } \mathbf{x}'} \frac{\rho(\mathbf{w}_i, \mathbf{w}_o) L_S(\mathbf{x}', \mathbf{x} - \mathbf{x}') v(\mathbf{x}, \mathbf{x}') \cos\theta_i \cos\theta'}{\|\mathbf{x} - \mathbf{x}'\|^2} dA$
- $v(\mathbf{x}, \mathbf{x}')$: visibility term
 - 1 if two points are mutually visible
 - 0 otherwise



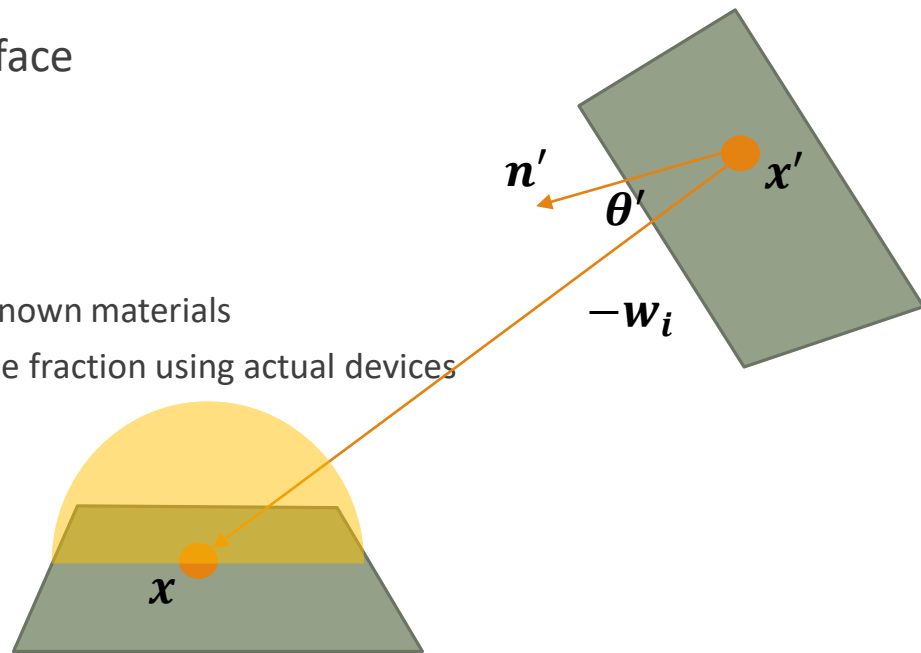
Global Illumination

- $L_S(\mathbf{x}, \mathbf{w}_o) = \int_{\text{all } x'} \frac{\rho(\mathbf{w}_i, \mathbf{w}_o) L_S(x', x-x') v(x, x') \cos\theta_i \cos\theta'}{\|x-x'\|^2} dA$
- $\rho(\mathbf{w}_i, \mathbf{w}_o)$: bidirectional reflectance distribution function (BRDF)
 - Reflected light with an outgoing direction \mathbf{w}_o / Incident light with an incoming direction \mathbf{w}_i
 - Determine appearance of a surface



Global Illumination

- $L_S(\mathbf{x}, \mathbf{w}_o) = \int_{\text{all } x'} \frac{\rho(\mathbf{w}_i, \mathbf{w}_o) L_S(x', x-x') v(x, x') \cos\theta_i \cos\theta'}{\|x-x'\|^2} dA$
- $\rho(\mathbf{w}_i, \mathbf{w}_o)$: bidirectional reflectance distribution function (BRDF)
 - Reflected light with an outgoing direction \mathbf{w}_o / Incident light with an incoming direction \mathbf{w}_i
 - Determine appearance of a surface
 - e.g. ideal diffuse surface
 - $\rho(\mathbf{w}_i, \mathbf{w}_o) = \rho$ (constant)
 - e.g. other surfaces?
 - Utilize a mathematical form for well-known materials
 - Build the 4D function by measuring the fraction using actual devices



Global Illumination

- $L_S(\mathbf{x}, w_o) = \int_{all\ x'} \frac{\rho(w_i, w_o) L_S(x', x-x') v(x, x') \cos\theta_i \cos\theta'}{\|x-x'\|^2} dA$
- How can we solve the integral?

