

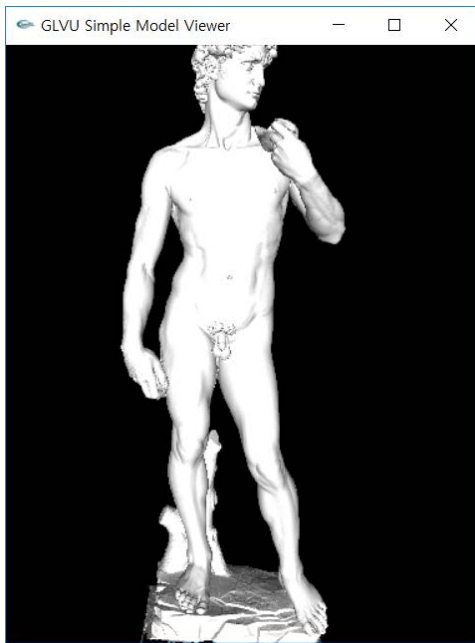
CT4510: Computer Graphics

Transformation

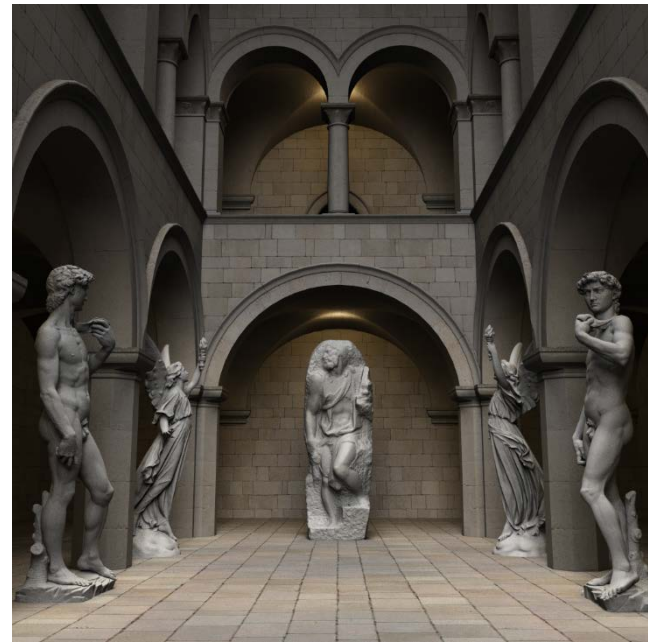
BOCHANG MOON

Transformation

- Fundamental operation to arrange objects in a 3D scene



Object



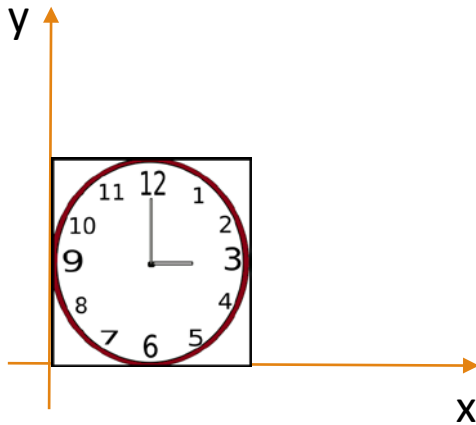
3D scene

Transformation: Scaling

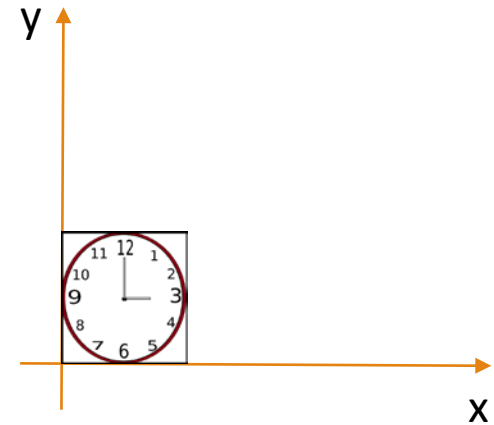
- Change length along the coordinate axes

