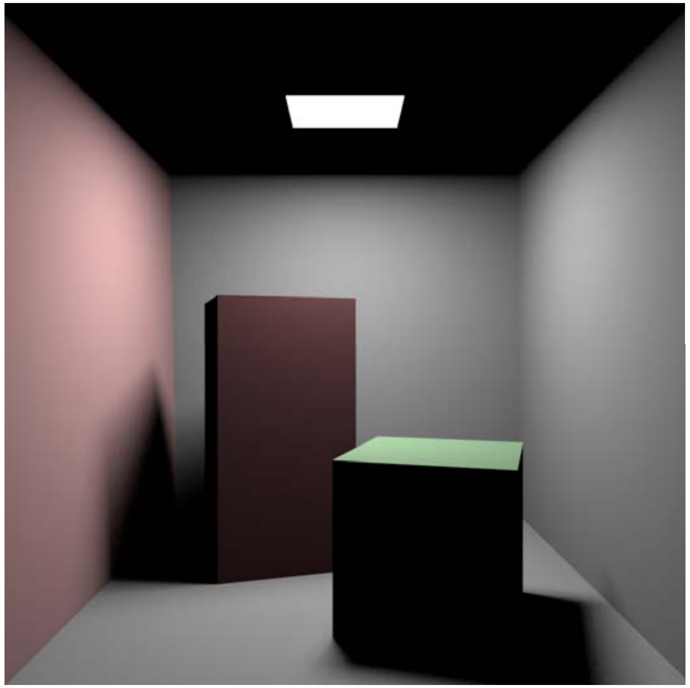
CT5510: Computer Graphics

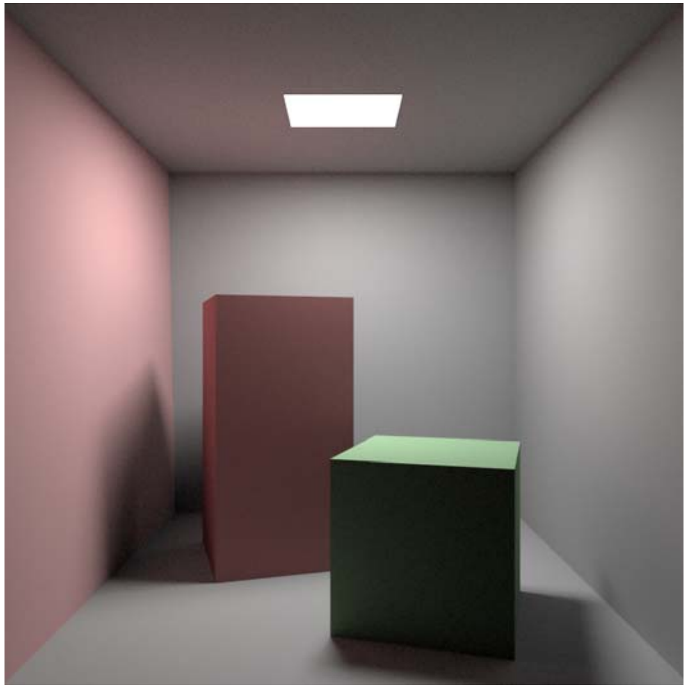
Global Illumination Methods

BOCHANG MOON

Global Illumination

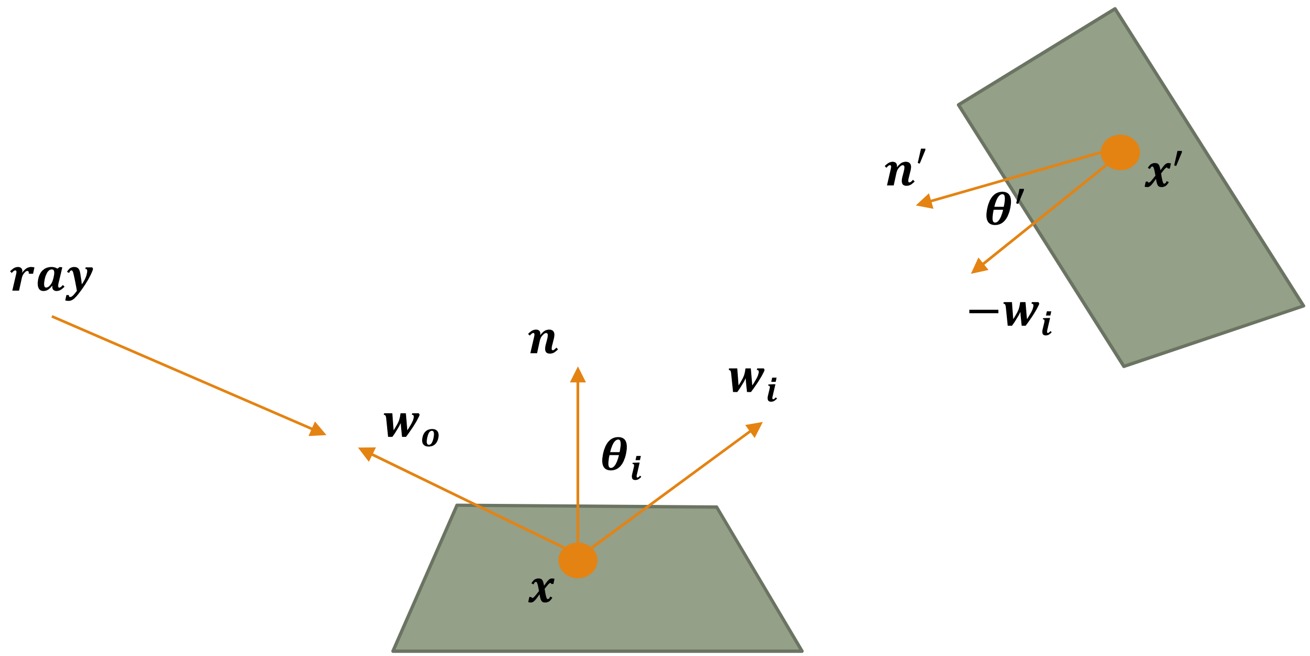
- Distributed ray tracing introduces a noticeable improvement on rendering quality, but it still misses some rendering effects





