CT5510: Computer Graphics

Whitted Ray Tracing

BOCHANG MOON

Whitted Ray Tracing

- Recursively perform the ray casting to simulate refraction and reflection

Whitted (1980)

An Improved Illumination Model [Whitted 80]

- Phong illumination model
  - $I = \sum_{i=1}^{\# \; of \; lights} \left\{ L_a^i k_a + L_d^i k_d \max(0, \boldsymbol{n} \cdot \boldsymbol{l^i}) + L_s^i k_s \max(0, \boldsymbol{r^i} \cdot \boldsymbol{v})^S \right\}$

- Whitted illumination model
  - $I = \sum_{i=1}^{\# \; of \; lights} \{ L_a^i k_a + L_d^i k_d \max(0, \boldsymbol{n} \cdot \boldsymbol{l^i}) \} + k_s S + k_t T$

  - S: intensity of light coming from a reflection ray
  - T: intensity of light that comes from a transmission ray
  - k_s, k_t: coefficients for specular and transmission

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  - S: intensity of light coming from a reflection ray
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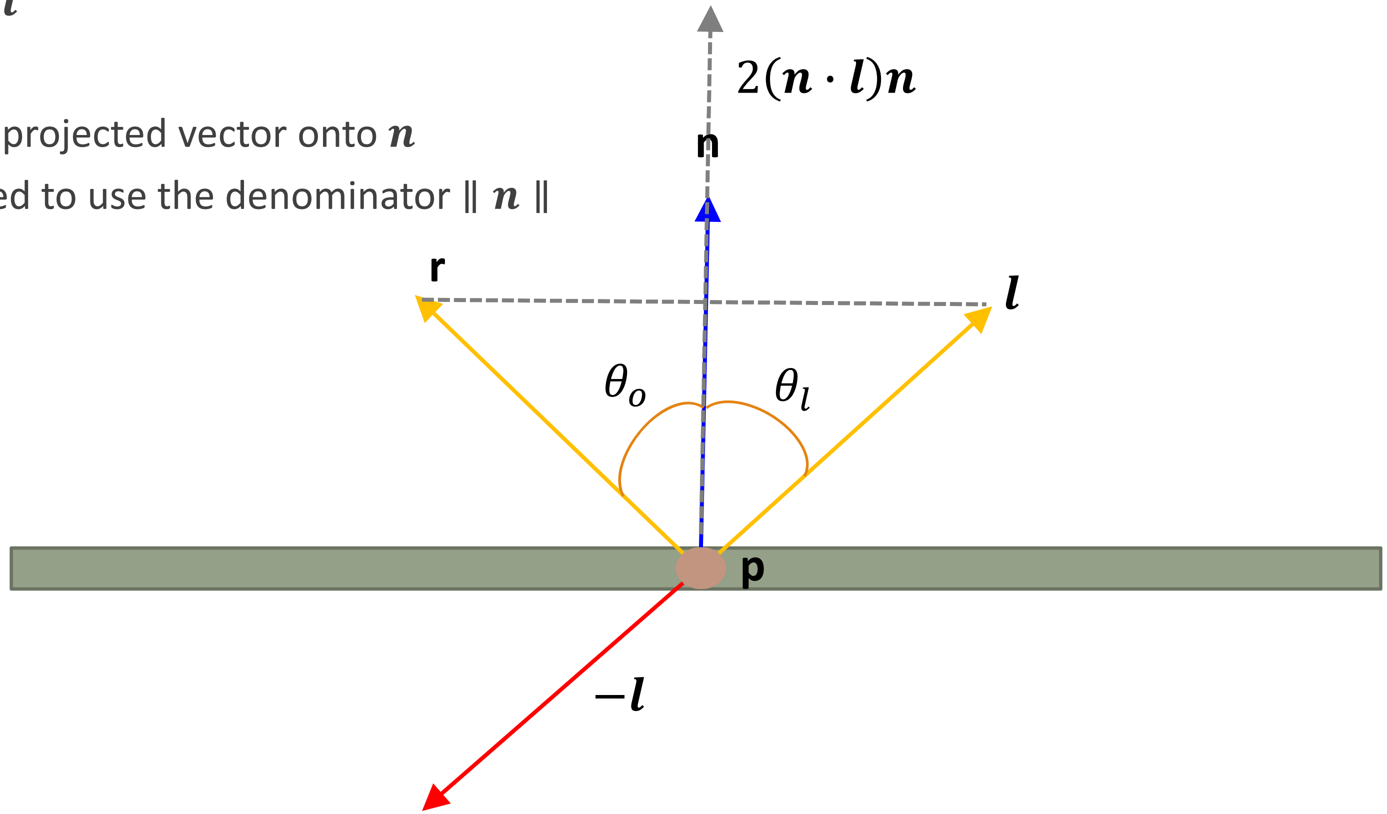
  - Q. how we can compute the rays?
    - Note that we need to generate the secondary rays to find S and T

