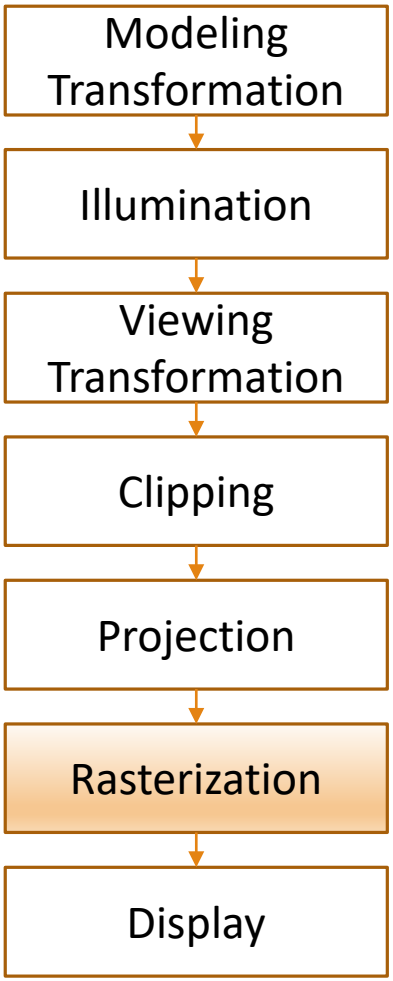
CT4510: Computer Graphics

Rasterization

BOCHANG MOON

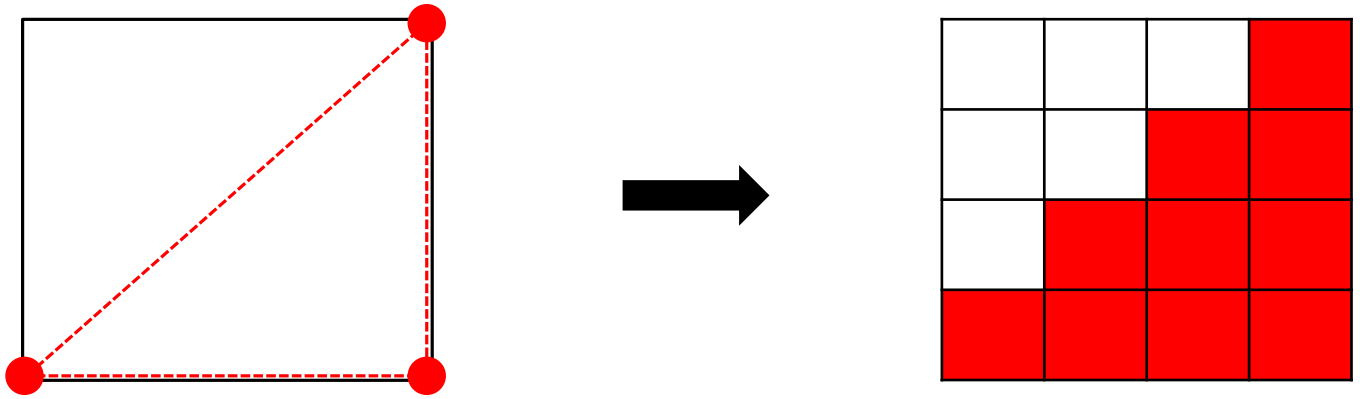
Rasterization



- Determine pixel values covered by primitives

Triangle Rasterization

- Draw a 2D triangles with 2D points p_0, p_1, p_2 in screen coordinates
  - Each vertex can contain multiple properties
    - position, color, etc.
  - For intermediate pixels, we need interpolate them.

