CT5202: Photorealistic Rendering

Advanced Monte Carlo Techniques

Lecturer: Bochang Moon

Overview

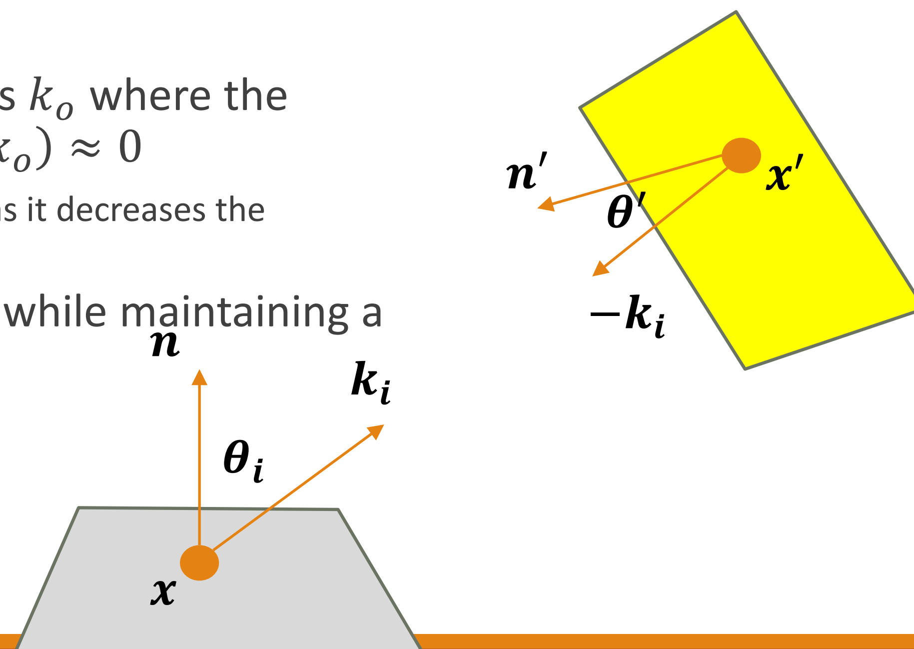
- Study important techniques to improve efficiency of MC estimators

- An efficiency measure for estimators F
  - $e(F) = \frac{1}{V(F)T(F)}$
  - To improve efficiency, we need to reduce the variance and time

- Russian Roulette
- Splitting
- Importance Sampling

Russian Roulette

- e.g. direct lighting
  - $L_s(x, k_o) = \int_{all\ x'} \frac{\rho(k_i,k_o)L_e(x',-k_i)v(x,x')cos\theta_i cos\theta'}{||x-x'||^2} dA'$
    - $\approx \frac{1}{N}\sum_{i=1}^{N} \frac{\rho(k_i,k_o)L_e(x',-k_i)v(x,x')cos\theta_i cos\theta' A}{p(x')||x-x'||^2}$
- Problems
  - Expensive to compute $v(x, x')$
  - Let suppose that there are some directions k_o where the integrand's value is almost 0 due to $\rho(k_i, k_o) \approx 0$
    - In this case, evaluating $v(x, x')$ is not a good idea, as it decreases the efficiency
  - Q. Can we somehow skip these directions while maintaining a correct answer?



Russian Roulette

- Given an estimator F, a new estimator F' with Russian Roulette can be given:
  - $F' = \begin{cases} \frac{F-qc}{1-q} & \xi > q \\ c & otherwise \end{cases}$
  - q: termination probability
  - c is usually chosen as 0

- Consistency check
  - $E(F') = (1-q)\left(\frac{E(F)-qc}{1-q}\right) + qc = E(F)$

- Properties
  - It does not reduce variance, but improves efficiency by skipping unimportant parts

Russian Roulette

- Examples
  - $L_s(k_o) \approx L_e(k_o) + \frac{\rho(k_i,k_o)L_f(k_i)cos\theta_i d\sigma_i}{p(k_i)}$
  - Problem: this is a recursive form, so ray depth can be infinite
  - Can apply the Russian Roulette to path tracing so that the ray depth can be reduced
  - $F' = \begin{cases} \frac{F-qc}{1-q} & \xi > q \\ c & otherwise \end{cases}$
  - Given a termination probability $q = 0.5\ and\ c = 0$, we can terminate the reflection with the probability. In this case, we need to scale the radiance value with $\frac{1}{1-q}$

