

CT5202: Photorealistic Rendering

Sampling

Lecturer: Bochang Moon

Sampling

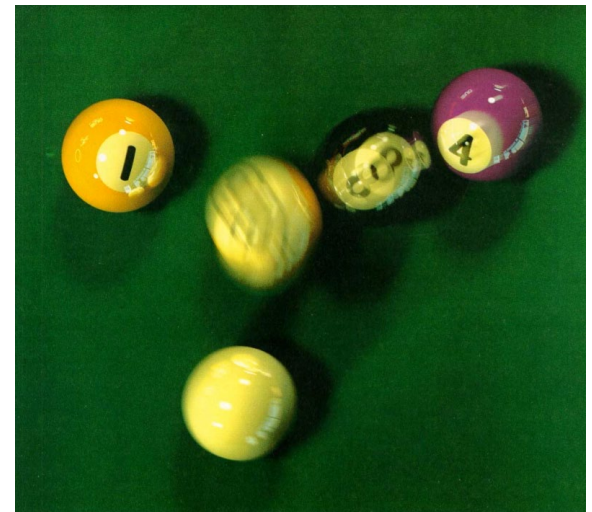
- In the previous lectures,
 - We studied light transport & MC integration
- To solve the MC integration, we need draw random samples. This process is called sampling.
- Why should we do sampling in rendering?

Review: Distributed Ray Tracing

- Motivation
 - The classical ray tracing produces very clean images (look fake)
 - Perfect focus
 - Perfect reflections
 - Sharp shadows
- [Cook et al. 1984]
 - The main idea is to replace the single ray with a distribution of rays
- Add randomness to rendering
 - Antialiasing
 - Soft shadows
 - Depth-of-field
 - Motion blur
 - Glossy reflections



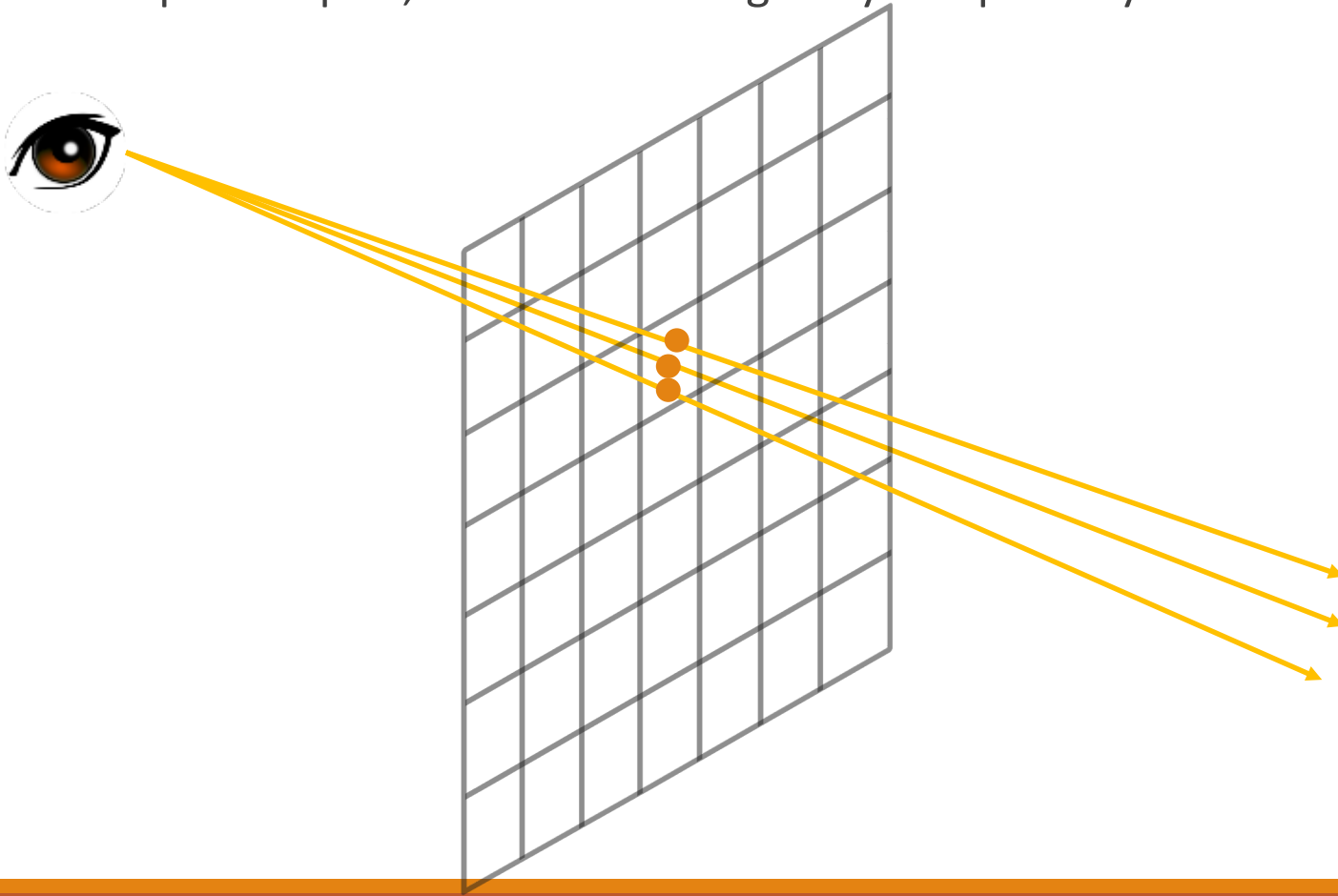
[Whitted 1980]