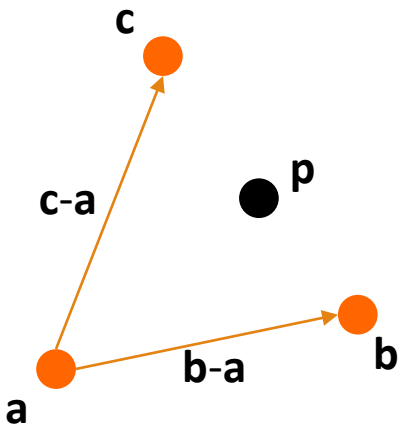


Background: Barycentric Coordinates

- Usually assign some properties (e.g., color) to each vertex in primitives and interpolate those values

- $\boldsymbol{p} = \boldsymbol{a} + \beta(\boldsymbol{b} - \boldsymbol{a}) + \gamma(\boldsymbol{c} - \boldsymbol{a})$
- $= (1 - \beta - \gamma)\boldsymbol{a} + \beta\boldsymbol{b} + \gamma\boldsymbol{c}$
- $= \alpha\boldsymbol{a} + \beta\boldsymbol{b} + \gamma\boldsymbol{c} \qquad (\alpha \equiv 1 - \beta - \gamma)$

- $\boldsymbol{p}(\alpha, \beta, \gamma) = \alpha\boldsymbol{a} + \beta\boldsymbol{b} + \gamma\boldsymbol{c}$
  - with $\alpha + \beta + \gamma = 1$

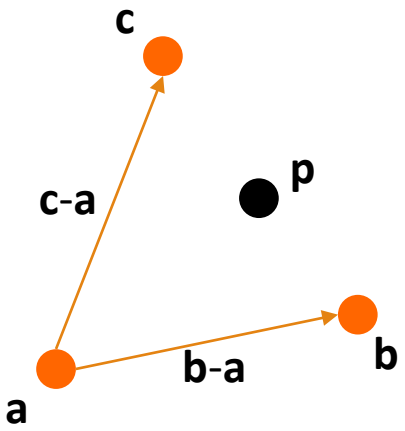


Background: Barycentric Coordinates

- $\boldsymbol{p}(\alpha, \beta, \gamma) = \alpha \boldsymbol{a} + \beta \boldsymbol{b} + \gamma \boldsymbol{c}$
  - with $\alpha + \beta + \gamma = 1$

- Conditions for a point p inside the triangle
  - $0 < \alpha < 1$
  - $0 < \beta < 1$
  - $0 < \gamma < 1$

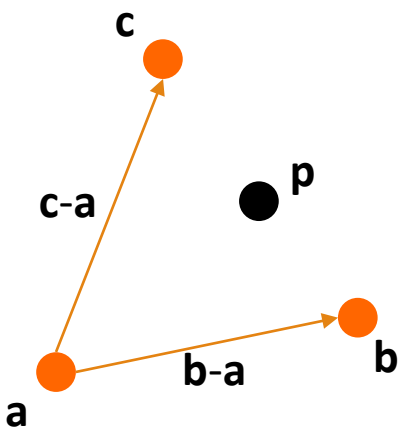
- Q. Conditions for a point on an edge?
- Q. Conditions for a point at a vertex?



Background: Barycentric Coordinates

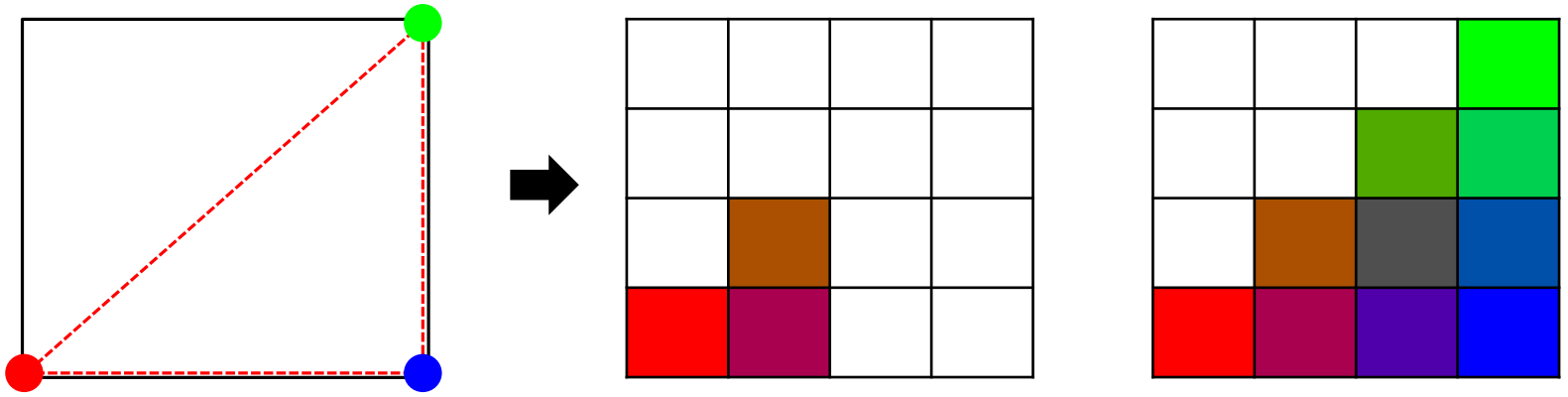
- $\boldsymbol{p}(\alpha, \beta, \gamma) = \alpha \boldsymbol{a} + \beta \boldsymbol{b} + \gamma \boldsymbol{c}$
  - with $\alpha + \beta + \gamma = 1$

