

CT4510: Computer Graphics

Whitted Ray Tracing

BOCHANG MOON

Whitted Ray Tracing

- Recursively perform the ray casting to simulate refraction and reflection



Whitted (1980)

An Improved Illumination Model [Whitted 80]

- Phong illumination model

- $$I = \sum_{i=1}^{\# \text{ of lights}} \{L_a^i k_a + L_d^i k_d \max(0, \mathbf{n} \cdot \mathbf{l}^i) + L_s^i k_s \max(0, \mathbf{r}^i \cdot \mathbf{v})^s\}$$

- Whitted illumination model

- $$I = \sum_{i=1}^{\# \text{ of lights}} \{L_a^i k_a + L_d^i k_d \max(0, \mathbf{n} \cdot \mathbf{l}^i)\} + k_s S + k_t T$$

- S: intensity of light coming from a reflection ray

- T: intensity of light that comes from a transmission ray

- k_s, k_t : coefficients for specular and transmission

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- S: intensity of light coming from a reflection ray

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- Q. how we can compute the rays?

- Note that we need to generate the secondary rays to find S and T

Specular Reflection

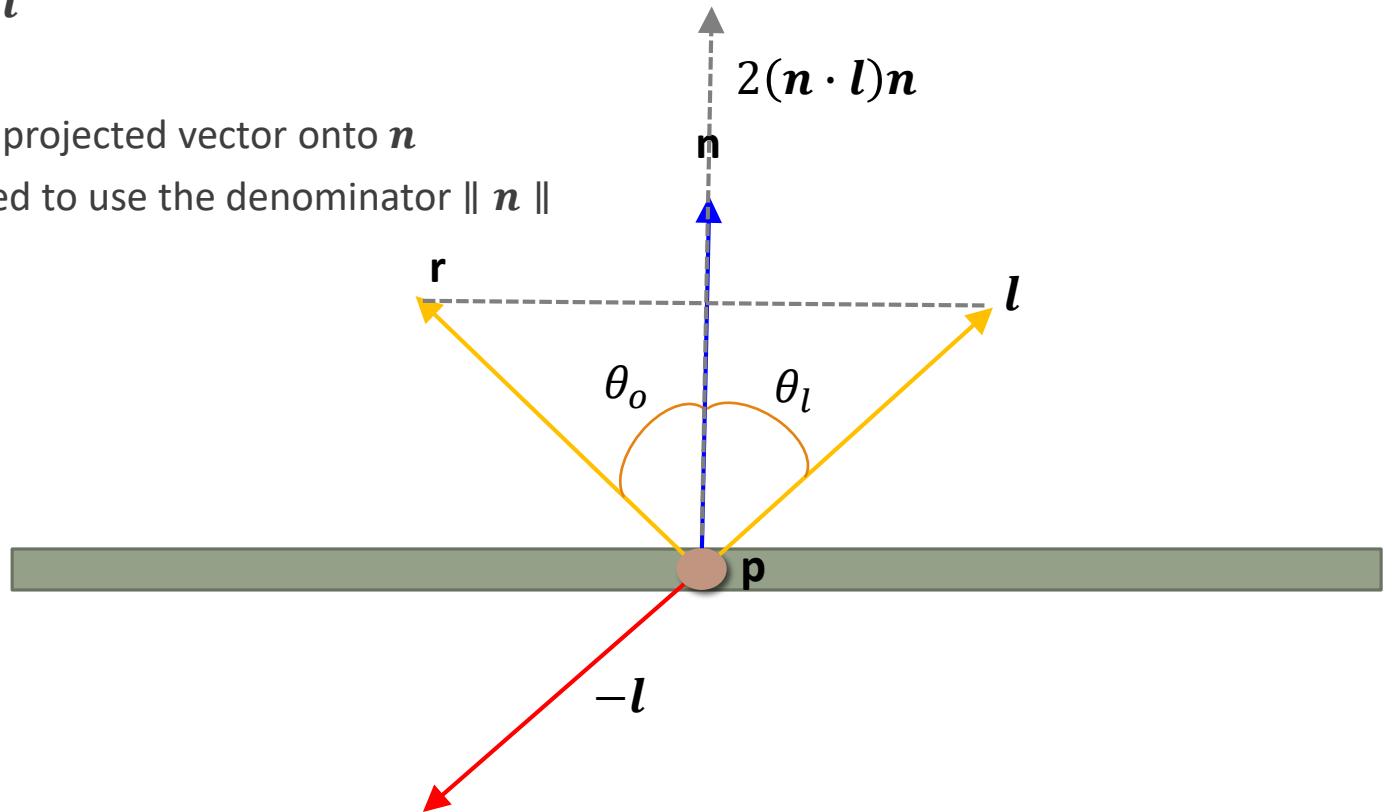
- The reflection vector, r , can be computed as the following:

