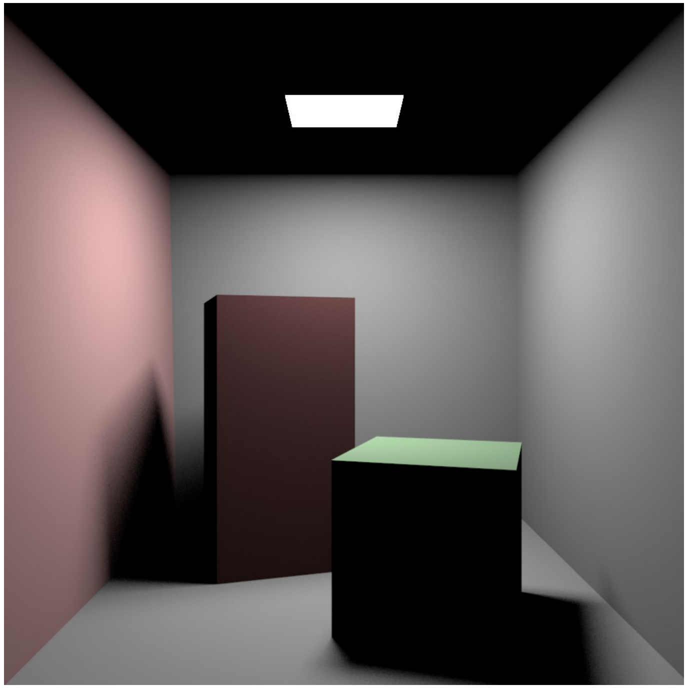
CT5202: Photorealistic Rendering

Global Illumination

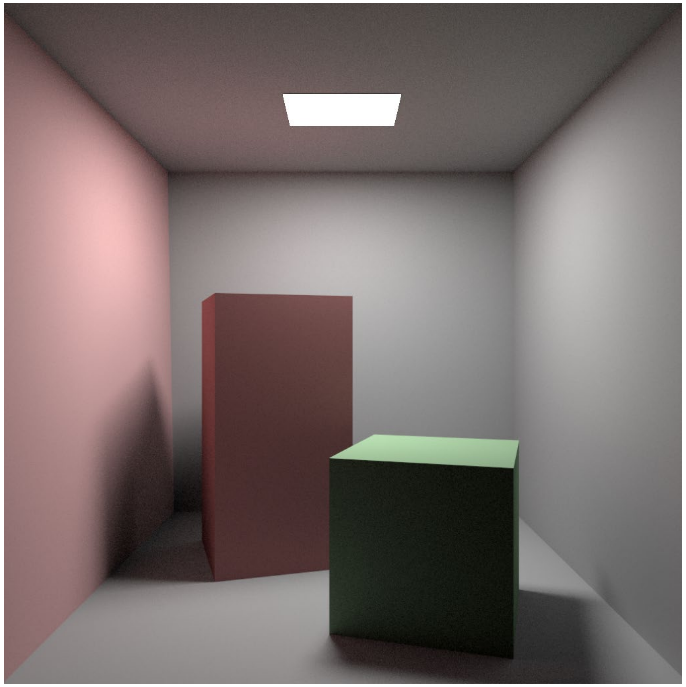
Lecturer: Bochang Moon

Global Illumination

- Global illumination methods consider both direct and indirect lighting



Without indirect lighting



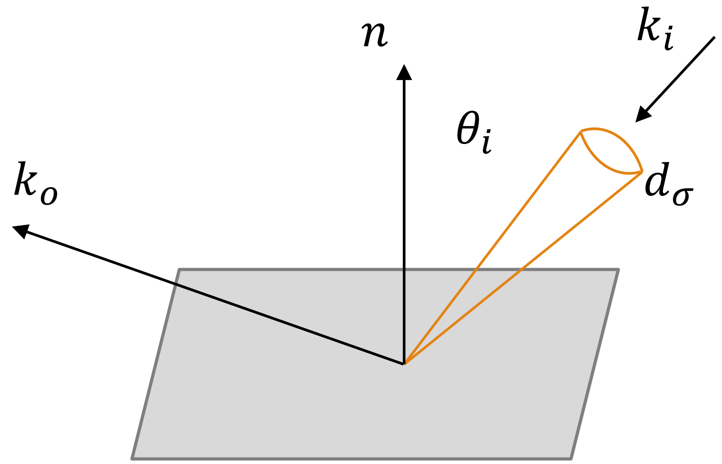
With indirect lighting

Radiosity

- $L_s(k_o) = \int_{all\ k_i} \rho(k_i, k_o) L_f(k_i) cos\theta_i d\sigma_i$

- Lambertian surfaces (ideal diffuse surface)
  - $\rho(k_i, k_o) = \frac{R}{\pi}$
  - R: diffuse reflectance