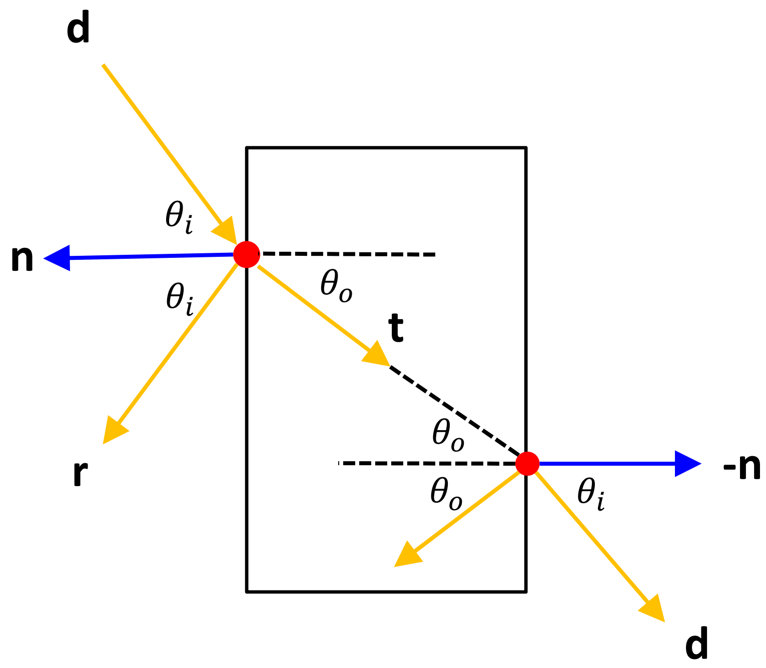
Specular Reflection

- The reflection vector, r, can be computed as the following:
  - $\boldsymbol{r} = 2(\boldsymbol{n} \cdot \boldsymbol{l})\boldsymbol{n} - \boldsymbol{l}$
  - $\boldsymbol{n} \cdot \boldsymbol{l}$: length of the projected vector onto $\boldsymbol{n}$
  - Note: we don't need to use the denominator $\| \boldsymbol{n} \|$
- e.g., mirror