- Q. Given a point **p**, how do we computer the coordinates?
  - $\boldsymbol{p} = \boldsymbol{a} + \beta(\boldsymbol{b} - \boldsymbol{a}) + \gamma(\boldsymbol{c} - \boldsymbol{a})$
  - Corresponding matrix form:
  - $\begin{bmatrix} x_b - x_a & x_c - x_a \\ y_b - y_a & y_c - y_a \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} x_p - x_a \\ y_p - y_a \end{bmatrix}$
  - $\alpha = 1 - \beta - \gamma$



Triangle Rasterization

- For all x do
  - For all y do
    - Compute α, β, γ for (x, y)
    - If ($\alpha \in [0,1]$ && $\beta \in [0,1]$ && $\gamma \in [0,1]$) then
      - $\boldsymbol{c} = \alpha \boldsymbol{c_0} + \beta \boldsymbol{c_1} + \gamma \boldsymbol{c_2}$ // Gouraud interpolation (Gouraud, 1971)
      - Draw a color **c** at a pixel (x, y)



Triangle Rasterization

- For all x within a bounding box do
  - For all y within a bounding box do
    - Compute α, β, γ for (x, y)
    - If ($\alpha \in [0,1]$ && $\beta \in [0,1]$ && $\gamma \in [0,1]$) then
      - $\boldsymbol{c} = \alpha \boldsymbol{c_0} + \beta \boldsymbol{c_1} + \gamma \boldsymbol{c_2}$ // Gouraud interpolation (Gouraud, 1971)
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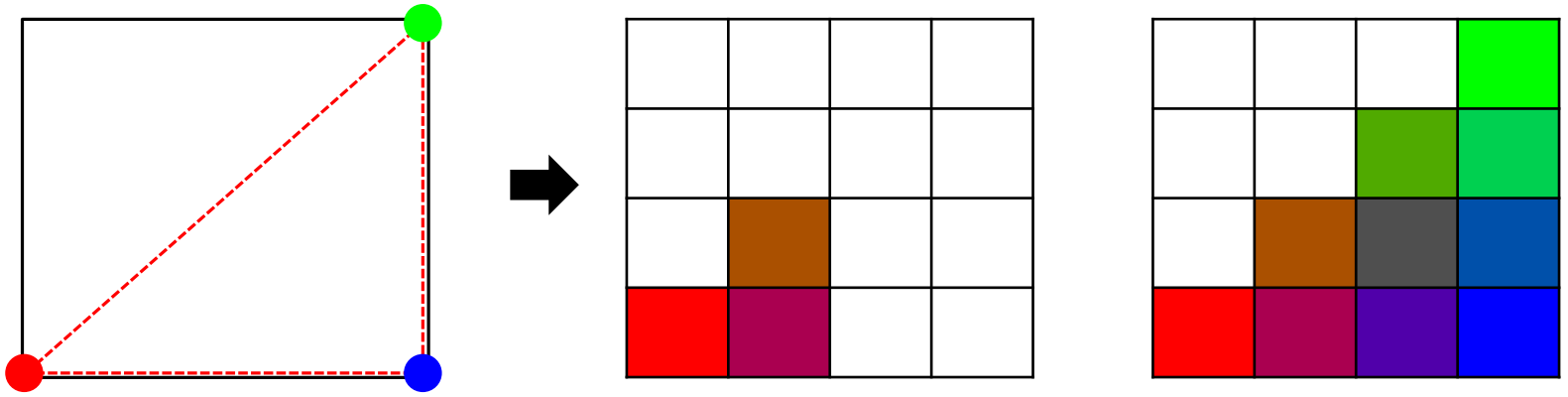
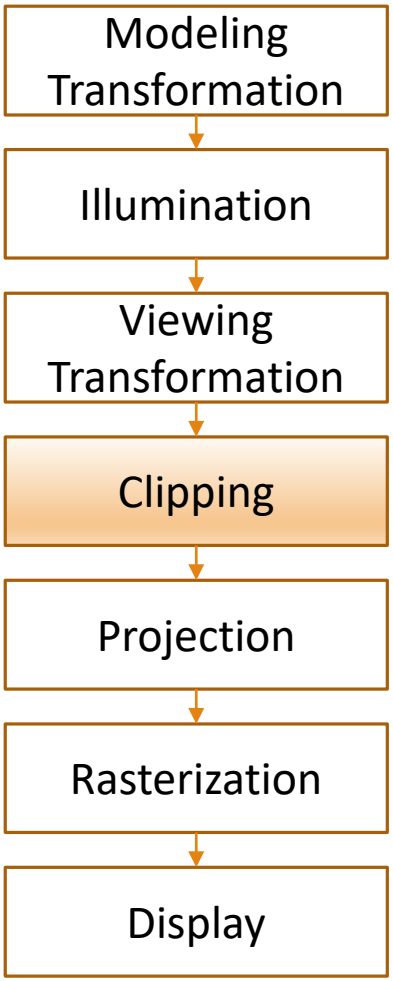
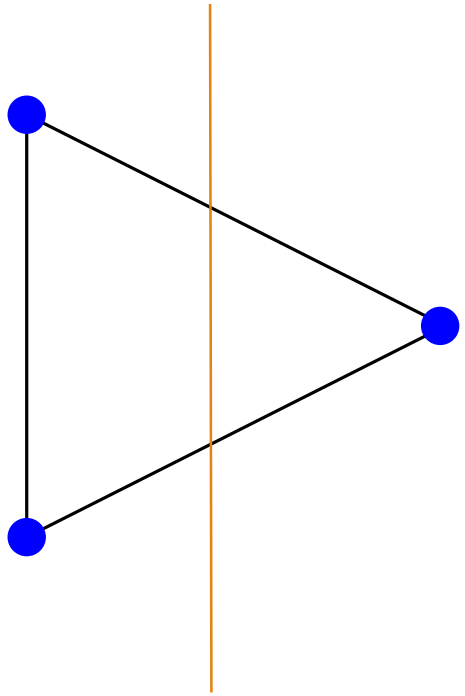


Clipping

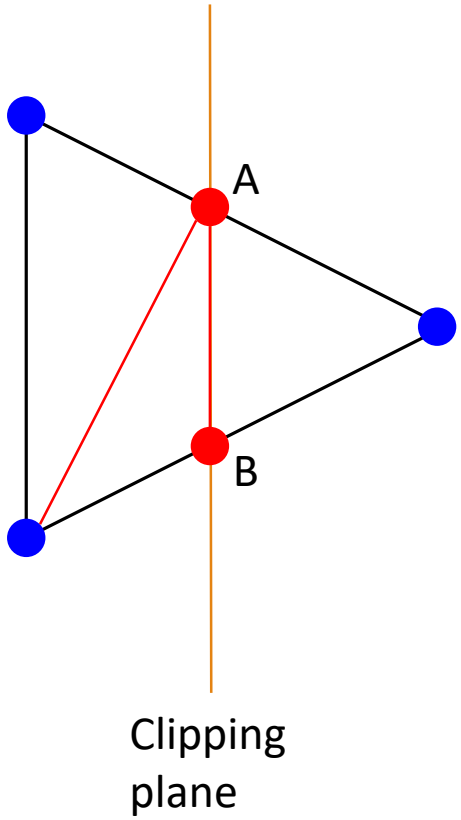

Modeling Transformation
Illumination
Viewing Transformation
Clipping
Projection
Rasterization
Display