- $scale(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$

- $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$



$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

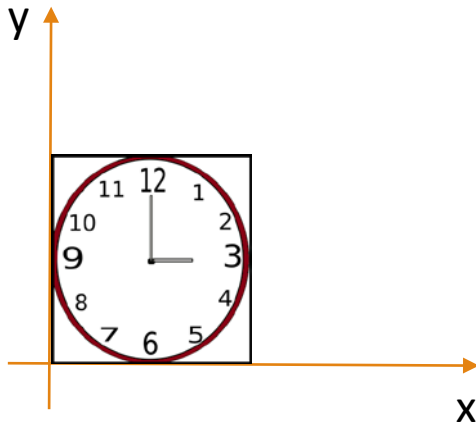


Transformation: Scaling

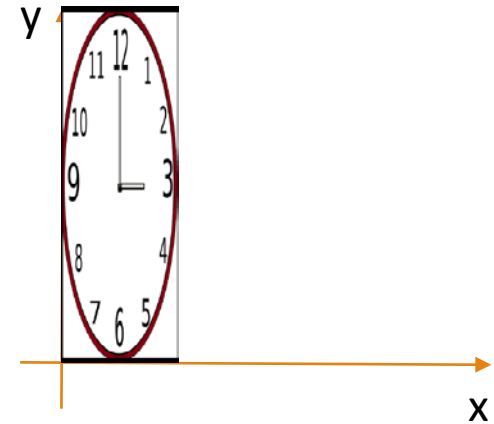
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- $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$



$$\begin{bmatrix} 0.5 & 0 \\ 0 & 2.0 \end{bmatrix}$$

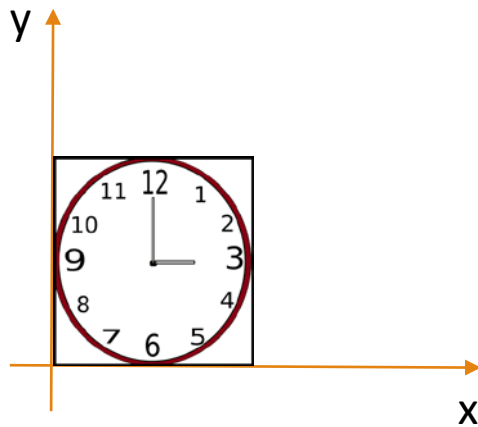


Transformation: Shearing

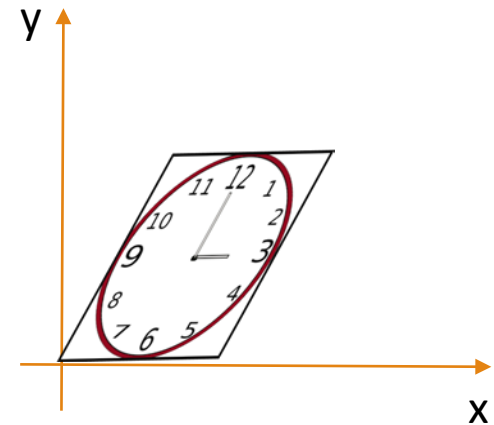
- Push objects sideways along horizontal or vertical direction

- $shear_x(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$

- $shear_y(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$



$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

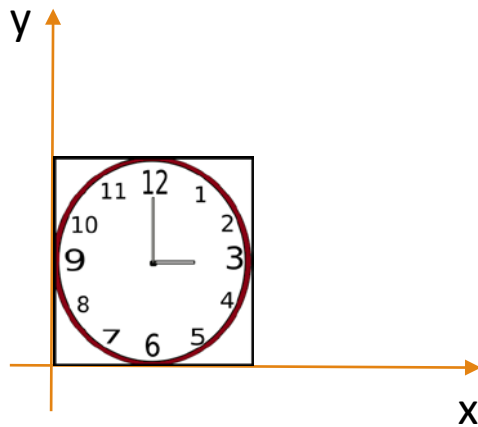


Transformation: Shearing

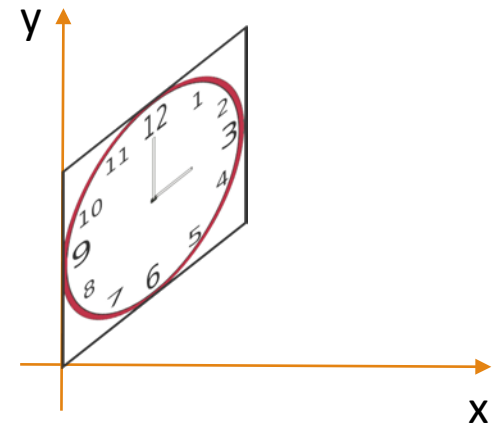
- Push objects sideways along horizontal or vertical direction