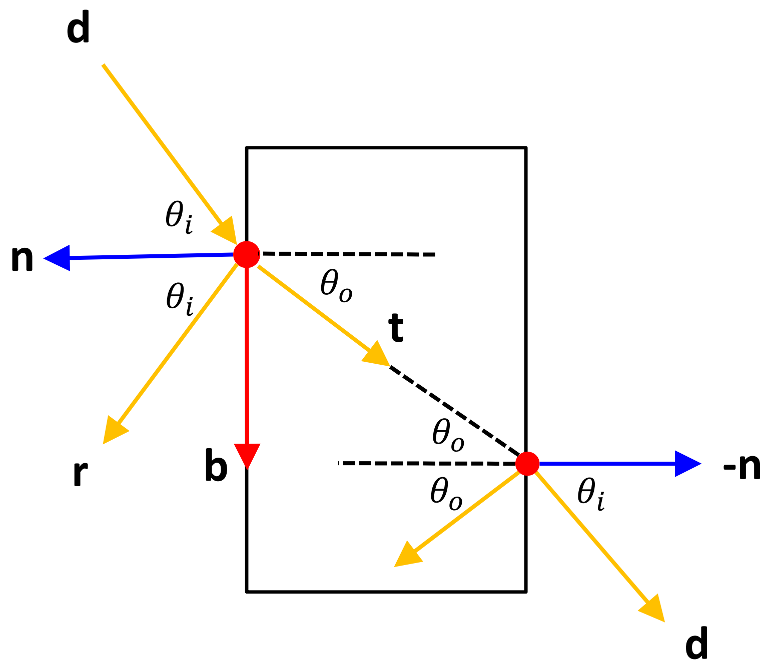
Specular Refraction

- Dielectric materials (e.g., diamonds, glass, water, and air)
  - Transparent objects that refract light
- Snell's law
  - $n_i sin\theta_i = n_o sin\theta_o$
    - n_i, n_o: indices of refraction
  - $n_i^2 \left(1 - \cos^2 \theta_i\right) = n_o^2 \left(1 - \cos^2 \theta_o\right)$
    - $\because$ *trigonometric identity* $(\sin^2\theta + \cos^2\theta = 1)$
  - $\cos^2 \theta_o = 1 - \frac{n_i^2 (1 - \cos^2 \theta_i)}{n_o^2}$

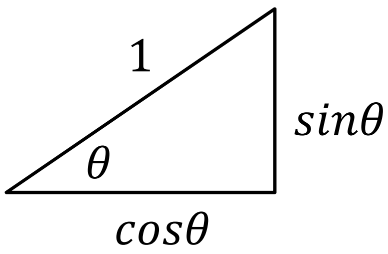


Specular Refraction

- Vectors **n** and **b** forms an orthonormal basis
- $\boldsymbol{t} = sin\theta_o \boldsymbol{b} - cos\theta_o \boldsymbol{n}$
  - Unknowns: ***t, b***
- $\boldsymbol{d} = sin\theta_i \boldsymbol{b} - cos\theta_i \boldsymbol{n},$
- $\boldsymbol{b} = \frac{\boldsymbol{d}+\boldsymbol{n}cos\theta_i}{sin\theta_i}$