Monte Carlo Integration

- A sampling method based on probability
- Draw random numbers and approximate an integral
- e.g., evaluate a 1D integral: $\int_a^b f(x)dx$
- $F_n = \frac{b-a}{n} \sum_{i=1}^n f(x_i)$
- $x_i \in [a, b]$: uniform random variable, $p(x) = \frac{1}{b-a}$
- $E[F_n] = \frac{b-a}{n} \sum_{i=1}^n E[f(x_i)]$
- $= \frac{b-a}{n} \sum_{i=1}^n \int_a^b f(x)p(x)dx$
- $= \frac{1}{n} \sum_{i=1}^n \int_a^b f(x)dx$
- $= \int_a^b f(x)dx$

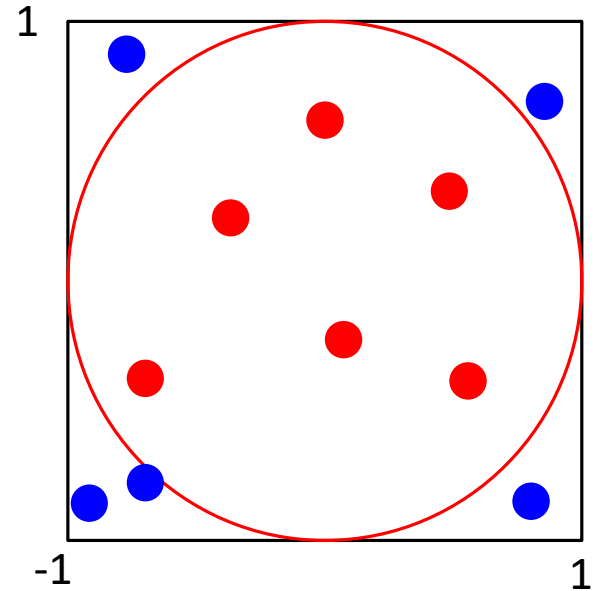
Monte Carlo Integration

- A sampling method based on probability
- Draw random numbers and approximate an integral
- e.g., evaluate a 1D integral: $\int_a^b f(x)dx$
- $F_n = \frac{b-a}{n} \sum_{i=1}^n f(x_i)$
- $x_i \in [a, b]$: from a probability density function $p(x)$
 - $F_n = \frac{1}{n} \sum_{i=1}^n \frac{f(x_i)}{p(x_i)}$
 - Indicate that we can control sampling density by adjusting $p(x)$
 - Require a good sampling strategy (e.g., *important sampling* – a research area)
 - The approximation error decreases as the number of samples increases