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- $shear_y(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$



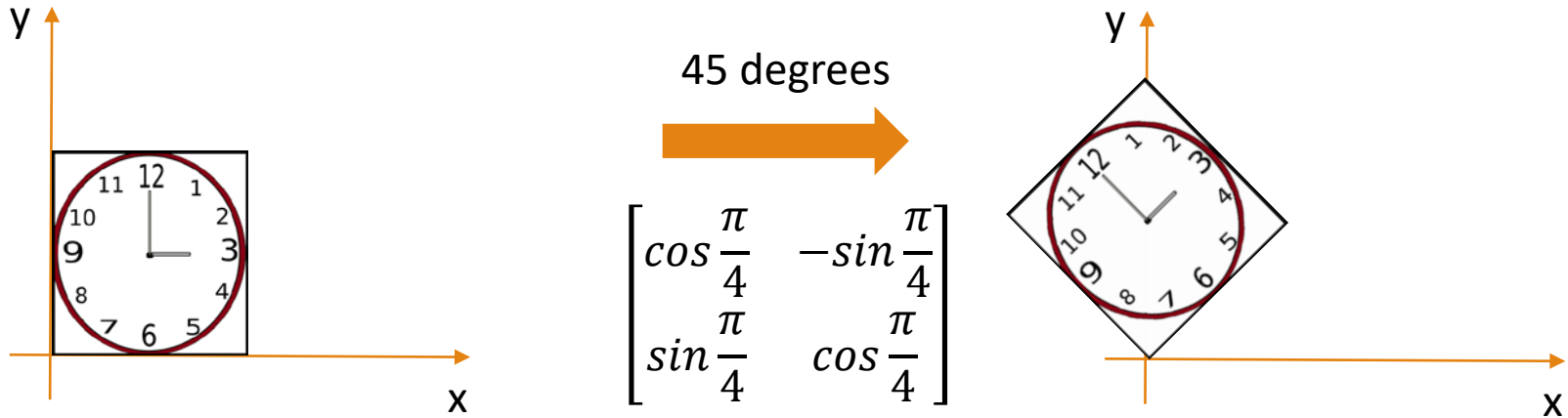
$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$



Transformation: Rotation

- Rotate objects counterclockwise

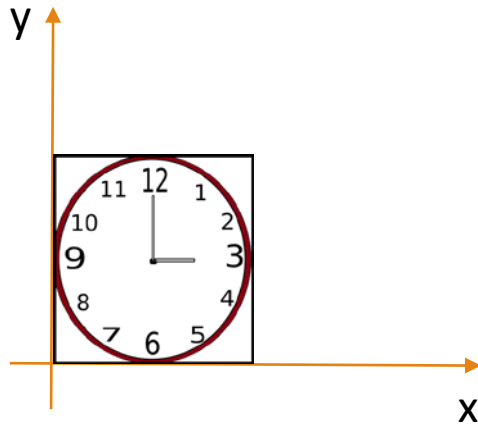
- $rotate(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$



Transformation: Rotation

- Rotate objects counterclockwise

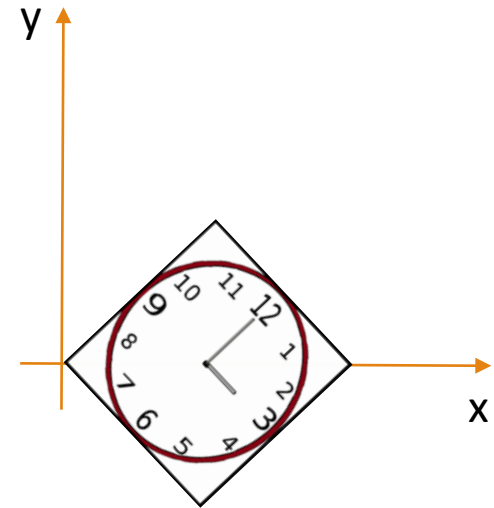
- $rotate(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$