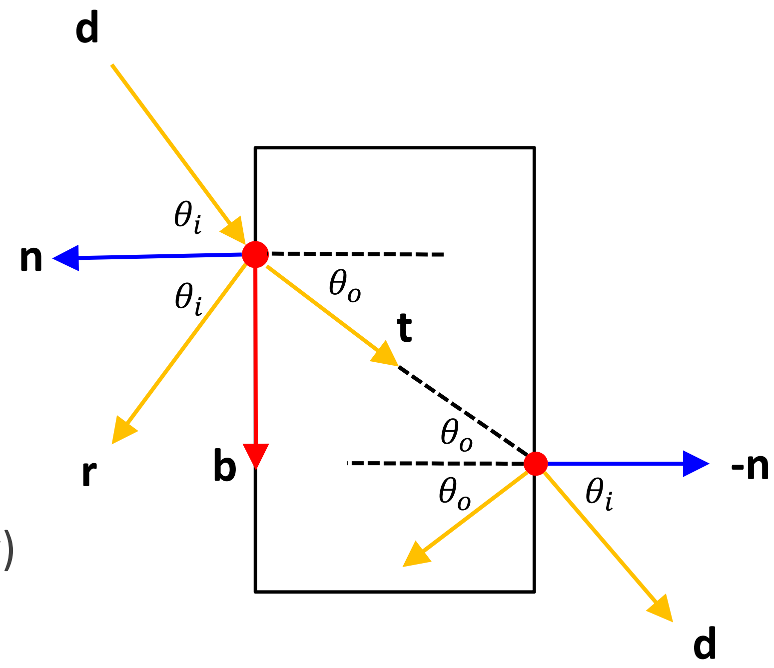
- Trigonometric functions



Specular Refraction

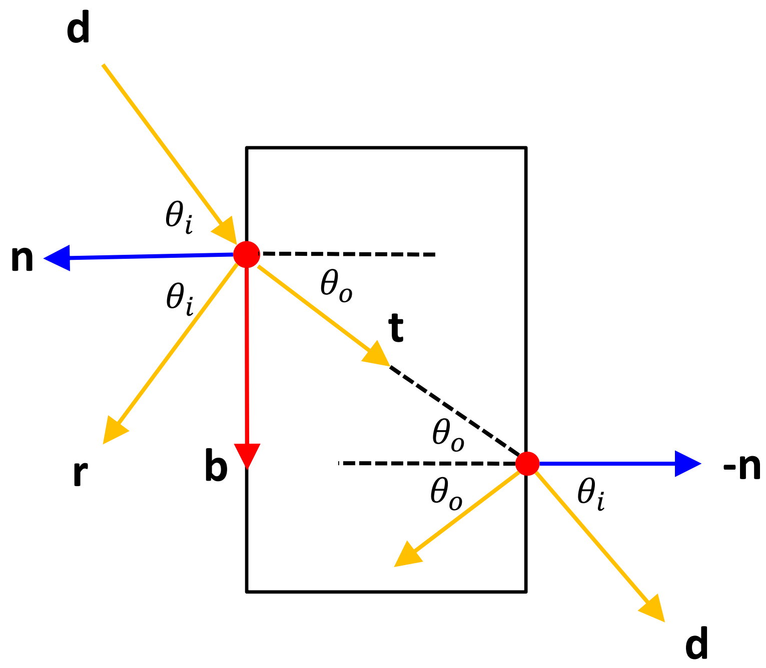
- $\boldsymbol{t} = sin\theta_o \boldsymbol{b} - cos\theta_o \boldsymbol{n}$
  - Unknowns: ***t, b***
- $\boldsymbol{b} = \frac{\boldsymbol{d} + \boldsymbol{n} cos\theta_i}{sin\theta_i}$
- By plug-in **b** into the equation for **t**:
- $\boldsymbol{t} = sin\theta_o \frac{\boldsymbol{d} + \boldsymbol{n} cos\theta_i}{sin\theta_i} - cos\theta_o \boldsymbol{n}$
- $= \frac{n_i(\boldsymbol{d} + \boldsymbol{n} cos\theta_i)}{n_o} - cos\theta_o \boldsymbol{n}$ (by Snell's law)
- $= \frac{n_i(\boldsymbol{d} - \boldsymbol{n}(\boldsymbol{d} \cdot \boldsymbol{n}))}{n_o} - \boldsymbol{n}\sqrt{1 - \frac{n_i^2(1 - (\boldsymbol{d} \cdot \boldsymbol{n})^2)}{n_o^2}}$

  - $\cos^2\theta_o = 1 - \frac{n_i^2(1 - \cos^2\theta_i)}{n_o^2}$



Specular Refraction

- The transmitting ray's direction $\boldsymbol{t}$
- $\boldsymbol{t} = \frac{n_i(\boldsymbol{d}-\boldsymbol{n}(\boldsymbol{d}\cdot\boldsymbol{n}))}{n_o} - \boldsymbol{n}\sqrt{1-\frac{n_i^2(1-(\boldsymbol{d}\cdot\boldsymbol{n})^2)}{n_o^2}}$
- $1-\frac{n_i^2(1-(\boldsymbol{d}\cdot\boldsymbol{n})^2)}{n_o^2} < 0$
  - No refracted ray (i.e., all energy is reflected)
- Q. what are the amount of reflected and refracted energy (i.e., reflectivity)?