[Cook et al. 1984]

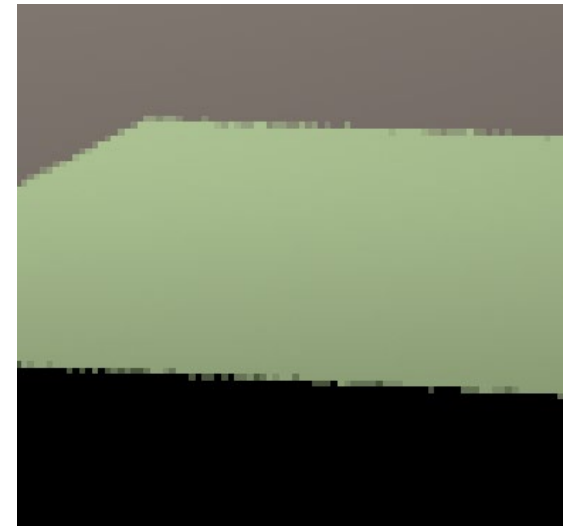
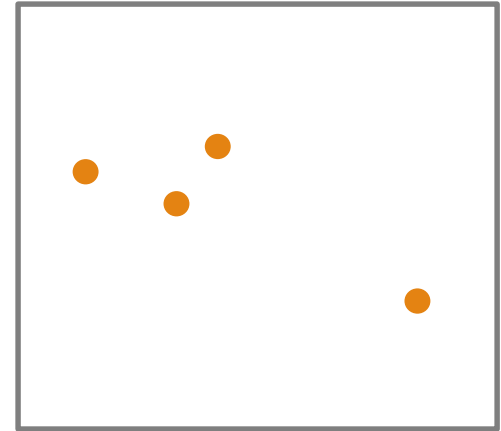
Review: Antialiasing

- To reduce image aliasing, we need to compute a pixel color by averaging multiple samples, instead of taking a ray sample only at the center point



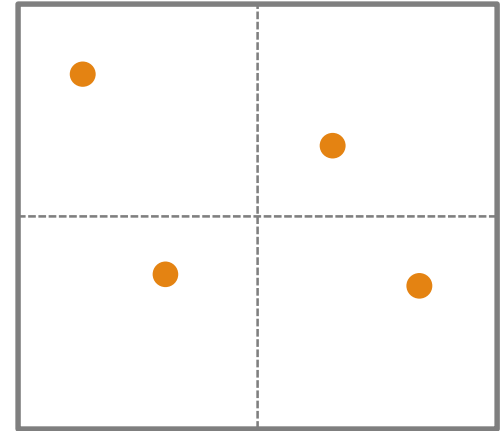
Review: Antialiasing

- e.g. four samples / pixel
- Random sampling: randomly generate n^2 rays
- for each pixel (x, y) do
 - $c(x, y) = 0$
 - for $p = 0$ to $n^2 - 1$ do
 - $c(x, y) = c(x, y) + \text{trace}(x + \epsilon_1, y + \epsilon_2)$
 - $c(x, y) = c(x, y) / n^2$
- $\epsilon \in [0,1)$ is a random number
- The regular pattern is converted into image noise