-45 degrees



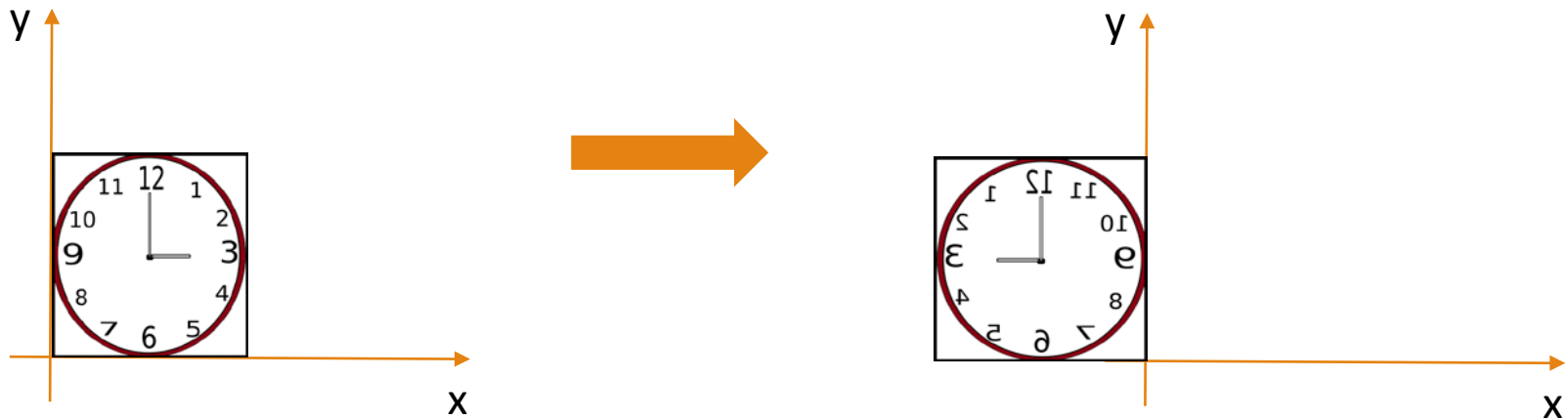
$$\begin{bmatrix} \cos \frac{-\pi}{4} & -\sin \frac{-\pi}{4} \\ \sin \frac{-\pi}{4} & \cos \frac{-\pi}{4} \end{bmatrix}$$



Transformation: Reflection

- Reflect objects across either of the coordinate axes

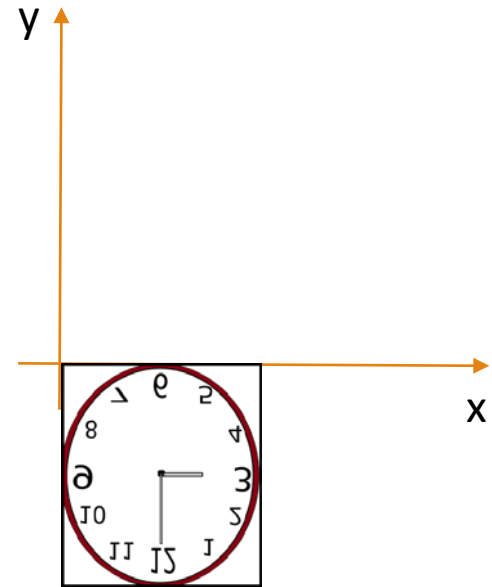
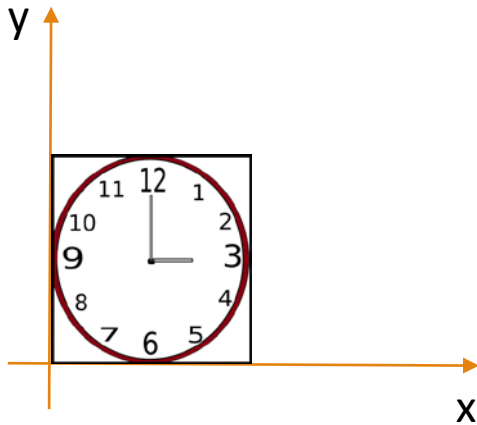
- $reflect_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$