Clipping plane

Clipping

- A parametric line connecting **a** and **c**:
  - $\boldsymbol{p}(t) = \boldsymbol{a} + t(\boldsymbol{c} - \boldsymbol{a})$

- Implicit plane through point **q** with normal **n**:
  - $f(\boldsymbol{p}) = (\boldsymbol{p} - \boldsymbol{q}) \cdot \boldsymbol{n} = 0$
  - Can be written as:
    - $f(\boldsymbol{p}) = \boldsymbol{p} \cdot \boldsymbol{n} + D = 0$
  - Recall that
    - $f(\boldsymbol{p}) < 0$: p is inside
    - $f(\boldsymbol{p}) = 0$: p is on the plane
    - $f(\boldsymbol{p}) > 0$: p is outside

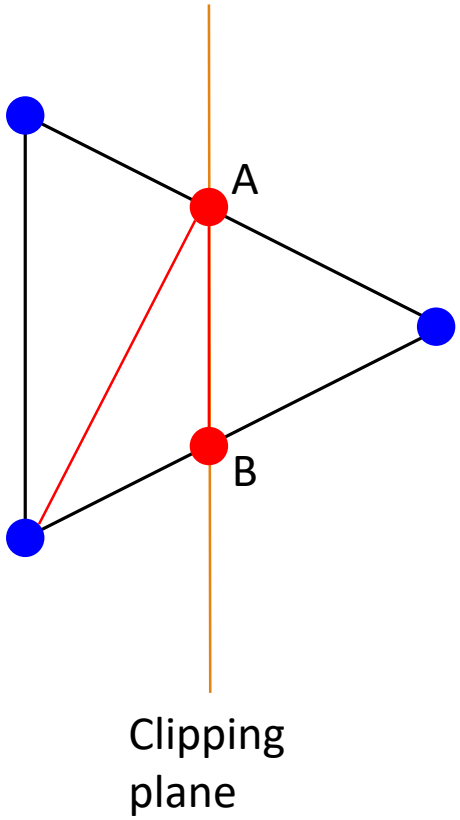


Clipping

- A parametric line connecting **a** and **c**:
  - $\boldsymbol{p}(t) = \boldsymbol{a} + t(\boldsymbol{c} - \boldsymbol{a})$

- Implicit plane through point **q** with normal **n**:
  - $f(\boldsymbol{p}) = \boldsymbol{p} \cdot \boldsymbol{n} + D = 0$

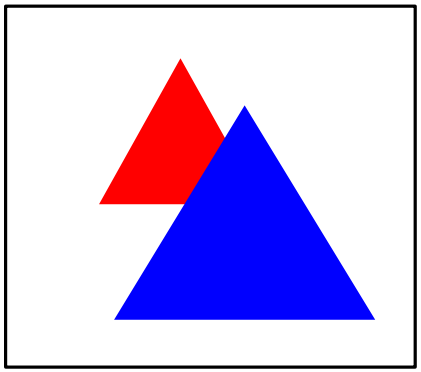
  - If the end points of a line have different signs,
    - we need to find the intersection point between the plane and line segment
    - $\big(\boldsymbol{a} + t(\boldsymbol{c} - \boldsymbol{a})\big) \cdot \boldsymbol{n} + D = 0$
    - Solving for t:
      - $t_A = -\frac{\boldsymbol{n} \cdot \boldsymbol{a} + D}{\boldsymbol{n} \cdot (\boldsymbol{c} - \boldsymbol{a})}$
    - $\boldsymbol{A} = \boldsymbol{a} + t_A(\boldsymbol{c} - \boldsymbol{a})$
    - A similar computation will give B.