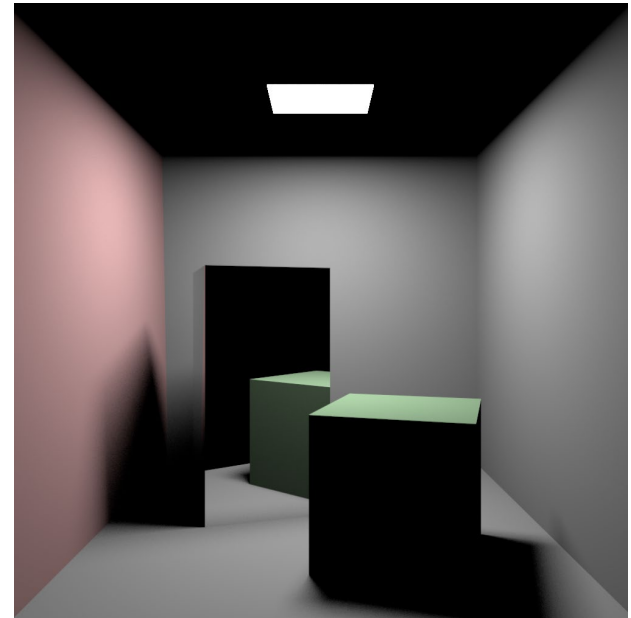
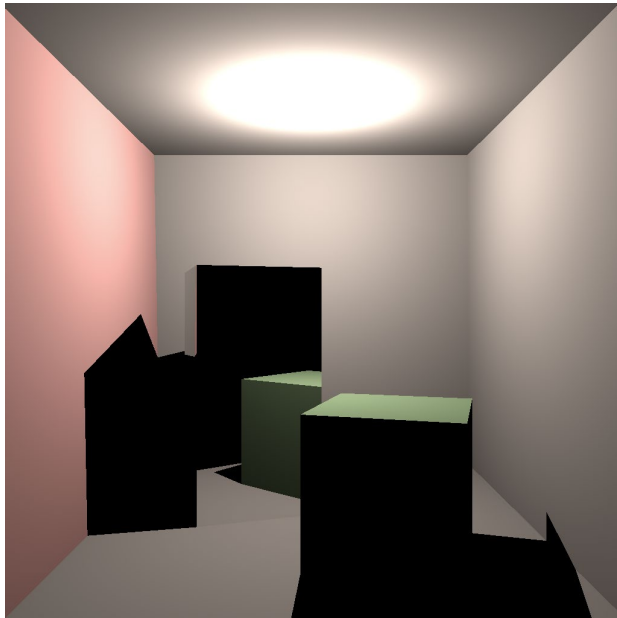


Review: Antialiasing

- e.g. four samples / pixel
- Jittering (stratified sampling): randomly generate a ray within each grid
- for each pixel (x, y) do
 - $c(x, y) = 0$
 - for $p = 0$ to $n - 1$ do
 - for $q = 0$ to $n - 1$ do
 - $c(x, y) = c(x, y) + \text{trace}\left(x + \frac{p+\epsilon_1}{n}, y + \frac{q+\epsilon_2}{n}\right)$
 - $c(x, y) = c(x, y)/n^2$
- This is a hybrid approach between the regular and random sampling