Transformation: Reflection

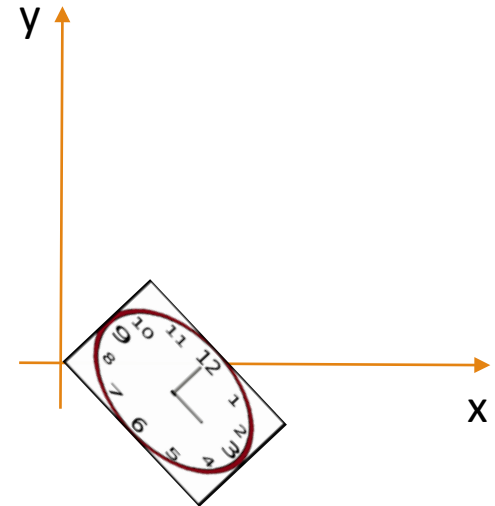
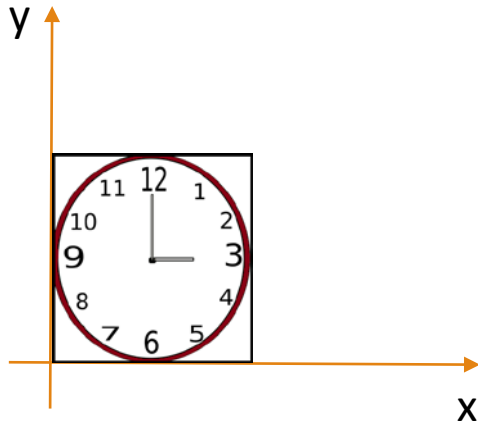
- Reflect objects across either of the coordinate axes

- $reflect_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$



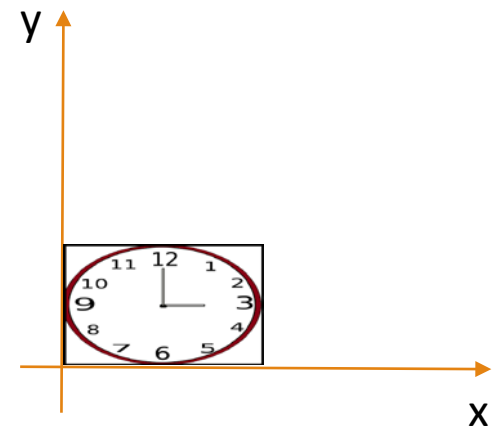
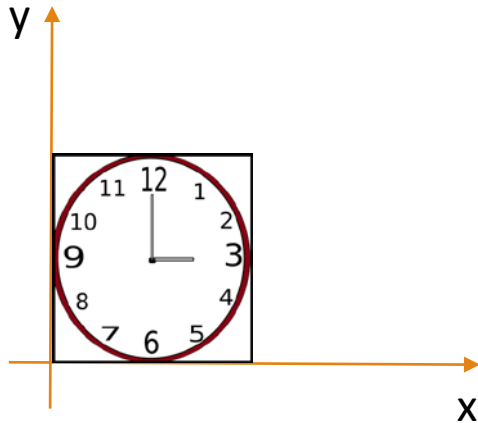
Transformation: Composition

- Apply more than one transformation to an object
- e.g. apply a scale S , and then a rotation R
 - $$\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = RS \begin{bmatrix} x_{old} \\ y_{old} \end{bmatrix}$$



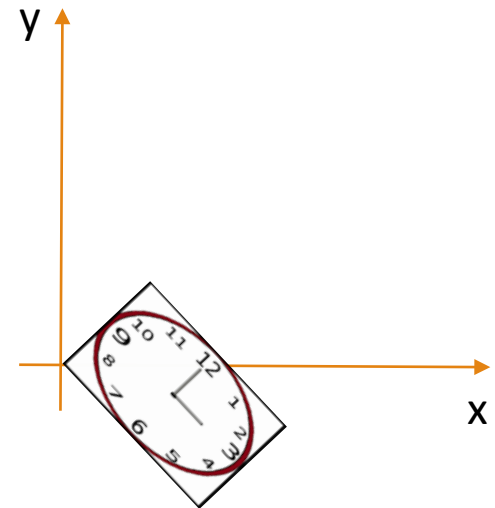
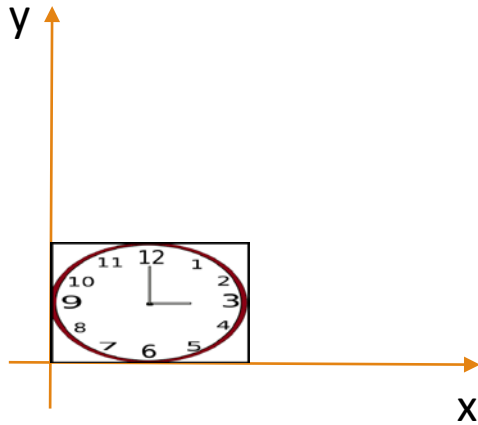
Transformation: Composition

- Apply more than one transformation to an object
- e.g. apply a scale S , and then a rotation R
 - $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S \begin{bmatrix} x_{old} \\ y_{old} \end{bmatrix}$