Monte Carlo Integration

- $f(x, y) = 1$ if $x^2 + y^2 \leq 1$
- $= 0$ otherwise

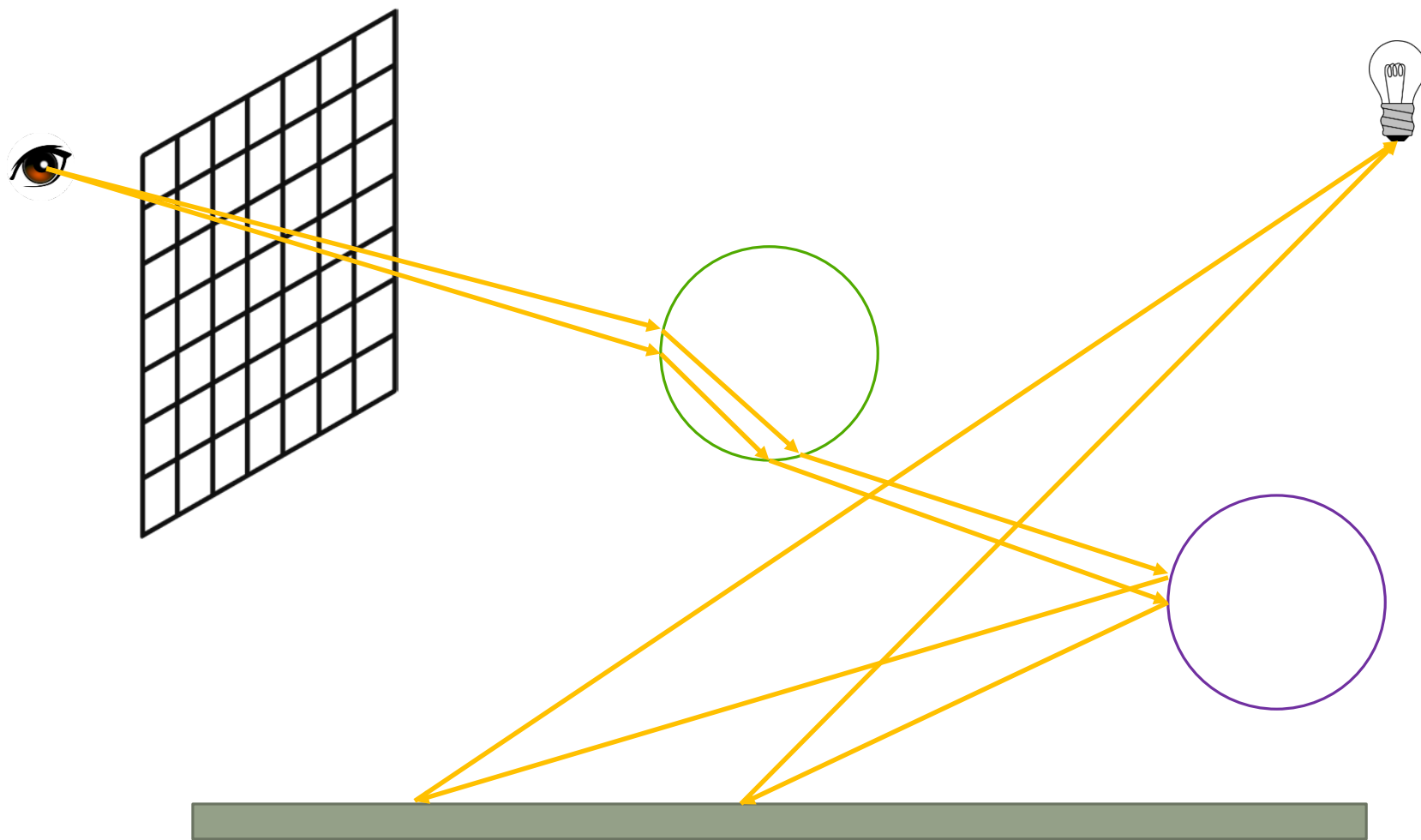
- $I = \int_{\Omega} f(x, y) dx dy = \pi$
- $\Omega = [-1, -1] \times [1, 1]$
- $I \approx \frac{V}{n} \sum_{i=1}^n f(x_i, y_i)$
- V is the volume of the Ω
 - $V = 4$
- (x_i, y_i) has a uniform pdf