- $r = 2(n \cdot l)n - l$

- $n \cdot l$: length of the projected vector onto n

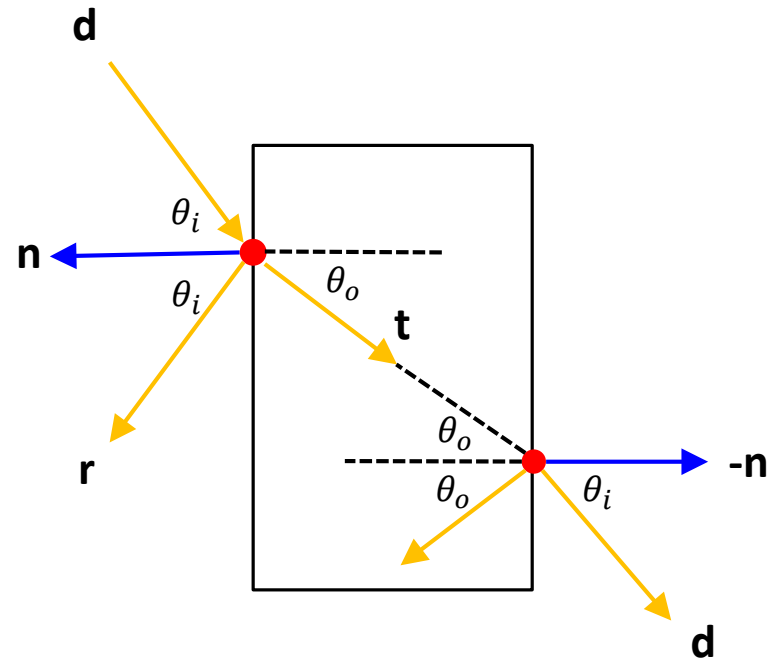
- Note: we don't need to use the denominator $\|n\|$

- e.g., mirror



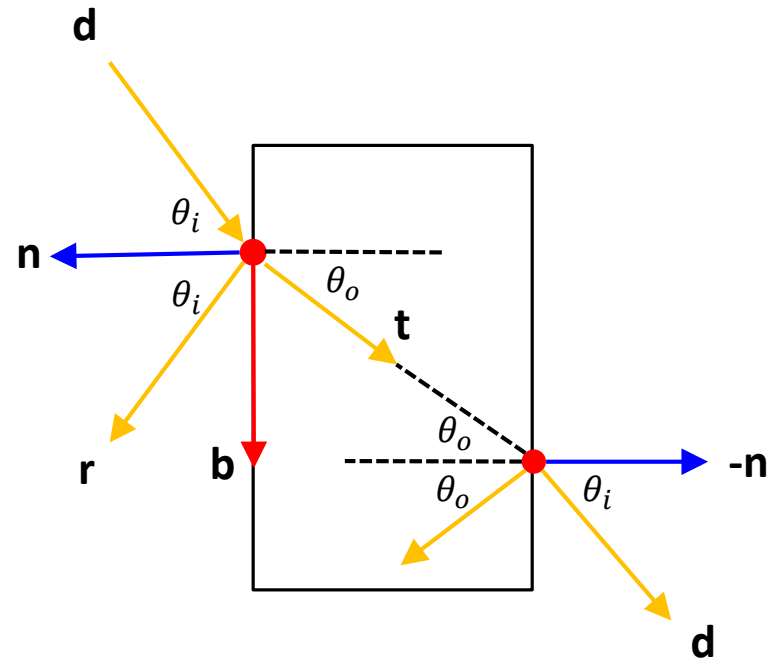
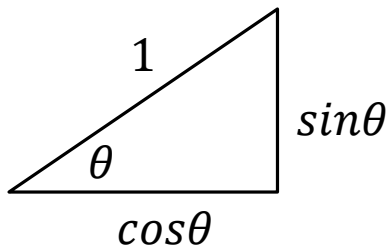
Specular Refraction