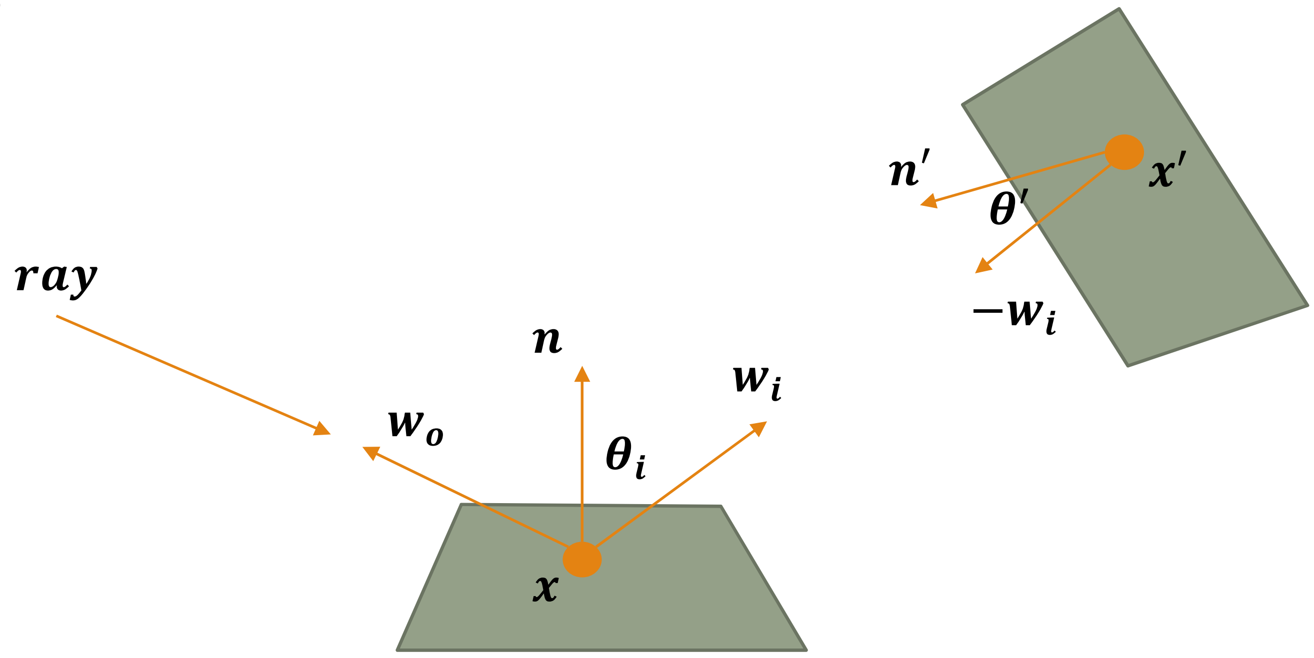
Global Illumination

- Rendering equation [Kajiya 86] (another form: [Immel and Cohen 86])
- $L_s(\boldsymbol{x}, w_o) = \int_{all\ x'} \frac{\rho(\boldsymbol{w_i},\boldsymbol{w_o}) L_s(\boldsymbol{x}',\boldsymbol{x}-\boldsymbol{x}') v(\boldsymbol{x},\boldsymbol{x}') cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x}'\|^2} dA$