Monte Carlo Integration

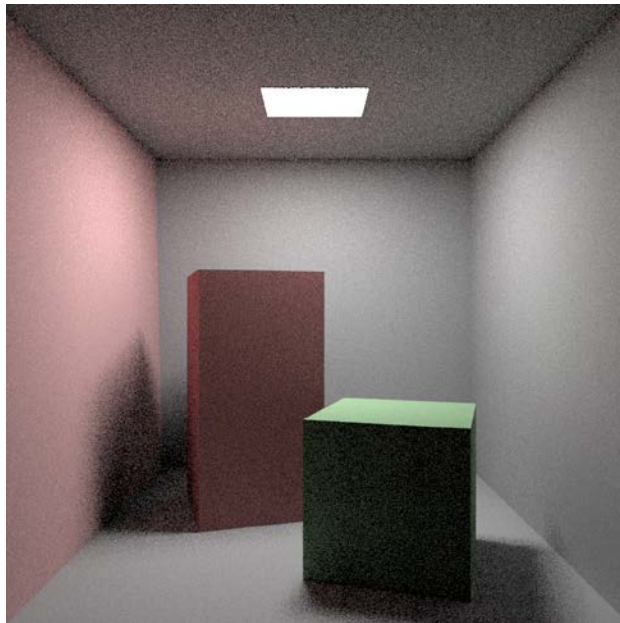
- $L_S(\mathbf{x}, w_o) = \int_{all\ x'} \frac{\rho(w_i, w_o) L_S(x', x-x') v(x, x') \cos\theta_i \cos\theta'}{\|x-x'\|^2} dA$
- Monte Carlo ray tracing
 - e.g., distributed ray tracing, path tracing
 - A random sample x_i is a high-dimensional vector
 - $x_i =$
[*random numbers on image plane, lens, time, area lights, for secondary rays*]
 - Essentially the integral is a high-dimensional integral
 - The pixel color is an output by averaging the colors from multiple light paths

Path Tracing

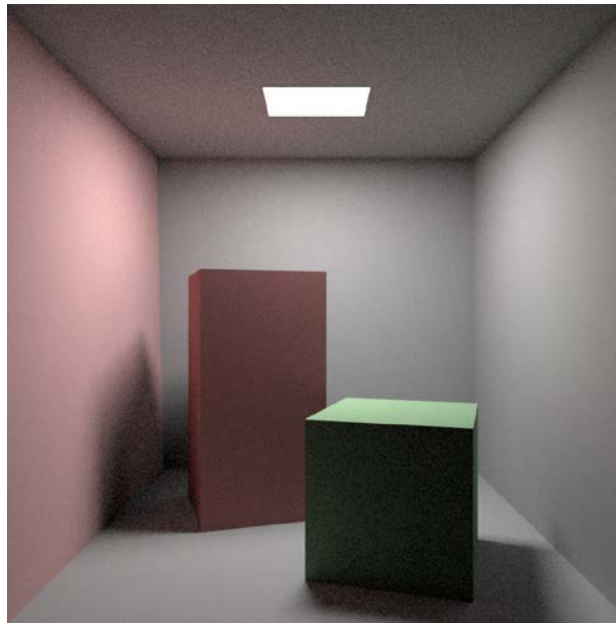


Path Tracing

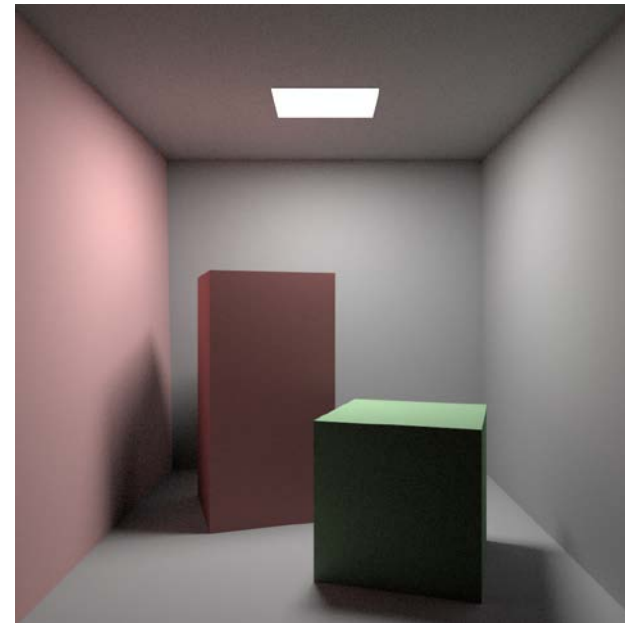
- A general rendering method that solves the full light transport equation (i.e., rendering equation)
- For each pixel color, it makes multiple ray paths, then averages the colors from the ray paths