- Assumption: all surfaces are Lambertian
- $L_s(k_o) = \int_{all\ k_i} \rho(k_i, k_o) L_f(k_i) cos\theta_i d\sigma_i$
- $= \frac{R}{\pi} \int_{all\ k_i} L_f(k_i) cos\theta_i d\sigma_i$

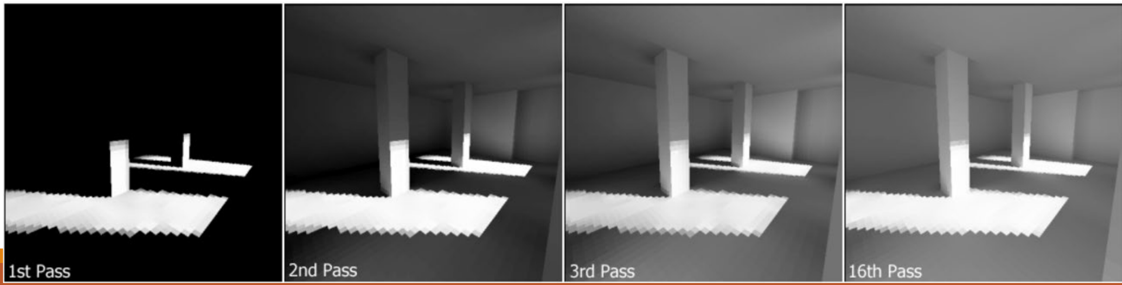


Radiosity

- $L_s(k_o) = \frac{R}{\pi} \int_{all\ k_i} L_f(k_i) cos\theta_i d\sigma_i$

- Finite element methods
  - Divide the scene into N small surfaces (patches) with unknown surface radiance L_i, reflectance R_i, and emitted radiance E_i
  - Then, the integral can be approximated with the N linear equations below:
  - $L_i = E_i + \frac{R_i}{\pi} \sum_{j=1}^{N} k_{ij} L_j$
  - k_{ij}: a constant related to the integral (form factor)
    - Fraction of light leaving a patch i arriving at a patch j
  - This results in N constant-colored polygons
  - Called *radiosity*

from wikipedia



Path Tracing

- $L_s(k_o) = L_e(k_o) + \int_{all\ k_i} \rho(k_i, k_o) L_f(k_i) cos\theta_i d\sigma_i$
- Monte Carlo integration
  - $\int_{x \in S} g(x) d\mu \approx \frac{1}{N} \sum_{i=1}^{N} \frac{g(x_i)}{p(x_i)}$
  - When N=1,
  - $L_s(k_o) \approx L_e(k_o) + \frac{\rho(k_i,k_o) L_f(k_i) cos\theta_i}{p(k_i)}$
  - Need to do:
    - Select a random direction k_i
    - Evaluate $L_f(k_i)$