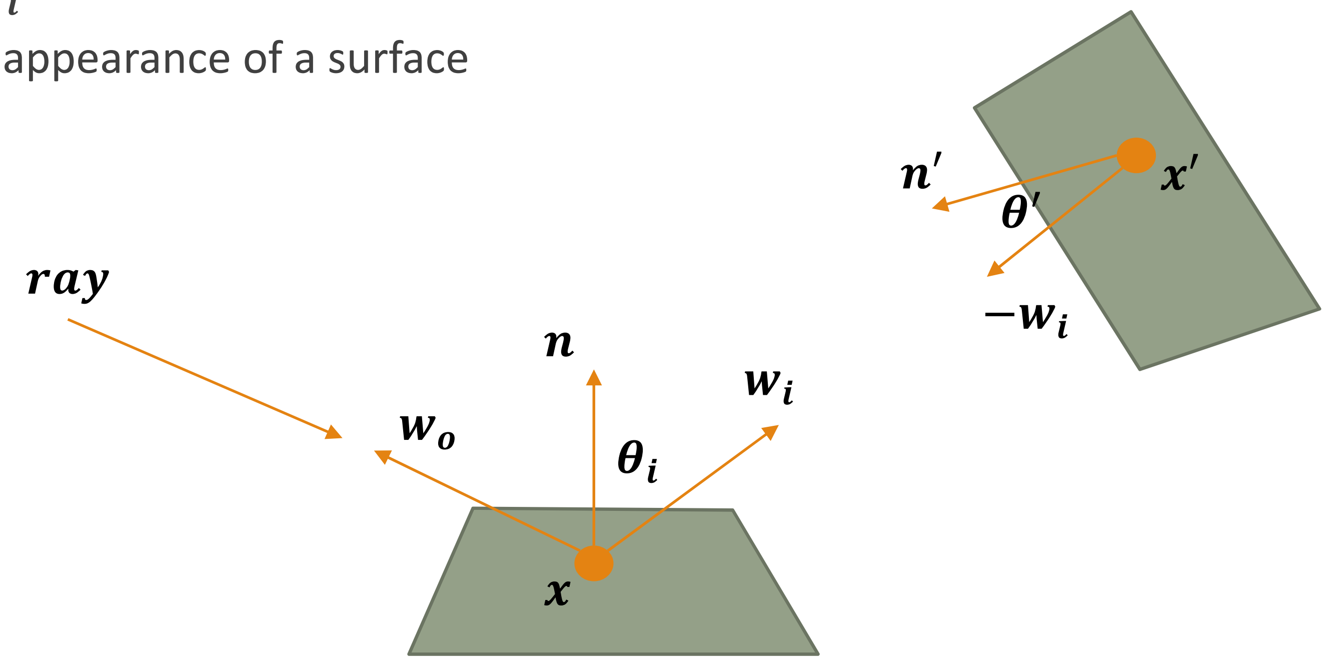
Global Illumination

- $L_s(\boldsymbol{x}, w_o) = \int_{all\ x'} \frac{\rho(\boldsymbol{w_i},\boldsymbol{w_o}) L_s(\boldsymbol{x'},\boldsymbol{x}-\boldsymbol{x'}) v(\boldsymbol{x},\boldsymbol{x'}) cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x'}\|^2} dA$
- $v(\boldsymbol{x}, \boldsymbol{x'})$: visibility term
  - 1 if two points are mutually visible
  - 0 otherwise



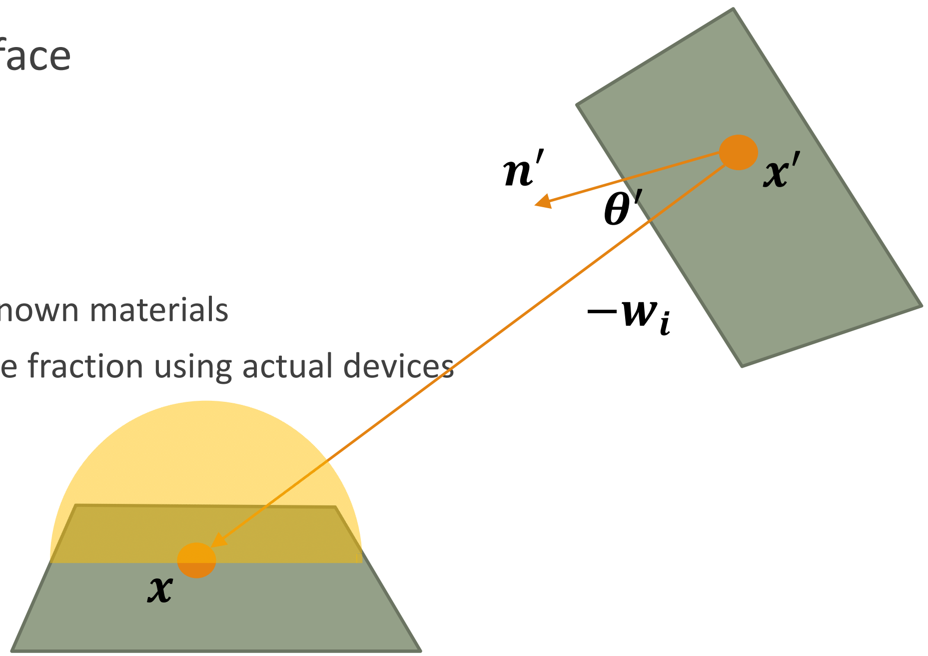
Global Illumination

- $L_s(\boldsymbol{x}, w_o) = \int_{all\ x'} \frac{\rho(\boldsymbol{w_i}, \boldsymbol{w_o}) L_s(\boldsymbol{x}', \boldsymbol{x} - \boldsymbol{x}') v(\boldsymbol{x}, \boldsymbol{x}') cos\theta_i cos\theta'}{\|\boldsymbol{x} - \boldsymbol{x}'\|^2} dA$
- $\rho(\boldsymbol{w_i}, \boldsymbol{w_o})$: bidirectional reflectance distribution function (BRDF)
  - Reflected light with an outgoing direction $\boldsymbol{w_o}$ / Incident light with an incoming direction $\boldsymbol{w_i}$
  - Determine appearance of a surface



Global Illumination

- $L_s(\boldsymbol{x}, w_o) = \int_{all\ \boldsymbol{x}'} \frac{\rho(\boldsymbol{w_i},\boldsymbol{w_o}) L_S(\boldsymbol{x}',\boldsymbol{x}-\boldsymbol{x}') v(\boldsymbol{x},\boldsymbol{x}') cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x}'\|^2} dA$
- $\rho(\boldsymbol{w_i}, \boldsymbol{w_o})$: bidirectional reflectance distribution function (BRDF)
  - Reflected light with an outgoing direction $\boldsymbol{w_o}$ / Incident light with an incoming direction $\boldsymbol{w_i}$
  - Determine appearance of a surface
  - e.g. ideal diffuse surface
    - $\rho(\boldsymbol{w_i}, \boldsymbol{w_o}) = \rho$ (constant)
  - e.g. other surfaces?
    - Utilize a mathematical form for well-known materials
    - Build the 4D function by measuring the fraction using actual devices



Global Illumination

- $L_s(\boldsymbol{x}, w_o) = \int_{all\ \boldsymbol{x'}} \frac{\rho(\boldsymbol{w_i},\boldsymbol{w_o}) L_S(\boldsymbol{x'}, \boldsymbol{x}-\boldsymbol{x'}) v(\boldsymbol{x},\boldsymbol{x'}) cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x'}\|^2} dA$

- How can we solve the integral?