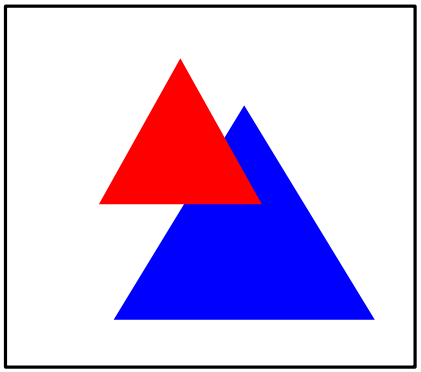


Z-Buffer for Hidden Surfaces

- In graphics, z-buffer (depth buffer) is used to draw closest primitives.

- Z-buffer
  - Store the depth at each pixel

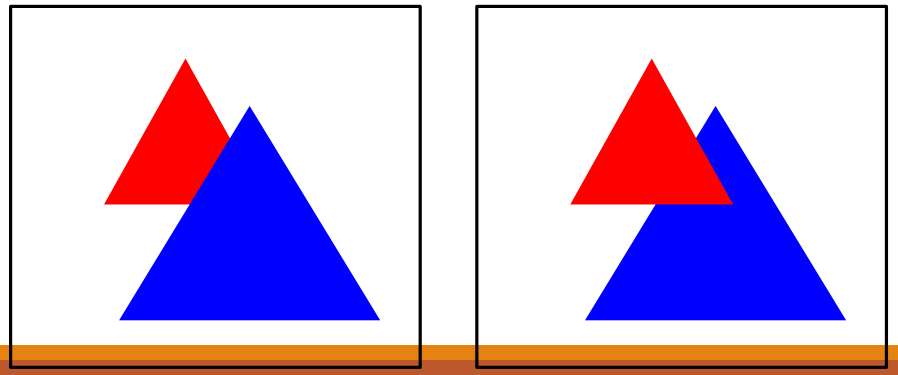




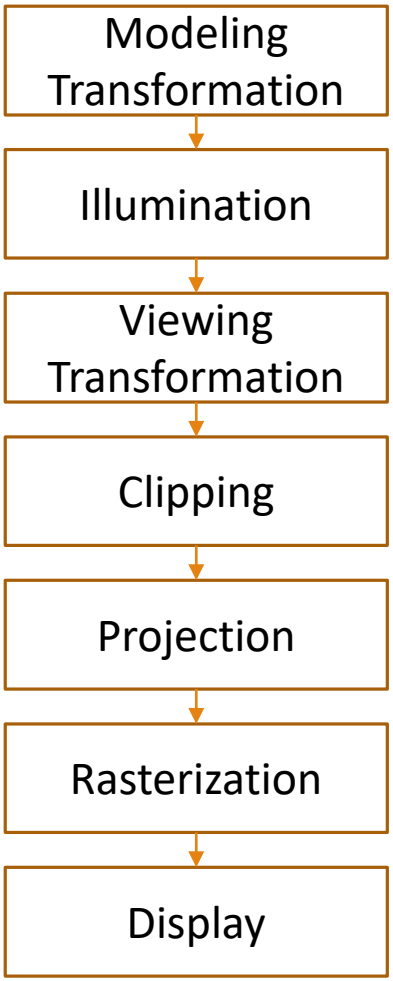
Z-Buffer for Hidden Surfaces

- In graphics, z-buffer (depth buffer) is used to draw closest primitives.

- Z-buffer
  - Store the depth at each pixel
- Algorithm:
  - Initialize z-buffer to the maximum depth (i.e., the depth of the far plane)
  - Interpolate the z-coordinate as a vertex attribute (similar to the color interpolation)
  - If the z value at (x, y) < z-buffer(x,y)
    - Fill the pixel color at (x, y)
    - Update the z-buffer(x,y)