Path Tracing

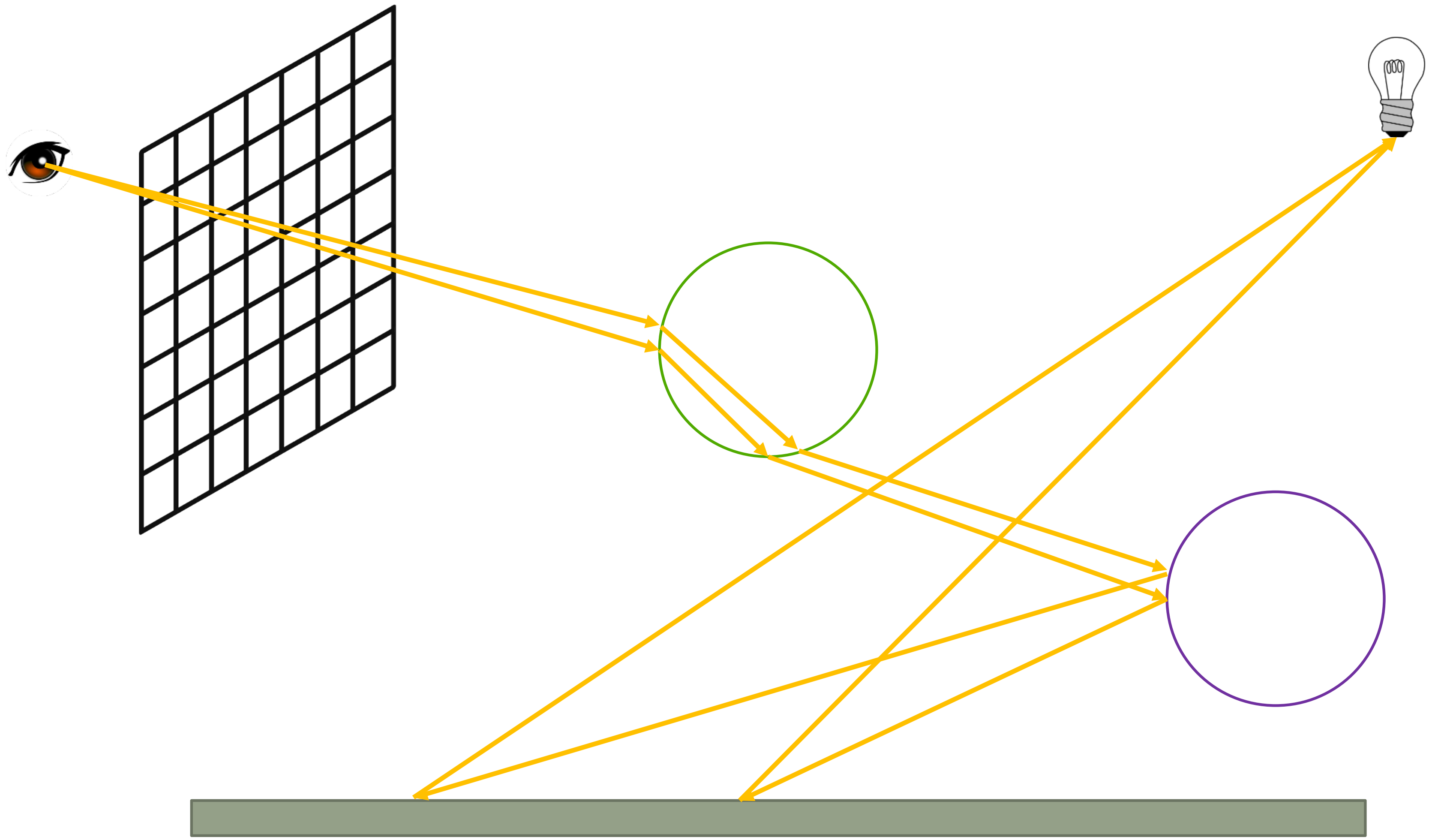
- $L_s(k_o) \approx L_e(k_o) + \frac{\rho(k_i,k_o)L_f(k_i)cos\theta_i d\sigma_i}{p(k_i)}$

- In case of the ideal diffuse surface:
  - $\rho = \frac{R}{\pi}$
  - When we choose a density function $p(k_i) = \frac{cos\theta_i}{\pi}$
    - $L_s(k_o) \approx L_e(k_o) + RL_f(k_i)$
    - Note that we can cancel out the cosign terms

Path Tracing

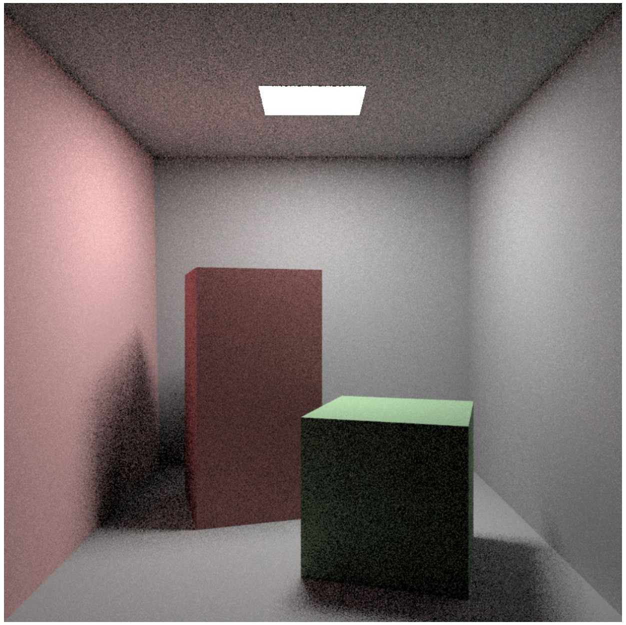
- Procedure
  - $RGB\ trace(ray\ a + tb, int\ depth)$
  - $if\ (ray\ hits\ at\ a\ point\ c)$
    - $RGB\ col = L_e(-b)$
    - $if\ (depth < maxdepth)$
      - $compute\ a\ random\ direction\ d$
      - $return\ col + R \times trace(c + sd, depth + 1)$
  - $else$
    - $return\ background\ color$