Monte Carlo Integration

- A sampling method based on probability
- Draw random numbers and approximate an integral
- e.g., evaluate a 1D integral: $\int_a^b f(x)dx$
- $F_n = \frac{b-a}{n}\sum_{i=1}^{n} f(x_i)$
- $x_i \in [a, b]$: uniform random variable, $p(x) = \frac{1}{b-a}$
- $E[F_n] = \frac{b-a}{n}\sum_{i=1}^{n} E[f(x_i)]$
- $= \frac{b-a}{n}\sum_{i=1}^{n} \int_a^b f(x)p(x)dx$
- $= \frac{1}{n}\sum_{i=1}^{n} \int_a^b f(x)dx$
- $= \int_a^b f(x)dx$

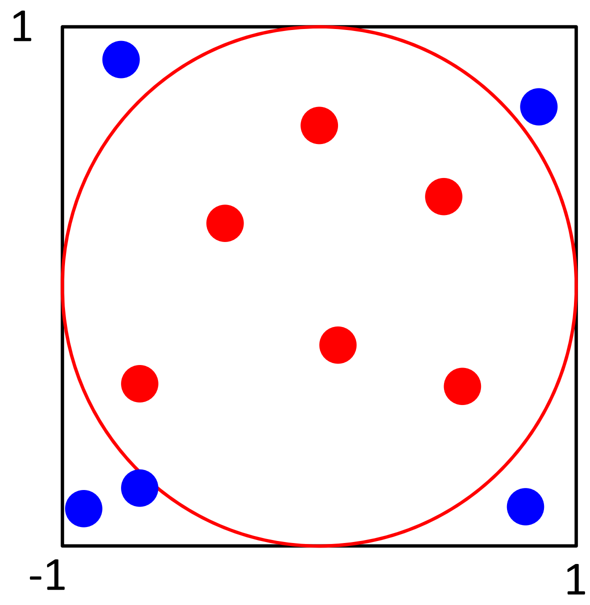
Monte Carlo Integration

- A sampling method based on probability
- Draw random numbers and approximate an integral
- e.g., evaluate a 1D integral: $\int_a^b f(x)dx$
- $F_n = \frac{b-a}{n} \sum_{i=1}^{n} f(x_i)$
- $x_i \in [a, b]$: from a probability density function $p(x)$
  - $F_n = \frac{1}{n} \sum_{i=1}^{n} \frac{f(x_i)}{p(x_i)}$
  - Indicate that we can control sampling density by adjusting $p(x)$
  - Require a good sampling strategy (e.g., *important sampling* – a research area)
  - The approximation error decreases as the number of samples increases

Monte Carlo Integration

- $f(x, y) = 1$ if $x^2 + y^2 \leq 1$
- $= 0$ otherwise

- $I = \int_{\Omega} f(x, y) dxdy = \pi$
- $\Omega = [-1, -1] \times [1,1]$
- $I \approx \frac{V}{n} \sum_{i=1}^{n} f(x_i, y_i)$
- V is the volume of the Ω
  - V = 4
- (x_i, y_i) has a uniform pdf