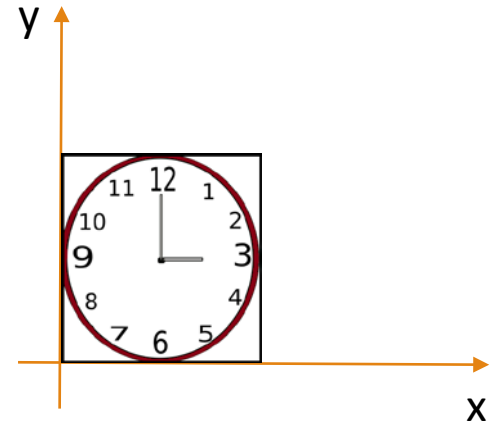
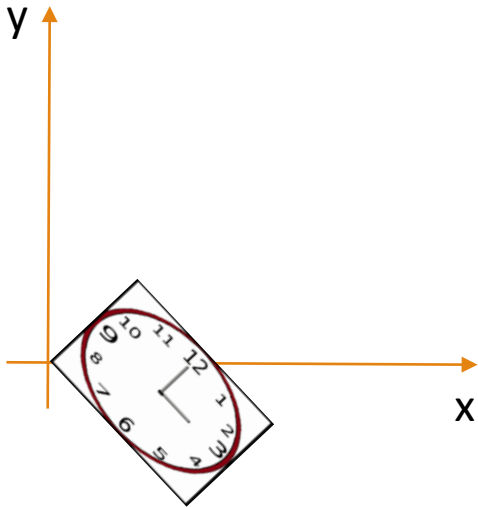
Transformation: Composition

- Apply more than one transformation to an object
- e.g. apply a scale S , and then a rotation R
 - $\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = R \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$



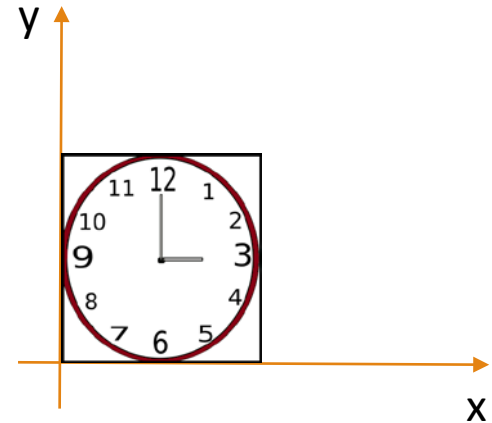
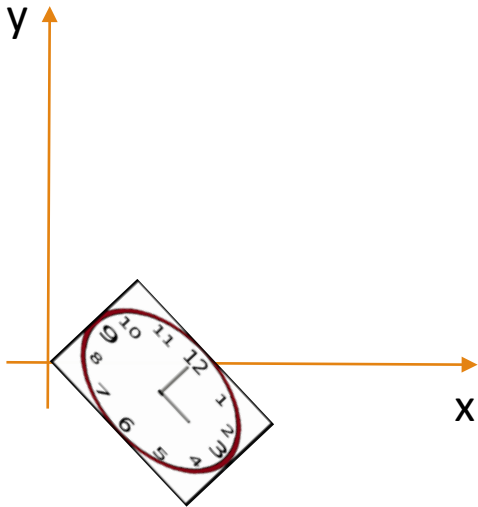
Inverse Transformation

- Your transformation
 - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
 - $x_{old} = T^{-1}x_{new}$