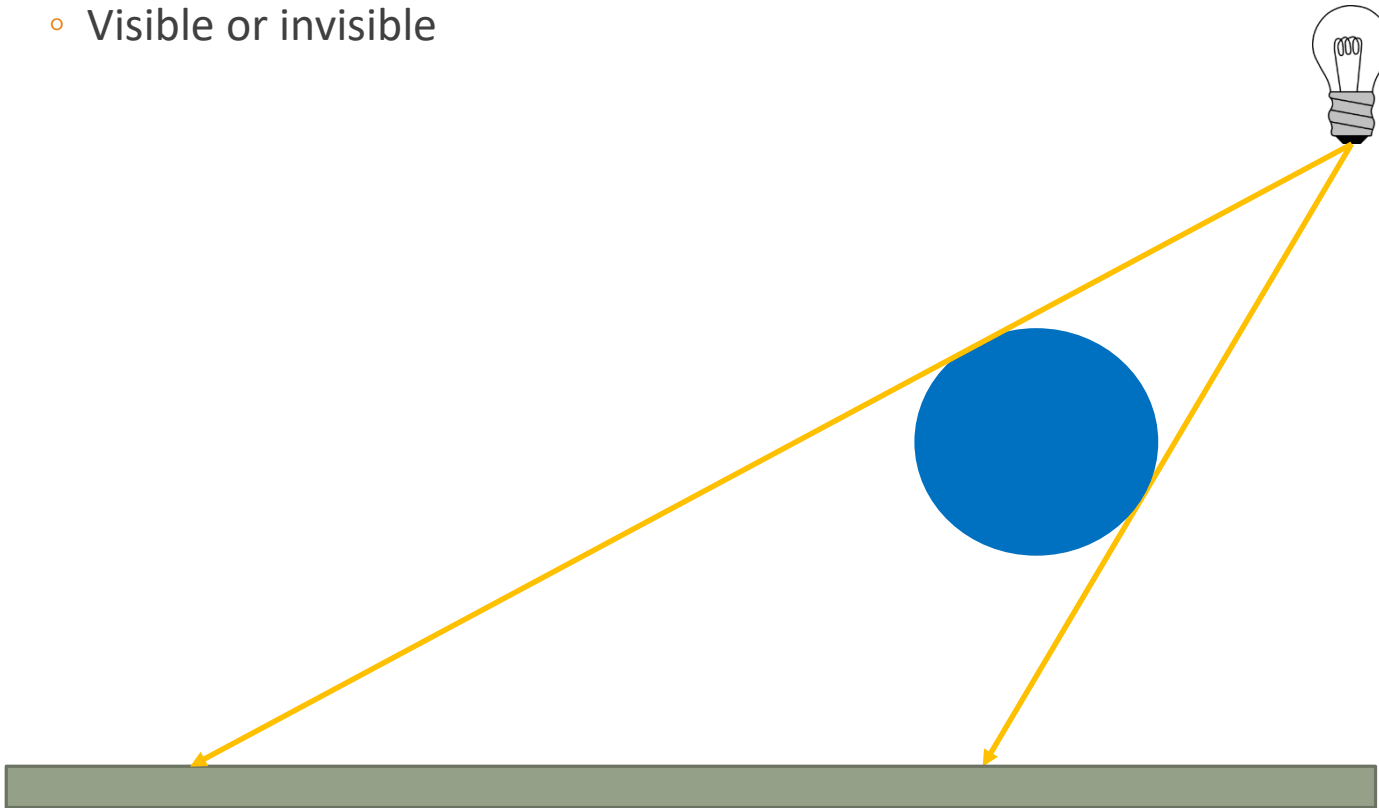


Review: Soft Shadows



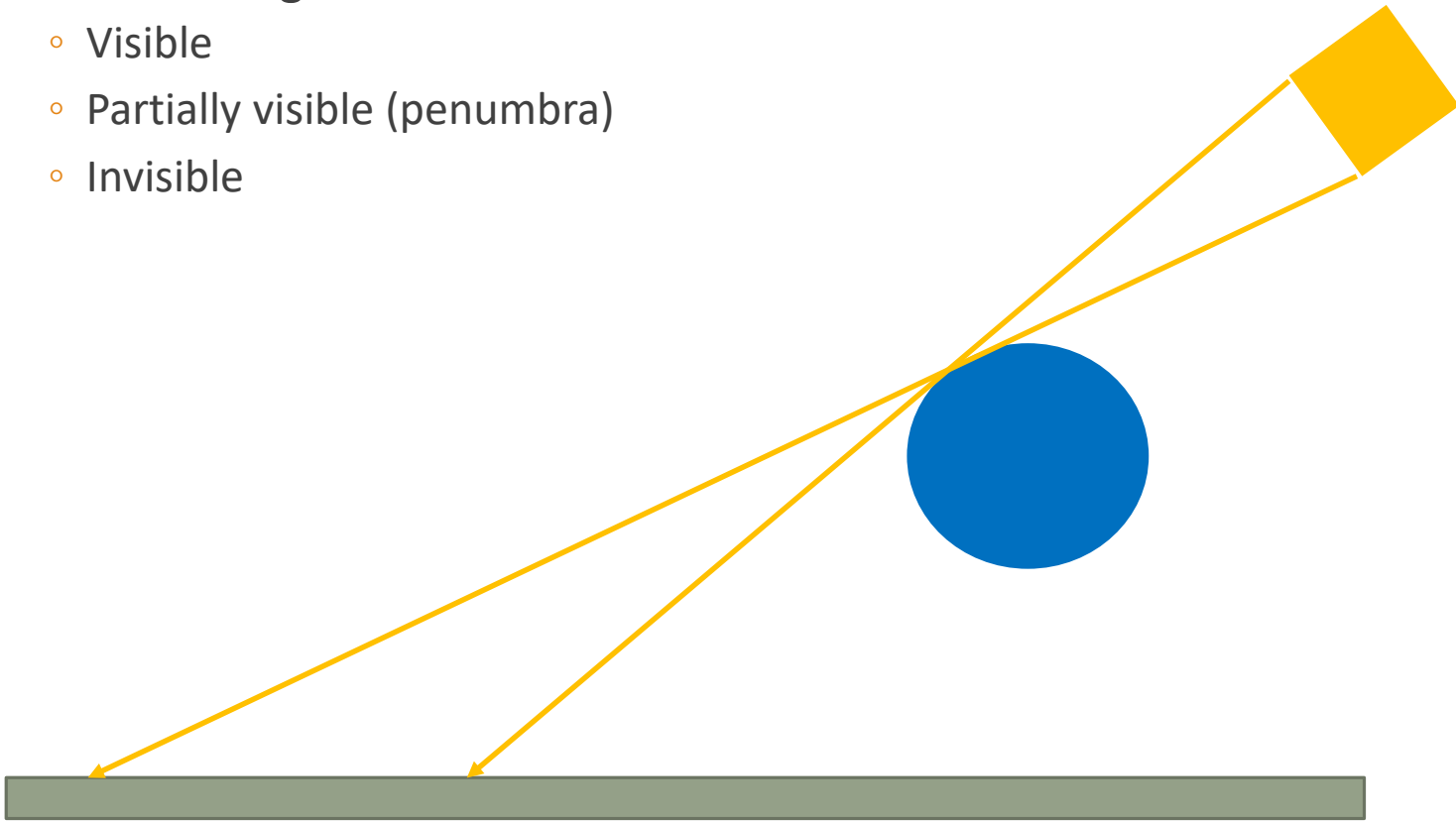
Review: Soft Shadows

- A point light source introduces hard shadows
 - Visible or invisible