4 samples / pixel (1.25 secs)



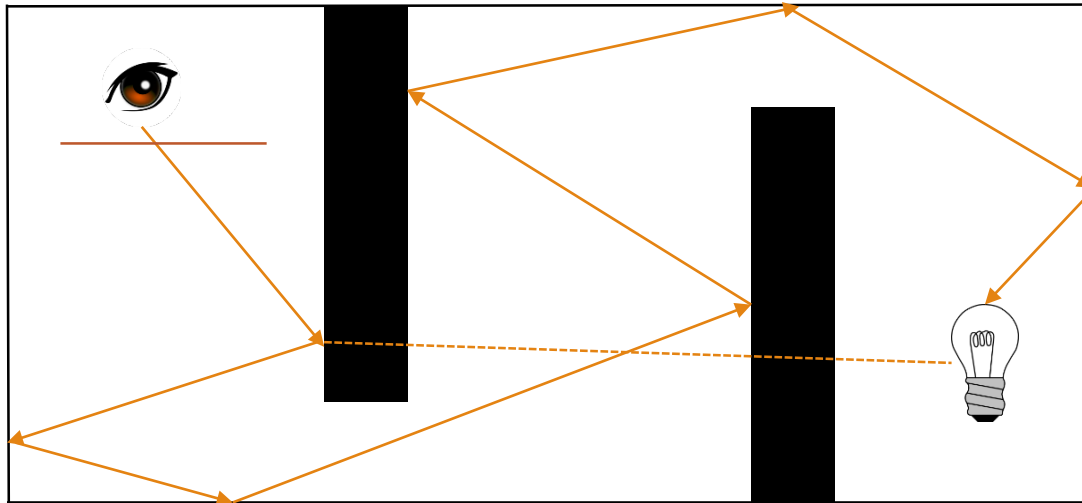
16 samples / pixel (5 secs)



64 samples / pixel (20 secs)

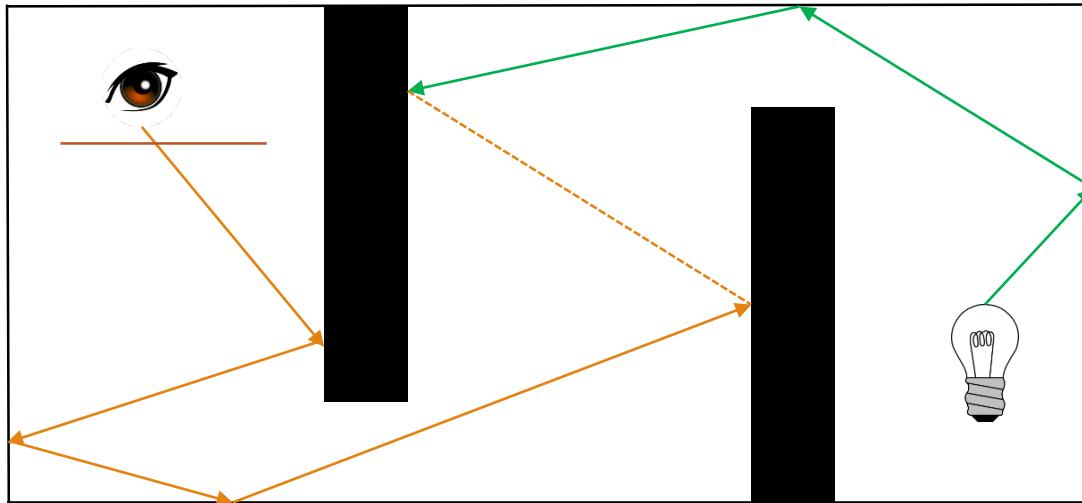
Path Tracing

- Find each light path starting from eye
 - Can be inefficient when light paths are difficult to reach from the eye