- Dielectric materials (e.g., diamonds, glass, water, and air)
 - Transparent objects that refract light
- Snell's law
 - $n_i \sin \theta_i = n_o \sin \theta_o$
 - n_i, n_o : indices of refraction
 - $n_i^2 (1 - \cos^2 \theta_i) = n_o^2 (1 - \cos^2 \theta_o)$
 - \because trigonometric identity ($\sin^2 \theta + \cos^2 \theta = 1$)
 - $\cos^2 \theta_o = 1 - \frac{n_i^2 (1 - \cos^2 \theta_i)}{n_o^2}$



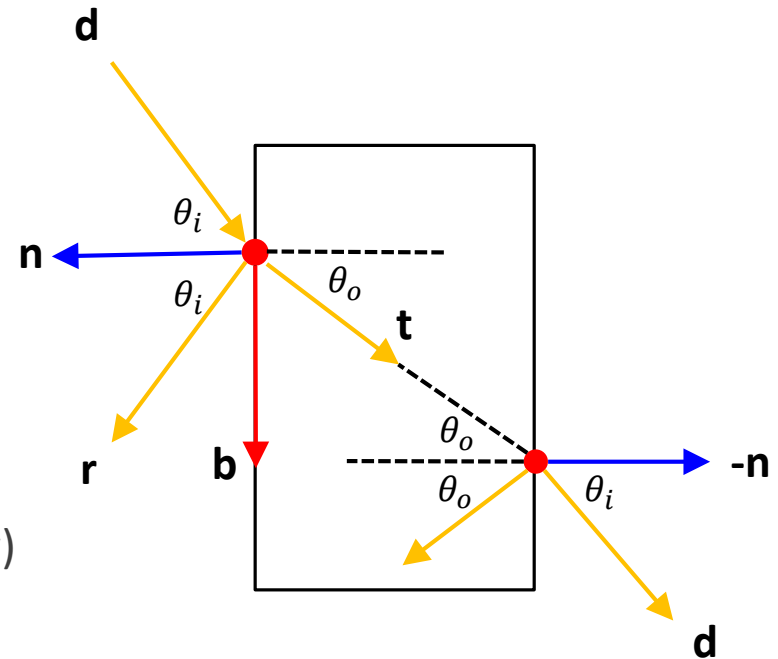
Specular Refraction

- Vectors \mathbf{n} and \mathbf{b} forms an orthonormal basis
- $\mathbf{t} = \sin\theta_o \mathbf{b} - \cos\theta_o \mathbf{n}$
 - Unknowns: \mathbf{t} , \mathbf{b}
- $\mathbf{d} = \sin\theta_i \mathbf{b} - \cos\theta_i \mathbf{n}$,
- $\mathbf{b} = \frac{\mathbf{d} + \mathbf{n} \cos\theta_i}{\sin\theta_i}$
- Trigonometric functions