Path Tracing

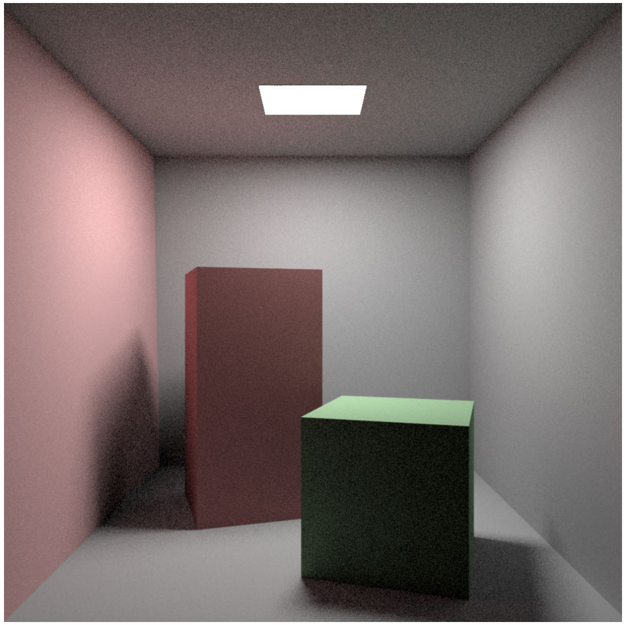


Path Tracing

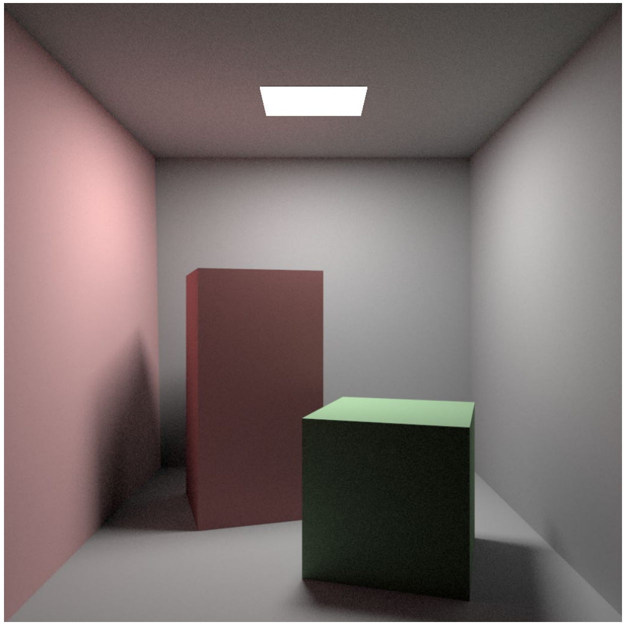
- A general rendering method that solves the full light transport equation (i.e., rendering equation)
- For each pixel color, it makes multiple ray paths, then averages the colors from the ray paths



4 samples / pixel (1.25 secs)



16 samples / pixel (5 secs)



64 samples / pixel (20 secs)

Problems in Naïve Path Tracing

- Issues: Hard to find a light path (very high variance)
  - i.e., the probability of hitting luminaries with small sizes is very low.

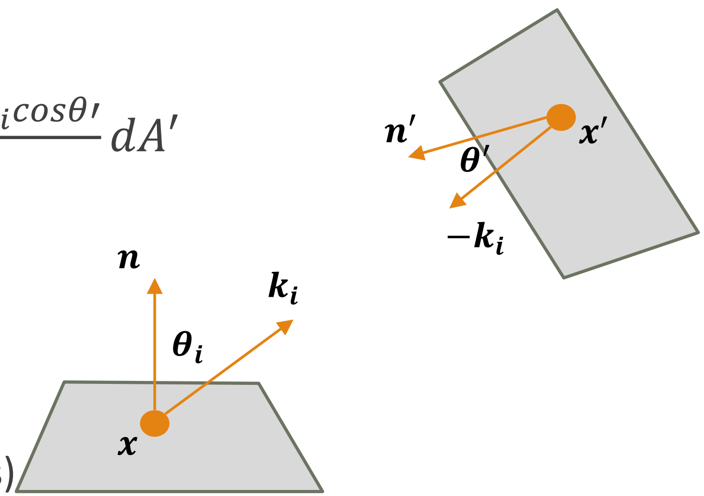
- A common practice for path tracing
  - Path tracing with direct lighting (sometimes referred to as path tracing with next event estimation)