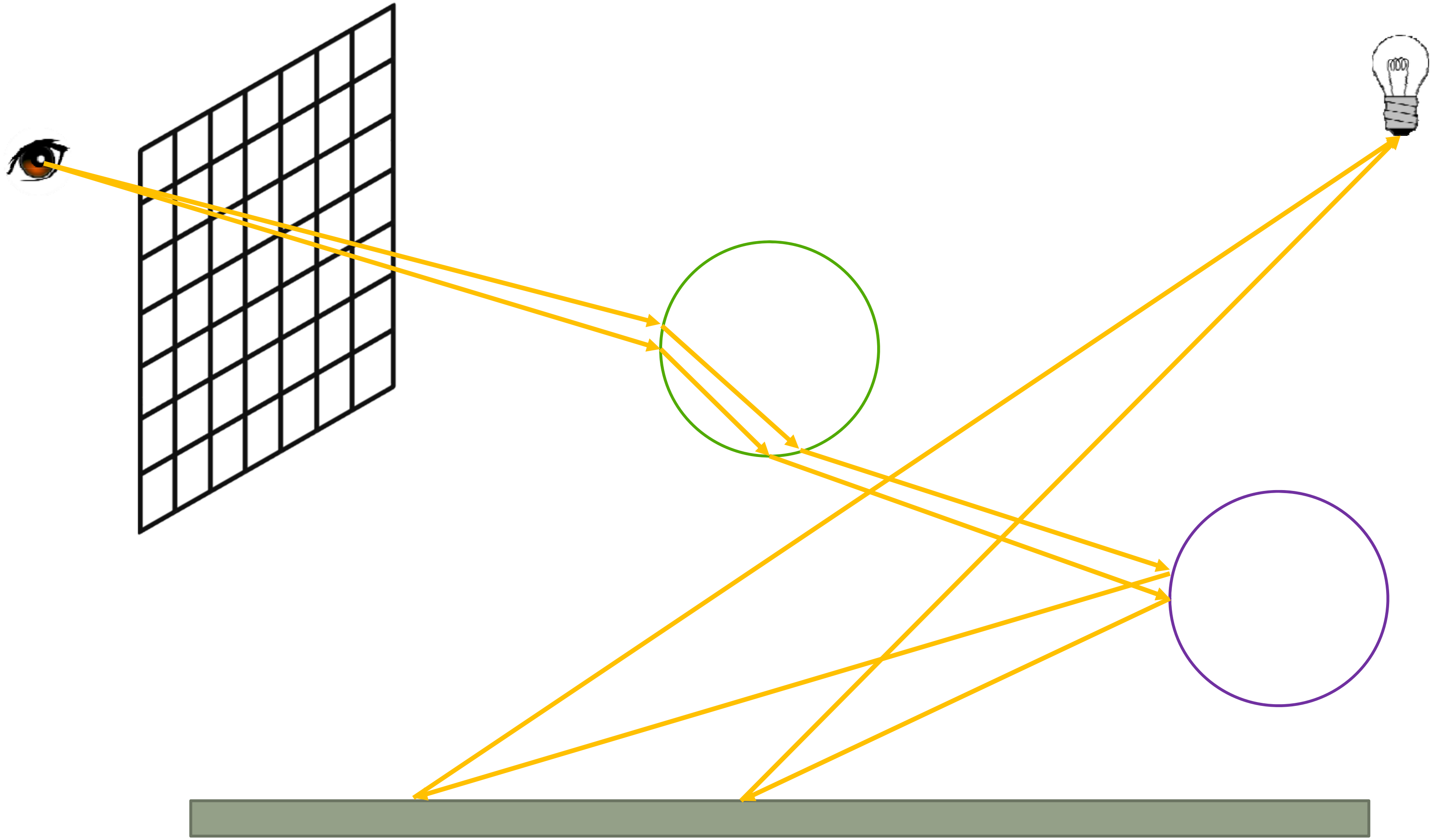


Monte Carlo Integration

- $L_s(\boldsymbol{x}, w_o) = \int_{all\ \boldsymbol{x'}} \frac{\rho(\boldsymbol{w_i},\boldsymbol{w_o})L_s(\boldsymbol{x'},\boldsymbol{x}-\boldsymbol{x'})v(\boldsymbol{x},\boldsymbol{x'})cos\theta_i cos\theta'}{\|\boldsymbol{x}-\boldsymbol{x'}\|^2} dA$

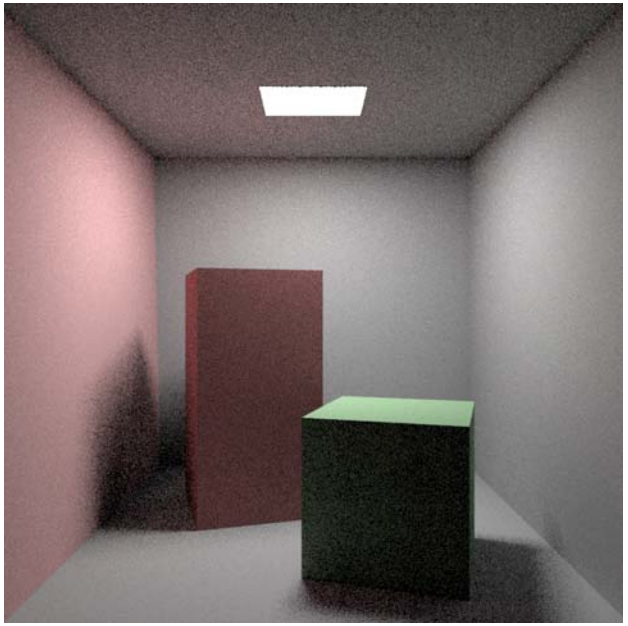
- Monte Carlo ray tracing
  - e.g., distributed ray tracing, path tracing
  - A random sample x_i is a high-dimensional vector
  - $x_i = [random\ numbers\ on\ image\ plane, lens, time, area\ lights, for\ secondary\ rays]$
  - Essentially the integral is a high-dimensional integral
  - The pixel color is an output by averaging the colors from multiple light paths