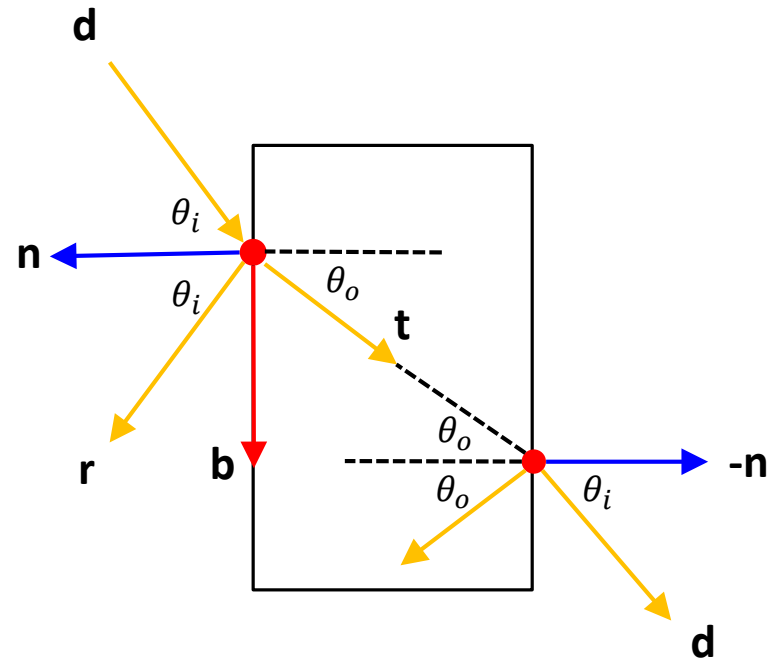
Specular Refraction

- $\mathbf{t} = \sin\theta_o \mathbf{b} - \cos\theta_o \mathbf{n}$
 - Unknowns: \mathbf{t} , \mathbf{b}
- $\mathbf{b} = \frac{\mathbf{d} + \mathbf{n} \cos\theta_i}{\sin\theta_i}$
- By plug-in \mathbf{b} into the equation for \mathbf{t} :
- $\mathbf{t} = \sin\theta_o \frac{\mathbf{d} + \mathbf{n} \cos\theta_i}{\sin\theta_i} - \cos\theta_o \mathbf{n}$
- $= \frac{n_i(\mathbf{d} + \mathbf{n} \cos\theta_i)}{n_o} - \cos\theta_o \mathbf{n}$ (by Snell's law)
- $= \frac{n_i(\mathbf{d} - \mathbf{n}(\mathbf{d} \cdot \mathbf{n}))}{n_o} - \mathbf{n} \sqrt{1 - \frac{n_i^2(1 - (\mathbf{d} \cdot \mathbf{n})^2)}{n_o^2}}$
- $\cos^2 \theta_o = 1 - \frac{n_i^2(1 - \cos^2 \theta_i)}{n_o^2}$