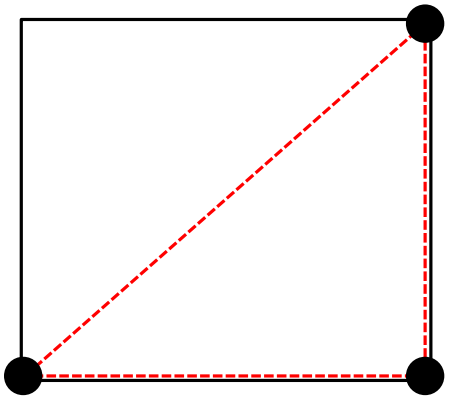


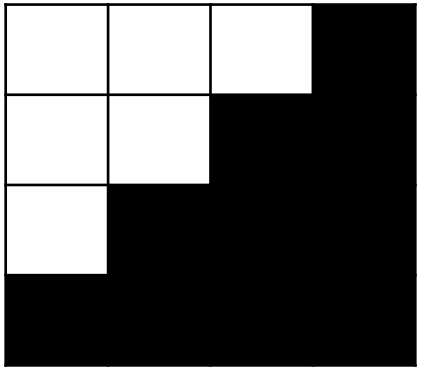
Review of the Rendering Pipeline



- So far, we have covered most pipeline components.
- The illumination part will be discussed later.

- Q. how do we handle aliasing?





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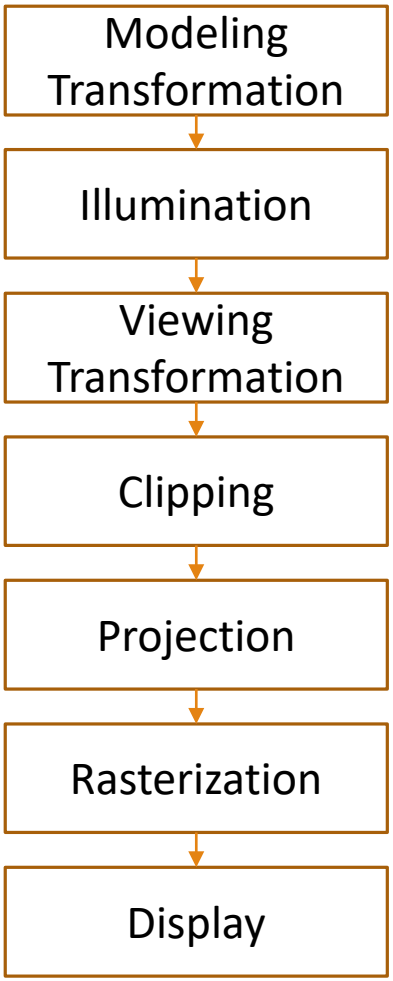
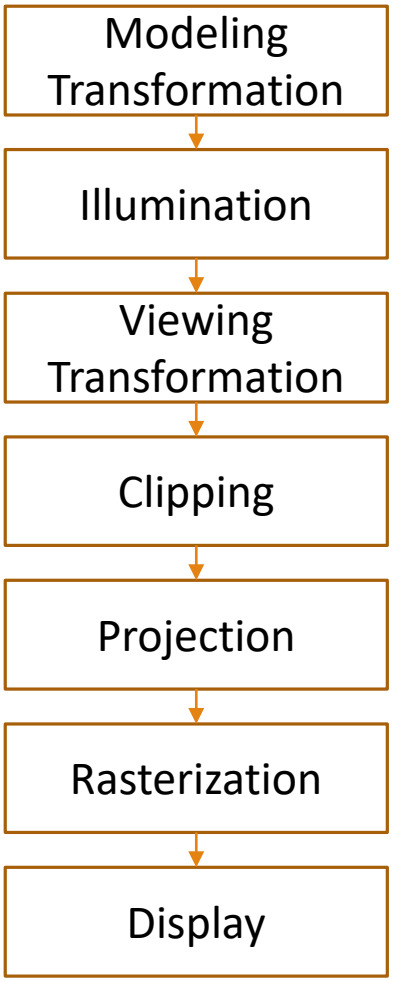


- So far, we have covered most pipeline components.
- The illumination part will be discussed later.

- Q. how do we handle aliasing?
  - Super-sampling: draw a larger image than a specified resolution, and average multiple colors with a filter (box filter)
  - Sophisticated filters can be designed.
    - See the video.
  - 3D game may provide an option to users for this anti-aliasing.