Path Tracing with Direct Lighting

- $\mathrm{L_s}(x, k_o) = \mathrm{L_s^D}(x, k_o) + \mathrm{L_s^I}(x, k_o)$
  - direct lighting $\mathrm{L_s^D}(x, k_o)$ + indirect lighting $\mathrm{L_s^I}(x, k_o)$

- $L_s^{\mathrm{D}}(k_o) = \int_{all\ x' \text{ in luminaries}} \frac{\rho(k_i,k_o)L_{\mathrm{e}}(x',x-x')v(x,x')cos\theta_i cos\theta'}{||x-x'||^2} dA'$
  - L_{e}: emitted radiance

- $L_s^{\mathrm{I}}(k_o) = \int_{all\ k_i} \rho(k_i, k_o) L_f^{\mathrm{R}}(k_i) cos\theta_i d\sigma_i$
  - $L_f^{\mathrm{R}}(k_i)$: field radiance (only reflected, i.e., not from luminaries)



Path Tracing with Direct Lighting

- $L_s^{\mathrm{D}}(x, k_o) = \int_{all\ x'} \frac{\rho(k_i,k_o)L_e(x',-k_i)v(x,x')cos\theta_i cos\theta\prime}{||x-x'||^2} dA'$

- Sample a point x' on a luminaire with density function p ($x' \sim p$)
- $L_s^{\mathrm{D}}(x, k_o) \approx \frac{\rho(k_i,k_o)L_e(x',-k_i)v(x,x')cos\theta_i cos\theta\prime}{p(x')\ ||x - x'||^2}$

- Pick a uniform random point x' from the luminaire
  - $p = \frac{1}{A}$ (A is the area of the luminaire)
  - $L_s^{\mathrm{D}}(x, k_o) \approx \frac{\rho(k_i,k_o)L_e(x',-k_i)v(x,x')cos\theta_i cos\theta' A}{||x - x'||^2}$

Path Tracing with Direct Lighting

- Other details:
  - Only applied when the surface point is non-specular.
  - e.g., narrow BRDFs
    - Sampling a point on luminaries is not effective
  - Uses shadow rays for checking occlusions between the surface point and sampled light point Shadow rays
    - Usually faster than finding the first intersection
  - Samples a point on luminaries can be done via inverse transform sampling when the shapes of luminaries are simple (e.g., spheres, triangles)

Path Tracing with Direct Lighting

- $L_s^{\mathrm{D}}(x, k_o) \approx \frac{\rho(k_i,k_o)L_e(x',-k_i)v(x,x')cos\theta_i cos\theta'}{p(x')\,\|\mathrm{x}-\mathrm{x}'\|^2}$
- Multiple light sources are given:
  - Generate one shadow ray per each light source, but this is not practical with many light sources
  - A common choice is to pick only a point x' and generate a shadow ray towards x'

Path Tracing with Direct Lighting

- $L_s^{D}(x, k_o) \approx \frac{\rho(k_i,k_o)L_e(x',-k_i)v(x,x')cos\theta_i cos\theta'}{p(x')\ ||x - x'||^2}$
- Determining $p(x')$ requires the following:
  - $p(x') = p(l)p(x'|l)$
    - Probability of selecting a luminary l: $p(l)$
    - Probability of sampling a point on the chosen light: $p(x'|l)$
  - How to select a light source l:
    - Uniform: the probability of selecting a light is equal.
    - Spatial: set the probability proportional to the light power (assume that all lights are visible against a point)
    - Visibility-aware selection: requires an estimation of visibility across surface points
    - Light clustering is a well-known approach for many lights, e.g., thousands of lights

Path Tracing with Direct Lighting

- Additional discussion:
  - Sampling a point on luminaries is often effective for Lambertian surfaces but not very effective on highly glossy surfaces
    - BRDF sampling for direct lighting can be better for such cases
    - How to combine Light sampling and BRDF sampling?
      - Multiple importance sampling (MIS) provides a solution; this will be covered later
  - When is direct lighting effective?
    - Intuitively, it is effective when lights are visible from most surfaces
    - A counter example

Image from https://benedikt-bitterli.me/resources/