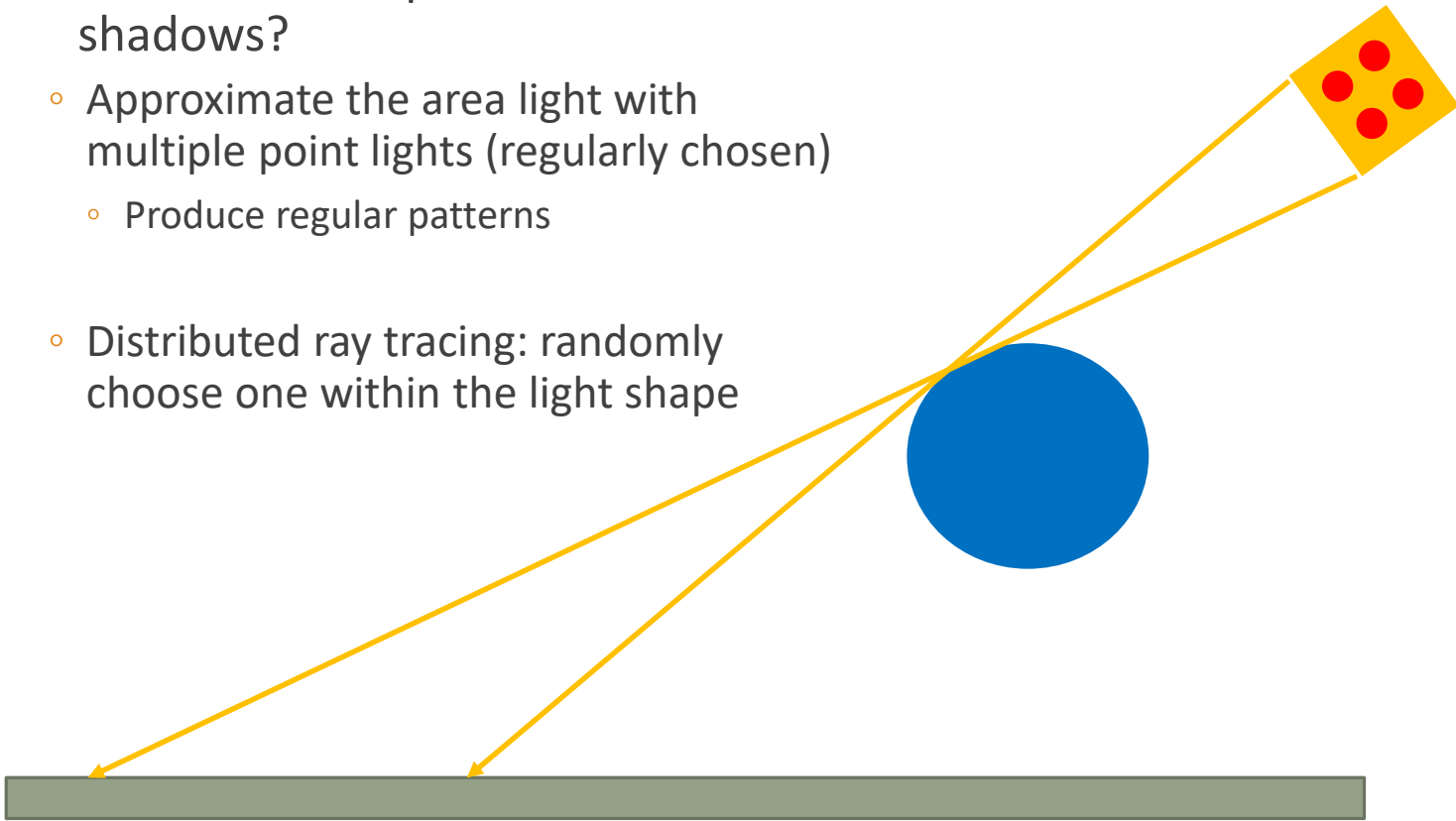
Review: Soft Shadows

- An area light source introduces soft shadows
 - Visible
 - Partially visible (penumbra)
 - Invisible



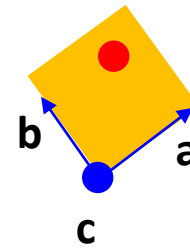
Review: Soft Shadows

- How can we implement the soft shadows?
 - Approximate the area light with multiple point lights (regularly chosen)
 - Produce regular patterns
 - Distributed ray tracing: randomly choose one within the light shape