Path Tracing

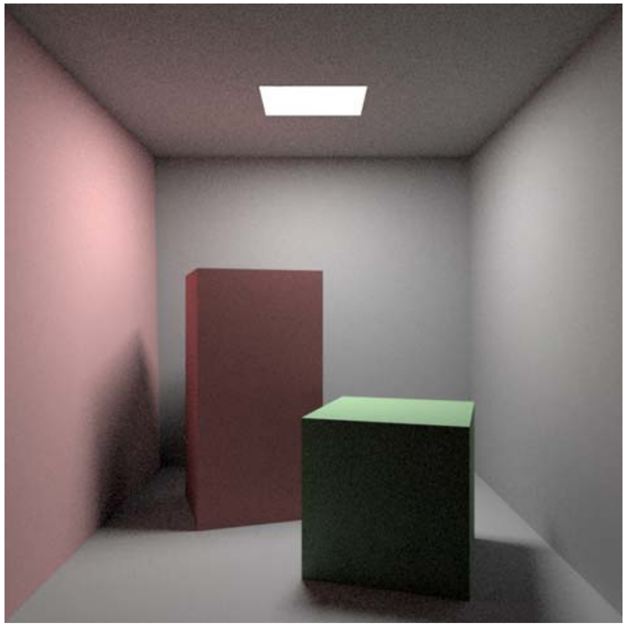


Path Tracing

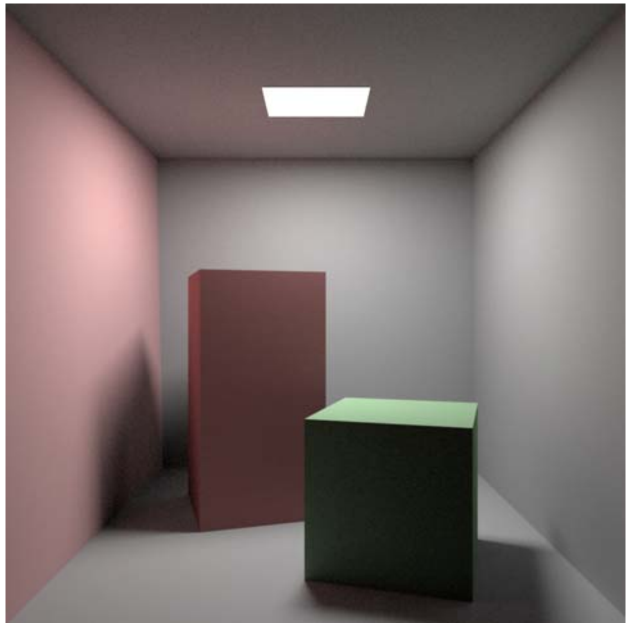
- A general rendering method that solves the full light transport equation (i.e., rendering equation)
- For each pixel color, it makes multiple ray paths, then averages the colors from the ray paths



4 samples / pixel (1.25 secs)



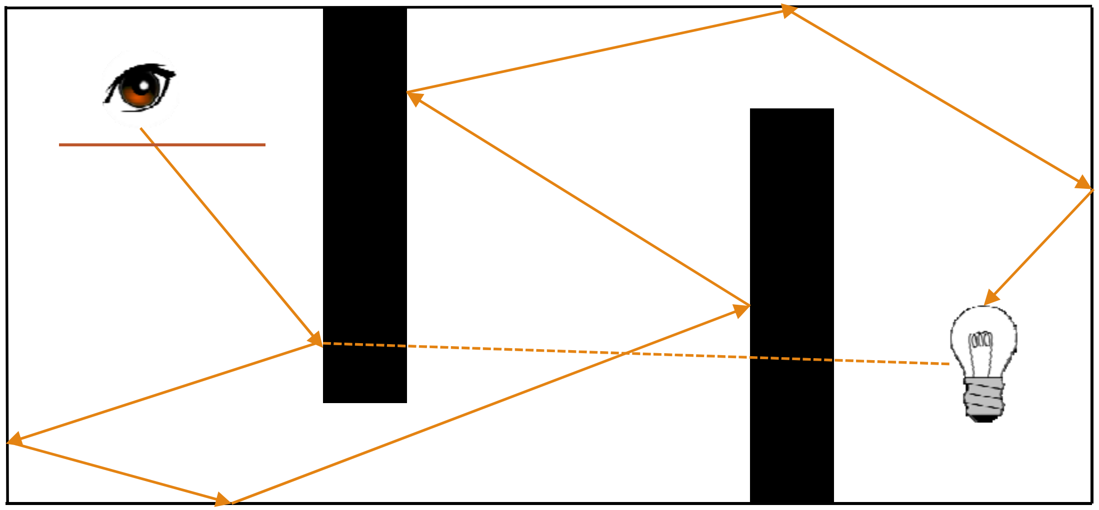
16 samples / pixel (5 secs)



64 samples / pixel (20 secs)

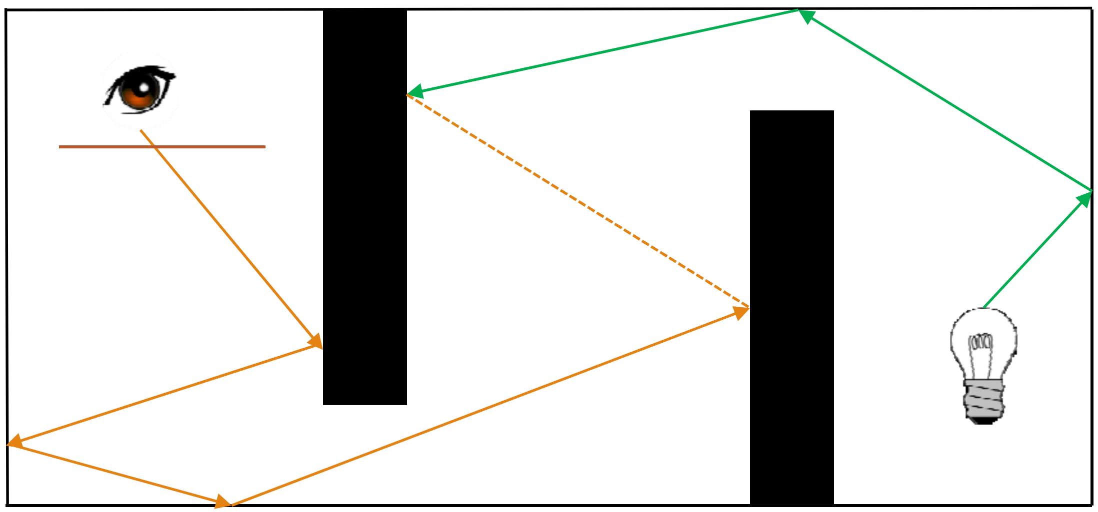
Path Tracing

- Find each light path starting from eye
  - Can be inefficient when light paths are difficult to reach from the eye