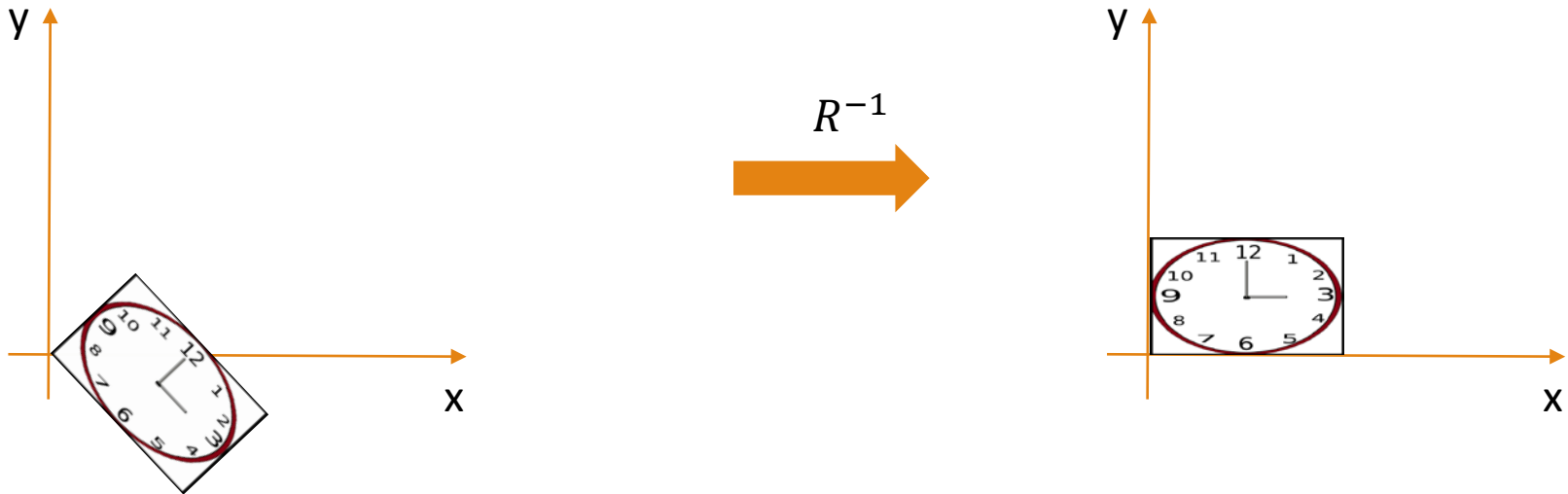
Inverse Transformation

- Your transformation
 - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
 - $x_{old} = T^{-1}x_{new} = (RS)^{-1}x_{new} = S^{-1}R^{-1}x_{new}$



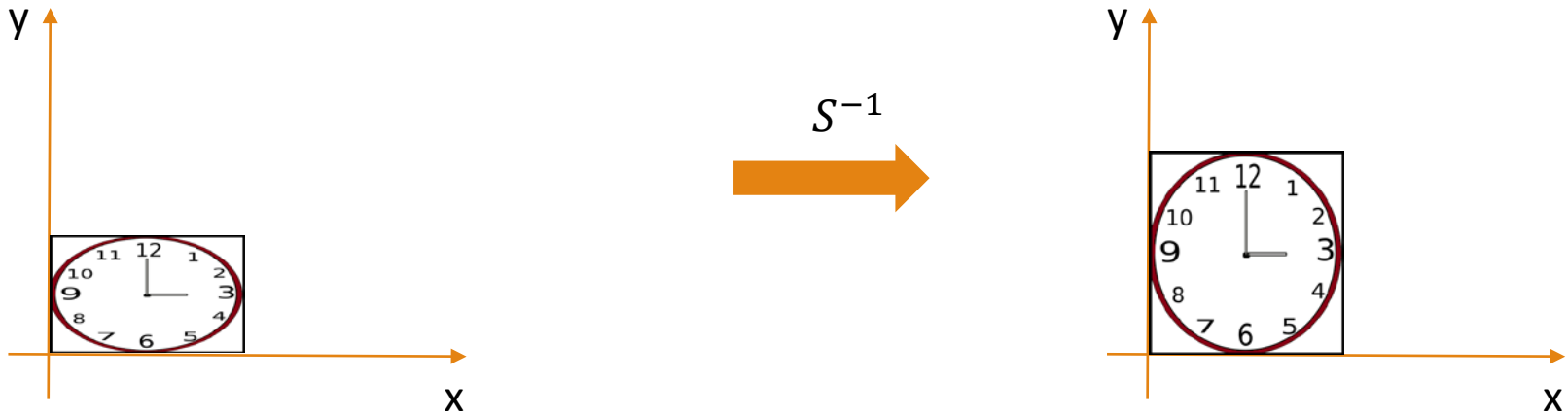
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Inverse Transformation

- Your transformation
 - $x_{new} = RSx_{old} = Tx_{old}$
- Undo your transformation
 - $x_{old} = T^{-1}x_{new} = (RS)^{-1}x_{new} = S^{-1}R^{-1}x_{new}$



Inverse Matrix

- $M = 2 \times 2$ matrix

- $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$

- $M^{-1} = \frac{1}{M_{11} \times M_{22} - M_{12} \times M_{21}} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix}$