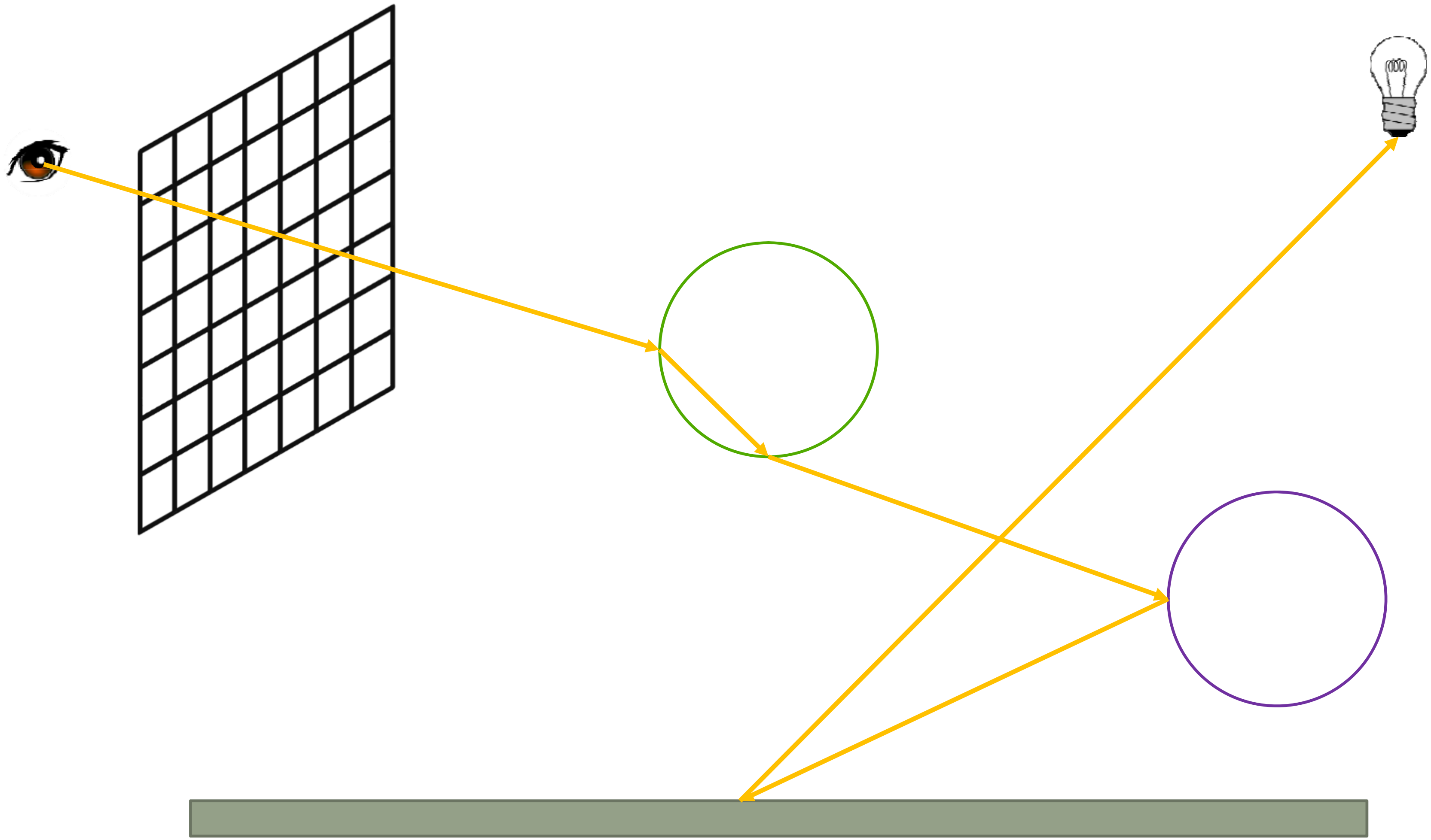


Specular Refraction

- The reflectivity varies with the incident direction of light and it is determined by *Fresnel equations* (from physics).

- Schlick approximation (1994) of the Fresnel equations
  - $R(\theta_i) = R_0 + (1 - R_0)(1 - cos\theta_i)^5$
  - $R_0 = \left(\frac{n_o - 1}{n_o + 1}\right)^2$

Whitted Ray Tracing



Whitted Ray Tracing

- For each pixel
  - Color c = (0, 0, 0)
  - Generate a primary ray (camera ray)
  - Find the closest intersection point between the ray and objects
  - If (there is a hit) then
    - Generate a shadow ray
    - If (there is no hit between the shadow ray and a light) then
      - c = c + shading()
    - Generate a secondary ray (reflection or refraction ray)
    - Go to the third line
  - Else
    - c = c + background color
  - Set the pixel color with c

- Issue: cannot trace a ray path that has infinite depth

Whitted Ray Tracing

- For each pixel
  - Color c = (0, 0, 0)
  - Generate a primary ray (with depth 0)
  - While (depth < d)
    - Find the closest intersection point between the ray and objects
    - If (there is a hit) then
      - Generate a shadow ray
      - If (there is no hit between the shadow ray and a light) then
        - c = c + shading()
      - Generate a secondary ray (reflection or refraction ray) // increase the ray depth +1
    - Else
      - c = c + background color
  - Set the pixel color with c

Whitted Ray Tracing

- Missing effects?
  - Soft shadow
  - Glossy reflection
  - Diffuse reflection
  - Depth-of-field effect
  - Motion blur
  - …

- Advanced ray tracing
  - Distributed ray tracing
  - Path tracing and photon mapping

Whitted (1980)

Further Readings

- Chapter 4, 13
- Papers
  - T. Whitted. An improved illumination model for shading display. Communications of the ACM, 23(6):343–349, 1980