Bidirectional Path Tracing

- Find each light sub-path from eye and light
- Connect the sub-paths

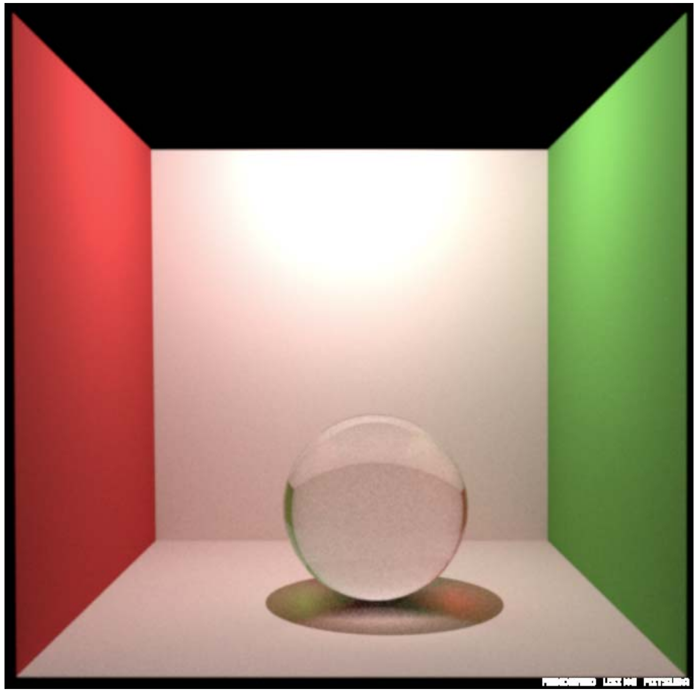


Path tracing, 64 samples per pixel

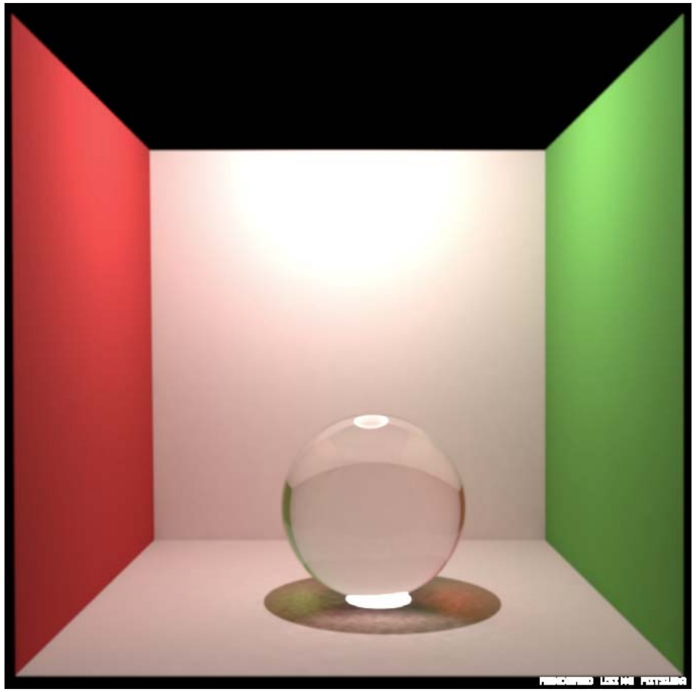
Bidirectional Path tracing, 64 samples per pixel

Photon Mapping

- [Jensen 1996]
- Motivation: it is hard to simulate some rendering effects (e.g., caustics) unless a large number of ray samples are used



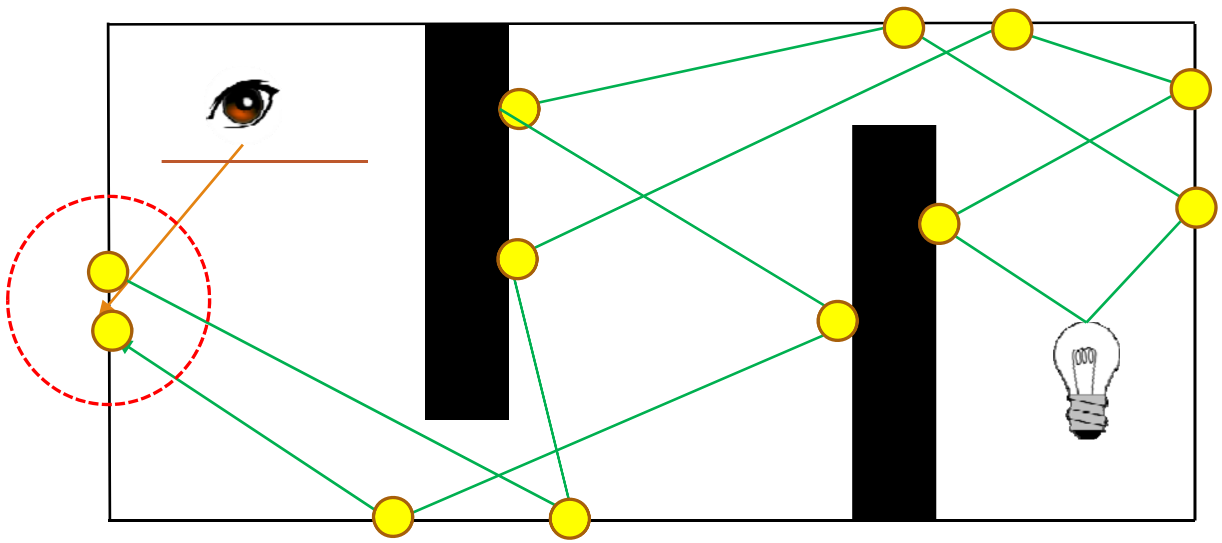
Path tracing



Photon mapping

Photon Mapping

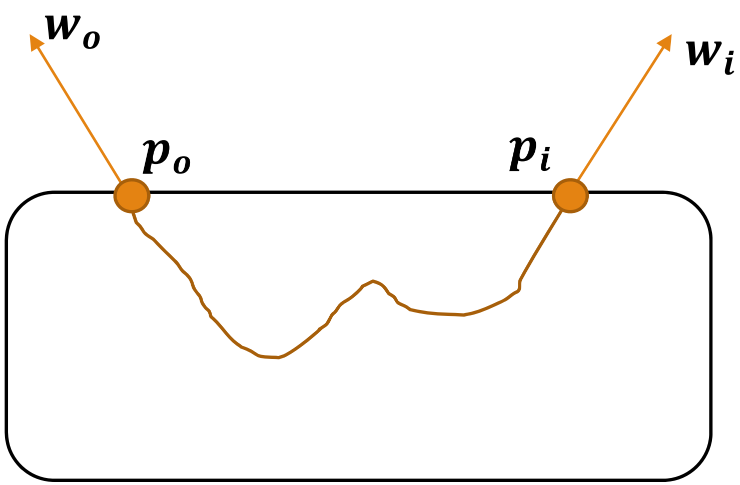
- Generate photons from each light and store them in kd-tree
- Trace primary rays and estimate a photon density at intersected points
  - Similar to the bidirectional path tracing, but the stored photons can be utilized for all eye rays (why?)