Bidirectional Path Tracing

- Find each light sub-path from eye and light
- Connect the sub-paths





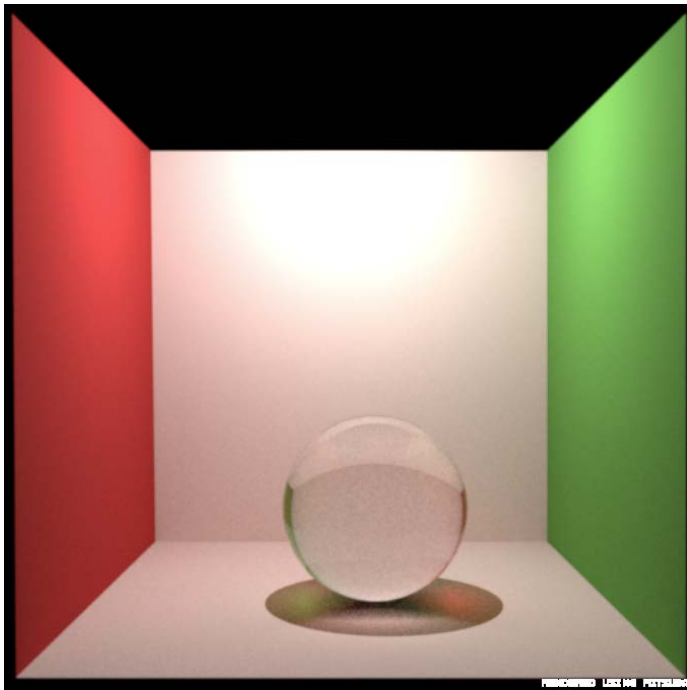
Path tracing, 64 samples per pixel



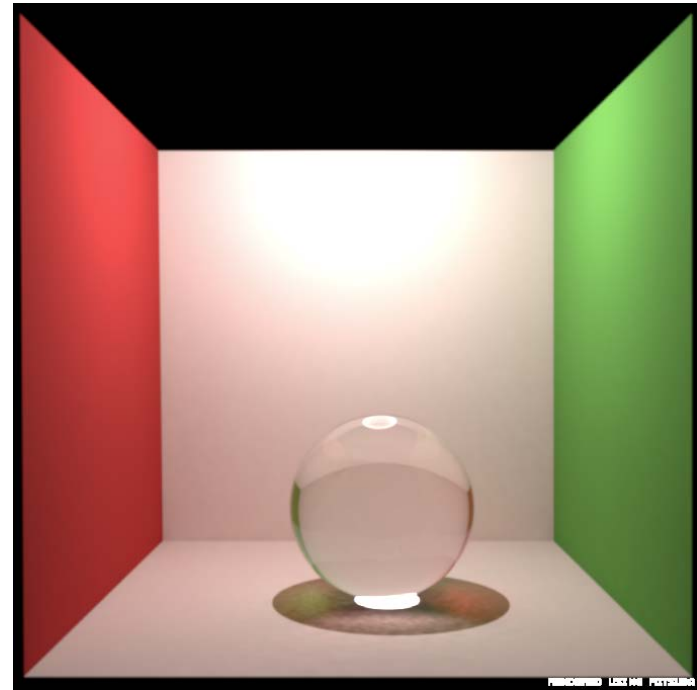
Bidirectional Path tracing, 64 samples per pixel

Photon Mapping

- [Jensen 1996]
- Motivation: it is hard to simulate some rendering effects (e.g., caustics) unless a large number of ray samples are used