Russian Roulette

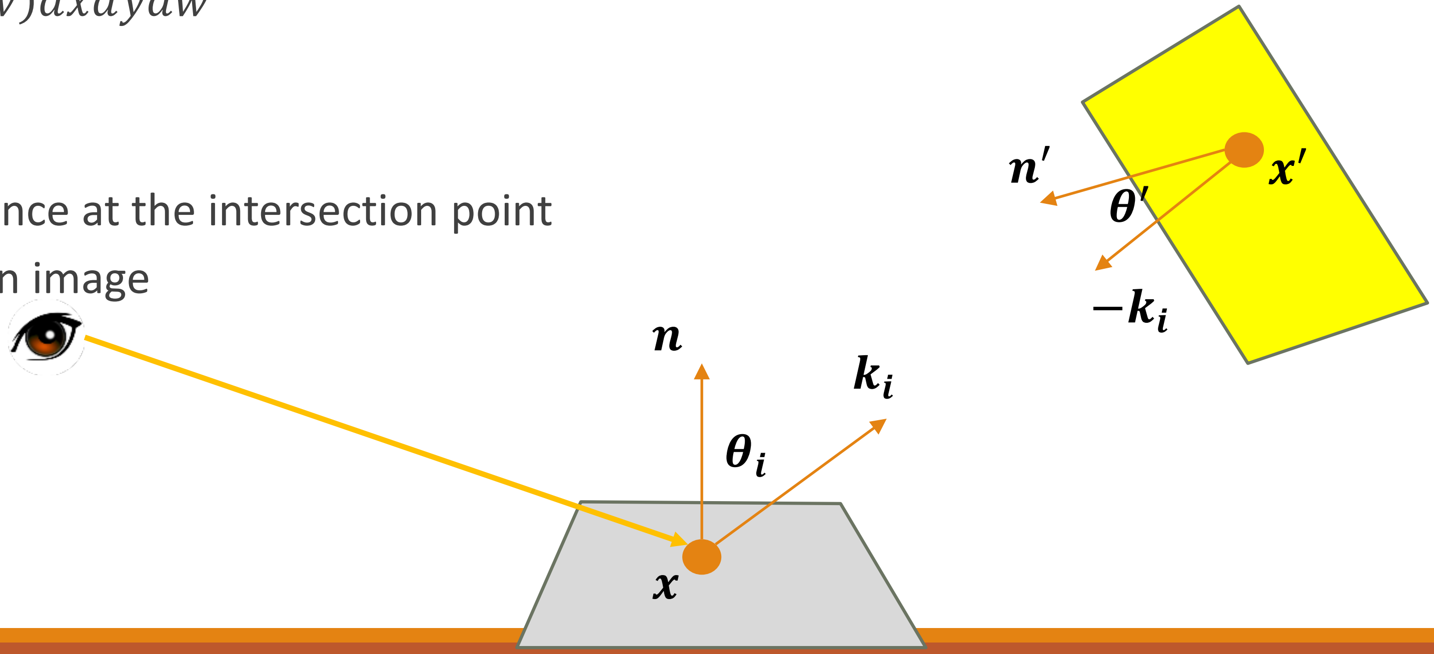
- Bad examples
  - Apply this to the camera rays with a termination probability $q = 0.99$
  - In this case,
    - Only trace 1% of the camera rays
    - Most of pixels are black and a few pixels are very bright, although its expectation is still correct
    - The variance of the estimator will be much higher than the original estimator

- Efficiency-optimized Russian roulette
  - A technique that optimizes the parameter

Splitting

- Splitting is a technique to increase the number of samples for improving the efficiency
  - Allocate more rays to important dimensions

- E.g., direct lighting with a shortened version
  - $\int_A \int_S L_d(x, y, w) dxdydw$
  - A: pixel area
  - S: light area
  - L_d: exitant radiance at the intersection point
  - (x, y): position on image



Splitting

- direct lighting with a shortened version
  - $\int_A \int_S L_d(x, y, w) dxdydw$
- A straightforward approach:
  - Generate N samples $(x_1, y_1, w_1), \dots (x_N, y_N, w_N)$
  - Evaluate $L_d(x_1, y_1, w_1), \dots, L_d(x_N, y_N, w_N)$
    - Need to generate N shadow rays
  - Average the radiance values