Specular Refraction

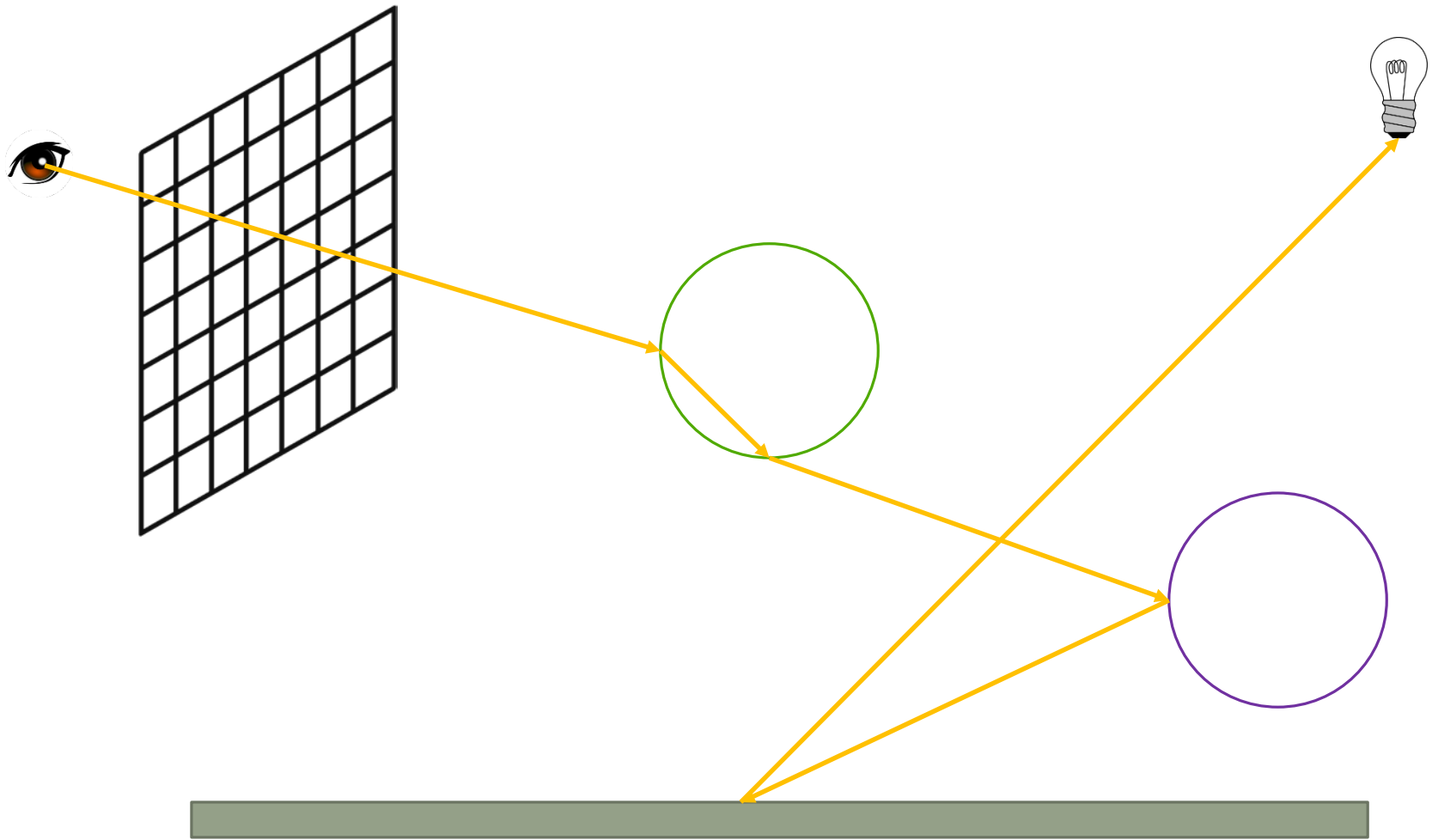
- The transmitting ray's direction \mathbf{t}
- $$\mathbf{t} = \frac{n_i(\mathbf{d} - \mathbf{n}(\mathbf{d} \cdot \mathbf{n}))}{n_o} - \mathbf{n} \sqrt{1 - \frac{n_i^2(1 - (\mathbf{d} \cdot \mathbf{n})^2)}{n_o^2}}$$
- $1 - \frac{n_i^2(1 - (\mathbf{d} \cdot \mathbf{n})^2)}{n_o^2} < 0$
 - No refracted ray (i.e., all energy is reflected)
- Q. what are the amount of reflected and refracted energy (i.e., reflectivity)?



Specular Refraction

- The reflectivity varies with the incident direction of light and it is determined by *Fresnel equations* (from physics).
- Schlick approximation (1994) of the Fresnel equations
 - $R(\theta_i) = R_0 + (1 - R_0)(1 - \cos\theta_i)^5$
 - $R_0 = \left(\frac{n_o - 1}{n_o + 1}\right)^2$