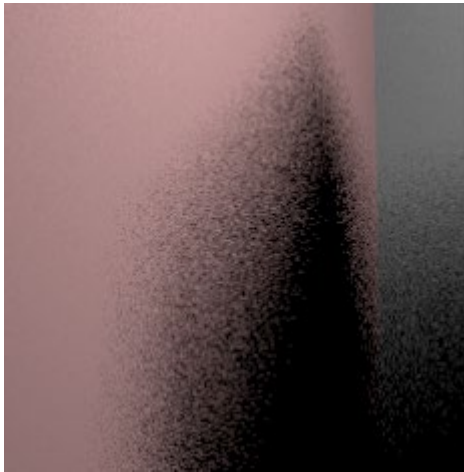


Review: Soft Shadows

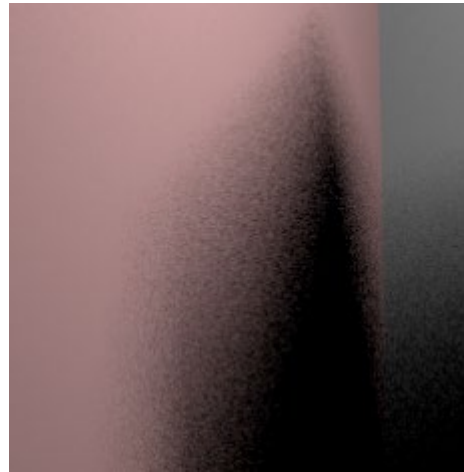
- e.g. area light defined as a parallelogram
 - Select a random point
 - $l_{pos} = c + \varepsilon_1 a + \varepsilon_2 b$
 - $\varepsilon_1, \varepsilon_2 \in [0,1)$
 - Generate a shadow ray from this point



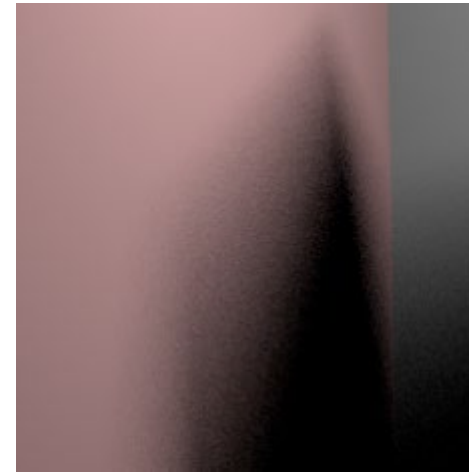
Review: Soft Shadows



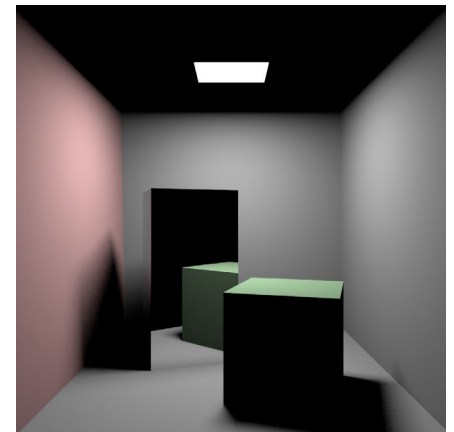
4 shadow rays / pixel



16 shadow rays / pixel

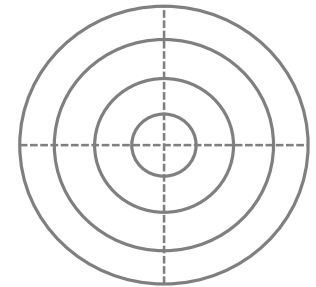
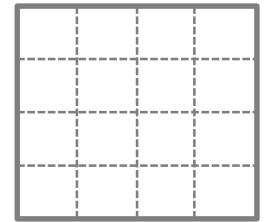


64 shadow rays / pixel



Random Point Selection

- Problem: draw N samples from $[0,1]^2$
 - i.e., provide $(x_1, y_1), \dots, (x_N, y_N)$
 - Techniques
 - Random sampling
 - Jittering
- Problem: draw N samples from a disk shape (a camera lens)
 - Randomly select u_i, v_i from $[0,1]$
 - $\phi_i = 2\pi u_i$
 - $r_i = v_i R$
- Note
 - This will cover all the area of a circle with a radius R
 - The downside is that the selected points will be distributed non-uniformly