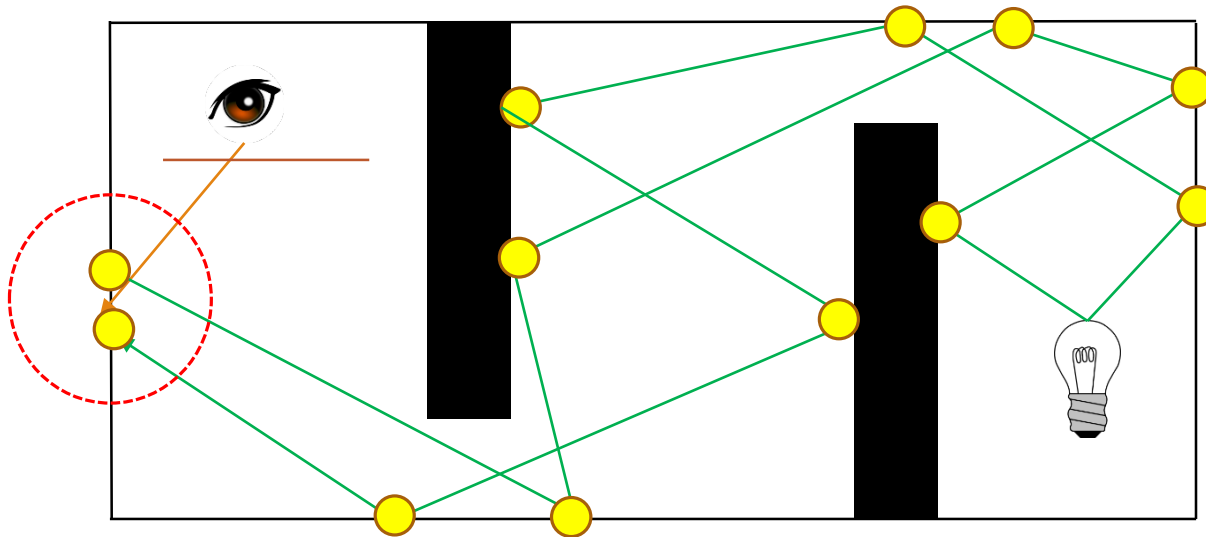
Path tracing



Photon mapping

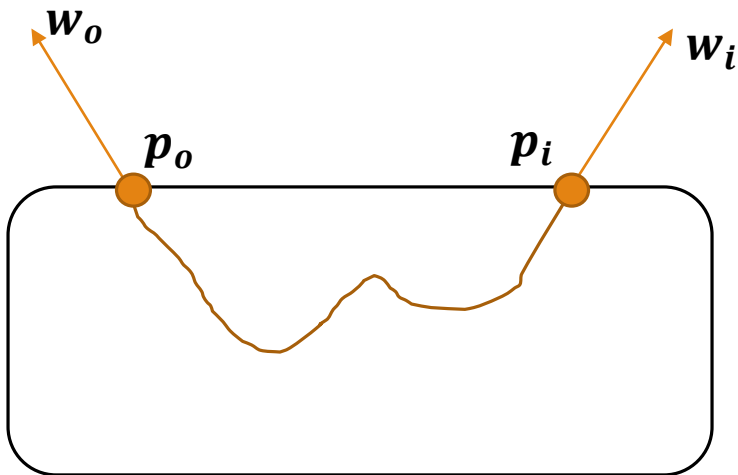
Photon Mapping

- Generate photons from each light and store them in kd-tree
- Trace primary rays and estimate a photon density at intersected points
 - Similar to the bidirectional path tracing, but the stored photons can be utilized for all eye rays (why?)



Advanced Topics

- Modeling complex materials
 - Bidirectional scattering-surface reflectance distribution function (BSSRDF)
- $\rho(\mathbf{w}_i, \mathbf{w}_o)$: BRDF
- $\rho(\mathbf{p}_i, \mathbf{w}_i, \mathbf{p}_o, \mathbf{w}_o)$: BSSRDF



Human head model with a BSSRDF

Model courtesy of Infinite Realities, Inc.