Whitted Ray Tracing



Whitted Ray Tracing

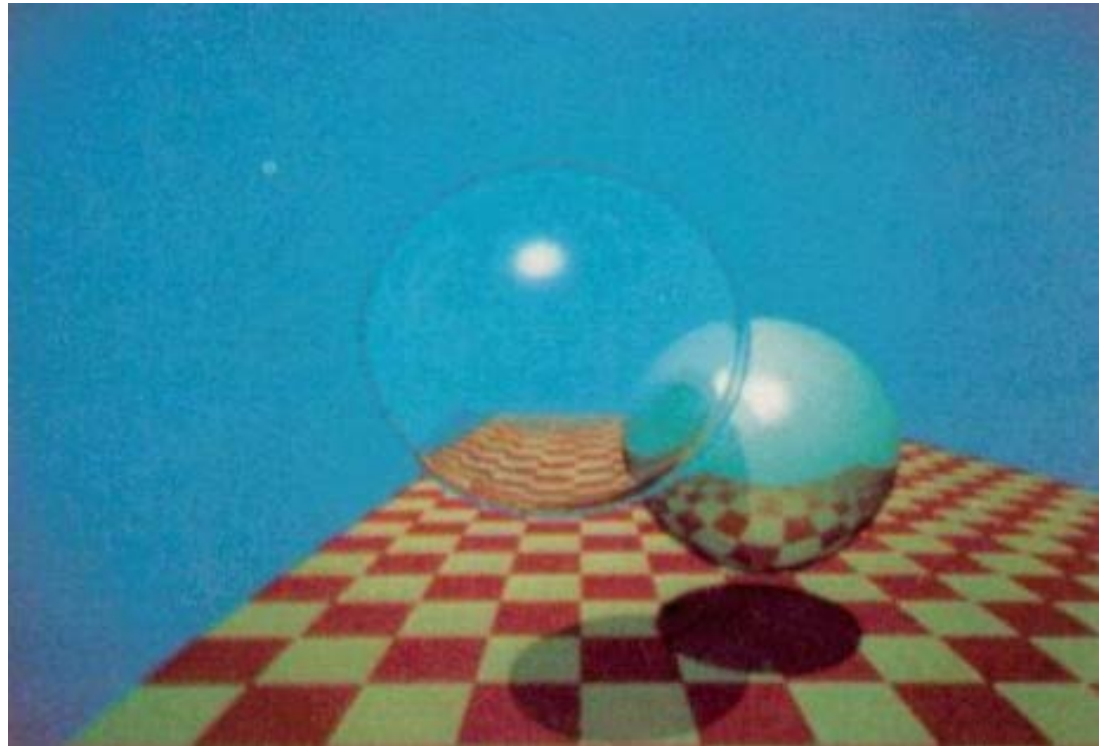
- For each pixel
 - Color $c = (0, 0, 0)$
 - Generate a primary ray (camera ray)
 - Find the closest intersection point between the ray and objects
 - If (there is a hit) then
 - Generate a shadow ray
 - If (there is no hit between the shadow ray and a light) then
 - $c = c + \text{shading}()$
 - Generate a secondary ray (reflection or refraction ray)
 - Go to the third line
 - Else
 - $c = c + \text{background color}$
 - Set the pixel color with c
- Issue: cannot trace a ray path that has infinite depth

Whitted Ray Tracing

- For each pixel
 - Color $c = (0, 0, 0)$
 - Generate a primary ray (with depth 0)
 - While (depth < d)
 - Find the closest intersection point between the ray and objects
 - If (there is a hit) then
 - Generate a shadow ray
 - If (there is no hit between the shadow ray and a light) then
 - $c = c + \text{shading}()$
 - Generate a secondary ray (reflection or refraction ray) // increase the ray depth +1
 - Else
 - $c = c + \text{background color}$
 - Set the pixel color with c

Whitted Ray Tracing

- Missing effects?
 - Soft shadow
 - Glossy reflection
 - Diffuse reflection
 - Depth-of-field effect
 - Motion blur
 - ...
- Advanced ray tracing
 - Distributed ray tracing
 - Path tracing and photon mapping



Whitted (1980)

Further Readings

- Chapter 4, 13
- Papers
 - T. Whitted. An improved illumination model for shading display. *Communications of the ACM*, 23(6):343–349, 1980