Inverse Transform Sampling

- 1D density function $f(x)$ with $x \in [x_{min}, x_{max}]$
- Q. Can we generate random numbers α_i so that they can have density $f(x)$ using a set of uniform random numbers $\xi_i \in [0,1]$?
- Cumulative probability distribution function $F(x)$:
 - $P(\alpha < x) = F(x) = \int_{x_{min}}^x f(x')d\mu$
- $\alpha_i = F^{-1}(\xi_i)$
- Example:
 - $y = x^2$ or $f(x) = x^2$ ($x > 0$)
 - $x = \sqrt{y}$ or $f^{-1}(x) = \sqrt{x}$

Inverse Transform Sampling

- 1D density function $f(x)$ with $x \in [x_{min}, x_{max}]$
- Q. Can we generate random numbers α_i so that they can have density $f(x)$ using a set of uniform random numbers $\xi_i \in [0,1]$?
- Problem: generate random points x_i that have the following density:
 - $f(x) = \frac{3x^2}{2}$ on $[-1,1]$
 - 1. $F(x) = \frac{x^3+1}{2}$
 - 2. $F^{-1}(x) = \sqrt[3]{2x-1}$
 - 3. $(x_1, \dots, x_N) = (\sqrt[3]{2\xi_1-1}, \dots, \sqrt[3]{2\xi_N-1})$
 - Note
 - ξ can be jittered samples

Inverse Transform Sampling

- 2D density function $f(x, y)$ with $(x, y) \in [x_{min}, x_{max}] \times [y_{min}, y_{max}]$
- $P(\alpha_x < x \text{ and } \alpha_y < y) = F(x, y) = \int_{y_{min}}^y \int_{x_{min}}^x f(x', y') d\mu(x', y')$
- Steps.
 - Choose x_i using a marginal distribution $F(x, y_{max})$
 - Choose y_i according to $\frac{F(x_i, y)}{F(x_i, y_{max})}$

Inverse Transform Sampling

- Problem: you want to sample from a 2D disk uniformly from the 2D uniform random numbers in Cartesian coordinates

- $$P(\alpha \in S) = \int_S \frac{1}{\pi R^2} dA$$

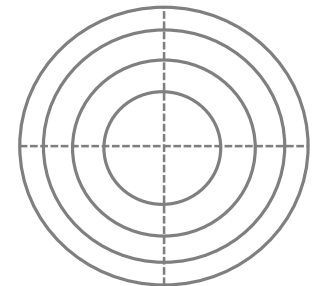
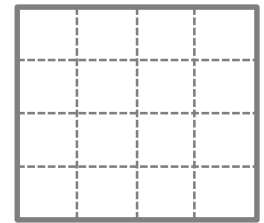
- Note: the differential area, $dA = r dr d\phi$

- Example:

- $$P(r < r_0 \text{ and } \phi < \phi_0) = F(r_0, \phi_0) = \int_0^{\phi_0} \int_0^{r_0} \frac{r dr d\phi}{\pi R^2} = \frac{\phi r^2}{2\pi R^2}$$

- Procedure.

- Generate two random numbers $\xi_1, \xi_2 \in [0,1]$
- Choose ϕ_i using a marginal distribution $F(r_{max}, \phi)$
 - $F(r_{max}, \phi) = \frac{\phi R^2}{2\pi R^2}$
 - $\phi = 2\pi\xi_1$
- Choose r_i according to $F(r|\phi_i) = \frac{F(r, \phi_i)}{F(r_{max}, \phi_i)}$
 - Similarly, $r = R\sqrt{\xi_2}$



Rejection

- Choose some random points according to a distribution and reject some of them
- Example: draw uniform random points within the unit circle
 - Choose uniform random samples $(x, y) \in [-1, 1]^2$
 - Reject the samples outside the circle
- Procedure
 - $i = 1$
 - *while* ($i < N$)
 - $x_i = -1 + 2 \times \text{rand}()$ // *rand()* will return uniform random sample in $[0, 1]$
 - $y_i = -1 + 2 \times \text{rand}()$
 - *if* ($x_i^2 + y_i^2 < r$)
 - $i = i + 1$

Rejection

- Choose some random points according to a distribution and reject some of them
- Pros
 - Very simple to code
- Cons
 - Can be very inefficient given complex scenarios