Advanced Topics

- Modeling complex materials
  - Bidirectional scattering-surface reflectance distribution function (BSSRDF)

- $\rho(\boldsymbol{w_i}, \boldsymbol{w_o})$: BRDF
- $\rho(\boldsymbol{p_i}, \boldsymbol{w_i}, \boldsymbol{p_o}, \boldsymbol{w_o})$: BSSRDF



Human head model with a BSSRDF
Model courtesy of Infinite Realities, Inc.
Image comes from pbrt.org

Advanced Topics

- Complex geometry (e.g., hair, fur) and scattering

[Jakob et al. 2010]

Model courtesy of Cem Yuksel
Image comes from pbrt.org

Advanced Topics

- Importance Sampling
  - $F_n = \frac{1}{n}\sum_{i=1}^{n}\frac{f(x_i)}{p(x_i)}$
  - Can we estimate an optimal pdf instead of the uniform distribution?

- Denoising

Path tracing,
16 samples per pixel

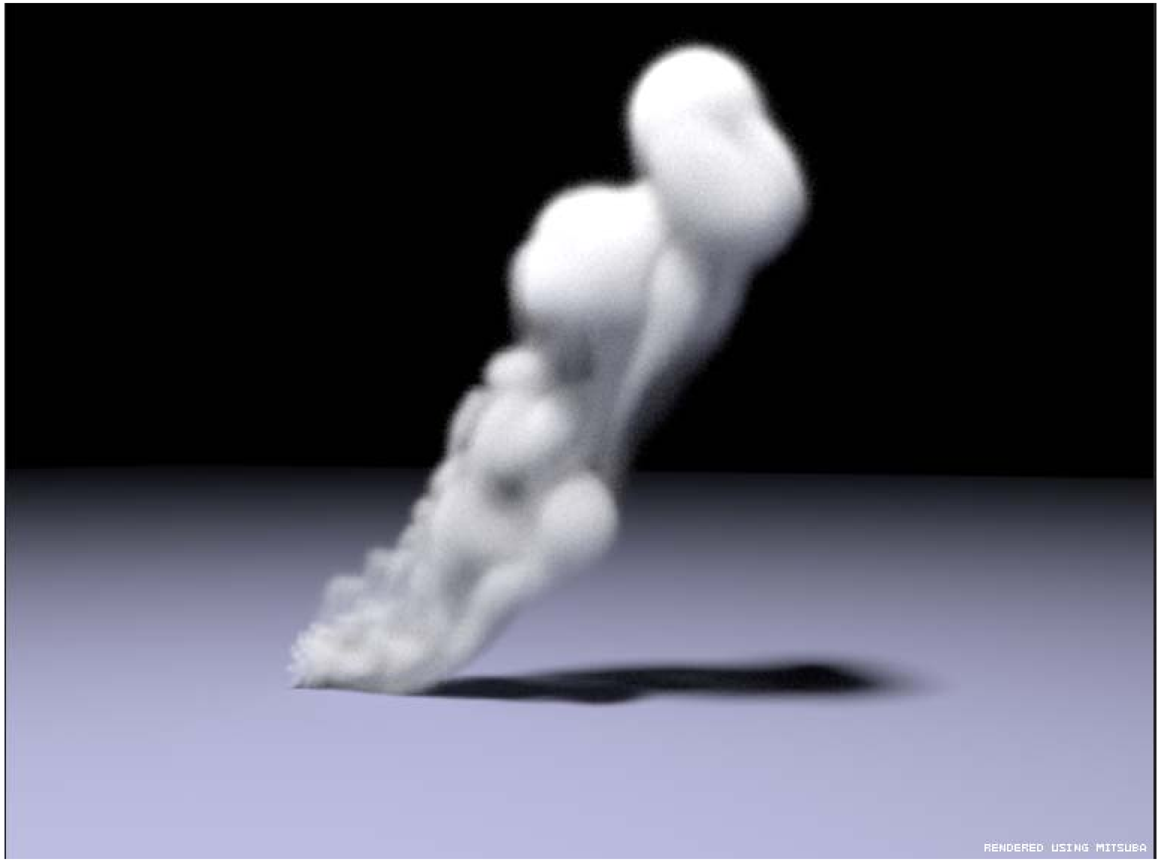
Denoised image,
16 samples per pixel

Path tracing
32K samples per pixel

Images from [Moon et al. 2016]

Advanced Topics

- Volume rendering



Graphics Topics

- Graphics Areas
  - Rendering
  - Modeling
  - Animation
  - User interaction
  - Augmented & virtual reality
  - Visualization
  - etc.

- Check out recent siggraph papers to know the active areas

Announcements

- Final exam
  - 1:00-2:30pm, June 14 (Wed.), 2017
  - The scope includes all materials that we discussed

- (potentially) related course?
  - Computer Graphics Trans-disciplinary Projects
  - Photorealistic Rendering