Review of the Rendering Pipeline



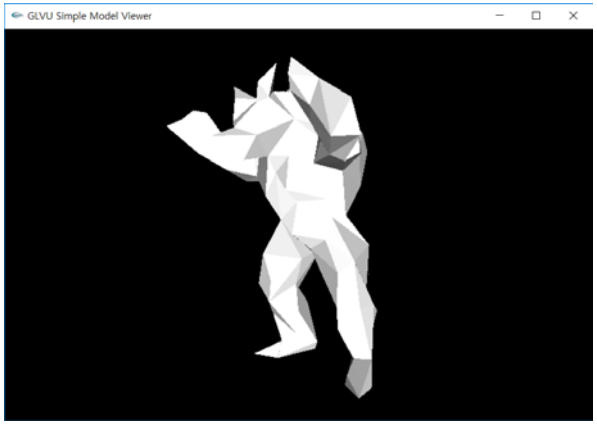
- So far, we have covered most pipeline components.
- The illumination part will be discussed later.

- Q. how do we handle aliasing?
- Q. do we have other optimization techniques?
  - Levels-of-detail (LOD)
    - Static LOD
    - Dynamic LOD
    - Stochastic Simplification of Aggregate Detail, [Cook et al. SIG 07]

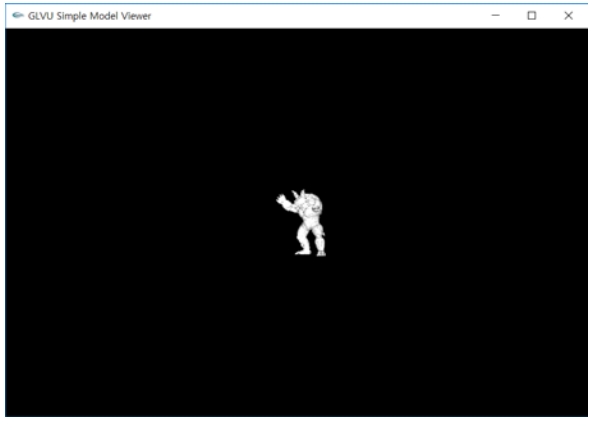
Review of the Rendering Pipeline

345944 tri.

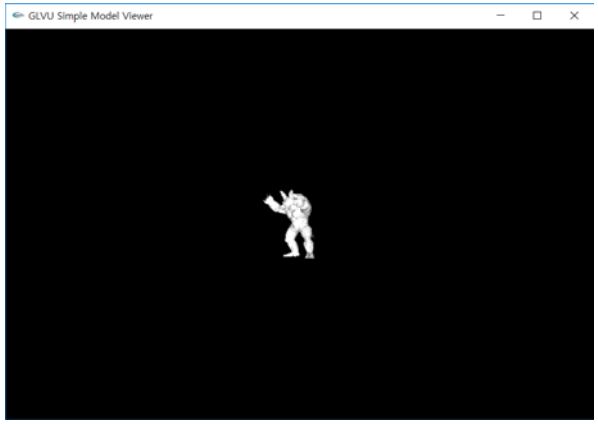
3000 tri.



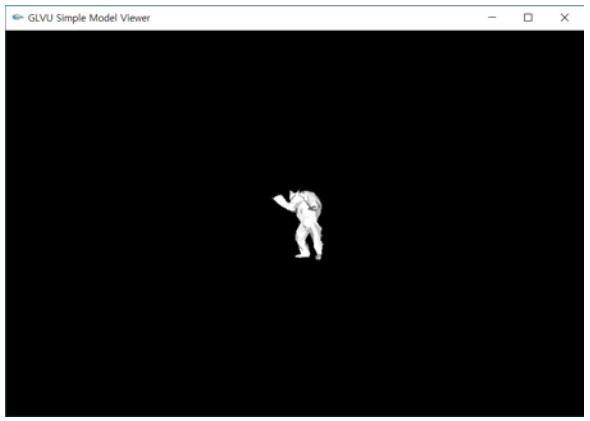
300 tri.



345944 tri.



3000 tri.



300 tri.

Further Readings

- Chapter 8
- Chapter 12.4