- Typically need a lot of samples given a large area light or many point lights
  - e.g., N = 100, 200 rays will be used (100 primary rays, 100 shadow rays)
  - Issue: 100 primary rays are often too much for a good antialiasing. Can we focus on

Splitting

- Typically need a lot of samples given a large area light or many point lights
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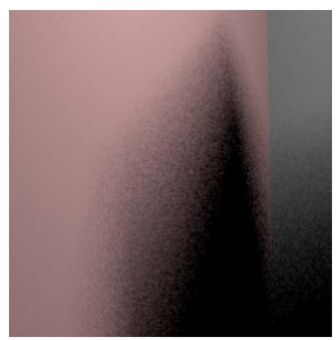
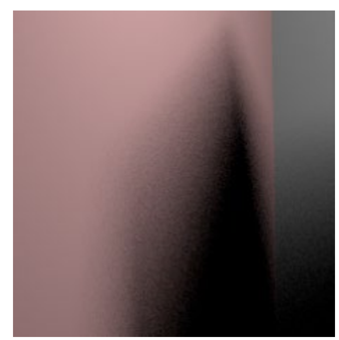
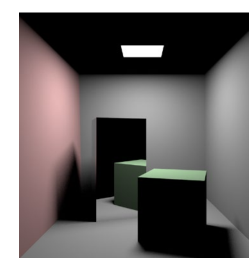
  - Issue: 100 primary rays are often too many for a good antialiasing.
  - Splitting
    - $\frac{1}{N}\frac{1}{M}\sum_{i=1}^{N}\sum_{j=1}^{M}\frac{L(x_i,y_i,w_{i,j})}{p(x_i,y_i)p(w_{i,j})}$
    - Are able to use 5 image samples and take 20 light samples per image sample. Total ray # = 5 + 5 x 20 = 105
    - Still use 100 shadow rays for high-quality soft shadows, but the total number of rays is reduced


4 shadow rays / pixel


16 shadow rays / pixel


64 shadow rays / pixel



Importance Sampling

- A variance reduction technique for Monte Carlo estimators
  - Allocate more samples to the important region where the integrand's value is high

- A Monte Carlo estimator:
  - $F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(X_i)}{p(X_i)}$

- e.g., $L_s(k_o) \approx L_e(k_o) + \frac{\rho(k_i,k_o)L_f(k_i)cos\theta_i d\sigma_i}{p(k_i)}$
  - An intuition is that if we take more samples in terms of k_i that makes $cos\theta_i$ high, we can reduce the variance of the estimator

Importance Sampling

- e.g., Evaluate an integral $\int f(x)dx$
- Note that we can choose an arbitrary pdf, $p(x)$
- What if we choose $p(x) \propto f(x)$ or $p(x) = cf(x)$
  - $c = \frac{1}{\int f(x)dx}$ a constant for normalization
  - $F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(X_i)}{p(X_i)} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{c}$
  - $V(F_N) = 0$
- Notes
  - In practice, we cannot choose $p(x)$ in this way, but it provides some intuition
  - If we choose $p(x)$ similarly compared to the shape of $f(x)$, we are able to reduce variance

Importance Sampling

- A counterexample:
  - Evaluate an integral $\int f(x)dx$
  - $p(x) = \begin{cases} 99.01 & x \in [0,0.01) \\ 0.01 & x \in [0.01,1] \end{cases}$
  - $f(x) = \begin{cases} 0.01 & x \in [0,0.01) \\ 1.01 & x \in [0.01,1] \end{cases}$
  - $\int f(x)dx = 1$
  - Most of samples will be taken from $[0,0.01)$, and $\frac{f(X_i)}{p(X_i)} \approx 0.0001$ which is far from 1
  - In this case, the variance will increase a lot

- Note
  - In practice, it is easy to apply an important sampling to the rendering, by considering only some terms
  - Taking account for all terms is ideal but this can be very challenging

More Topics?

- Evaluate an integral $\int f(x)g(x)dx$
- If we have each important sampling scheme for the functions $f(x)$ and $g(x)$, how can we combine the techniques?
- Multiple important sampling addresses this issue and this will be discussed later