Image comes from pbrt.org

Advanced Topics

- Complex geometry (e.g., hair, fur) and scattering



[Jakob et al. 2010]



Model courtesy of Cem Yuksel
Image comes from pbrt.org

Advanced Topics

- Importance Sampling

- $F_n = \frac{1}{n} \sum_{i=1}^n \frac{f(x_i)}{p(x_i)}$

- Can we estimate an optimal pdf instead of the uniform distribution?

- Denoising



Path tracing,
16 samples per pixel



Denoised image,
16 samples per pixel

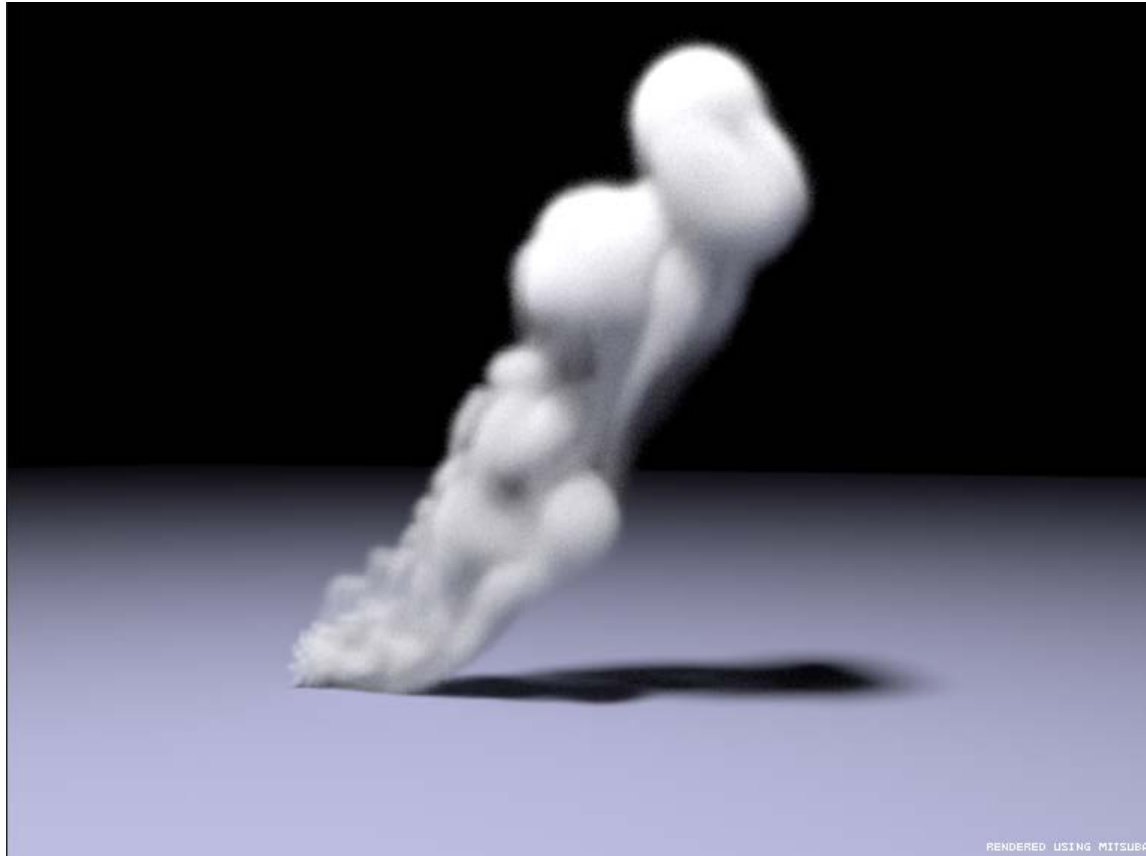


Path tracing
32K samples per pixel

Images from [Moon et al. 2016]

Advanced Topics

- Volume rendering



Graphics Topics

- Graphics Areas
 - Rendering
 - Modeling
 - Animation
 - User interaction
 - Augmented & virtual reality
 - Visualization
 - etc.
- Check out recent siggraph papers to know the active areas