CT5202: Photorealistic Rendering

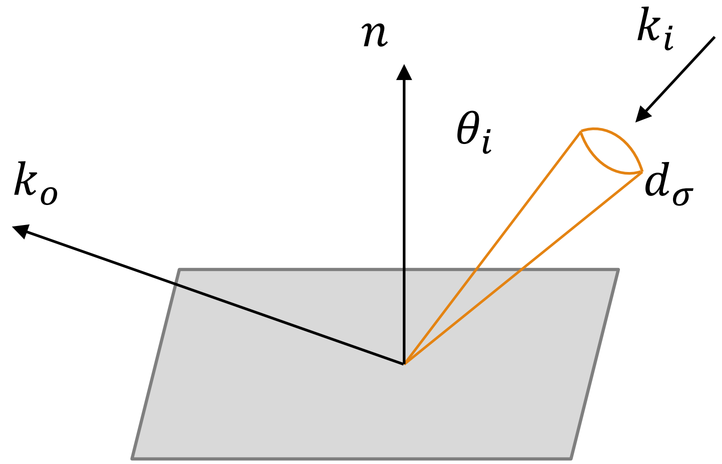
Light Transport Equation

Lecturer: Bochang Moon

Transport Equation

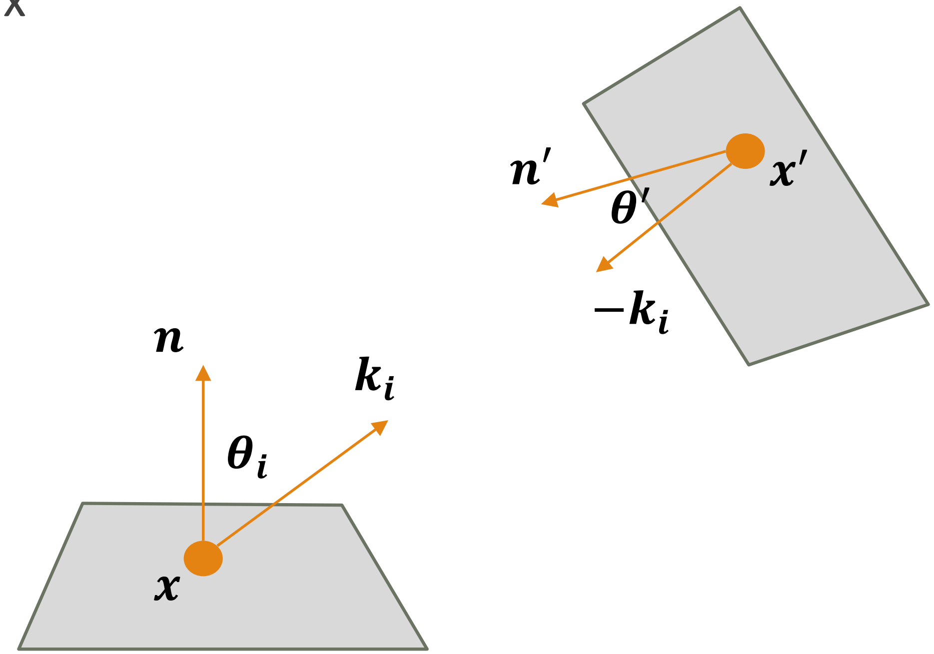
- $L_s(k_o) = \int_{all\ k_i} \rho(k_i, k_o) L_f(k_i) cos\theta_i d\sigma_i$
  - $L_f(k_i)$: field radiance from k_i direction
  - $L_s(k_o)$: surface radiance measured in k_o direction
  - *Rendering equation* [Immel, Cohen & Greenberg, 1986]

  - We can also write the equation with surface radiances only [Kajiya, 1986]



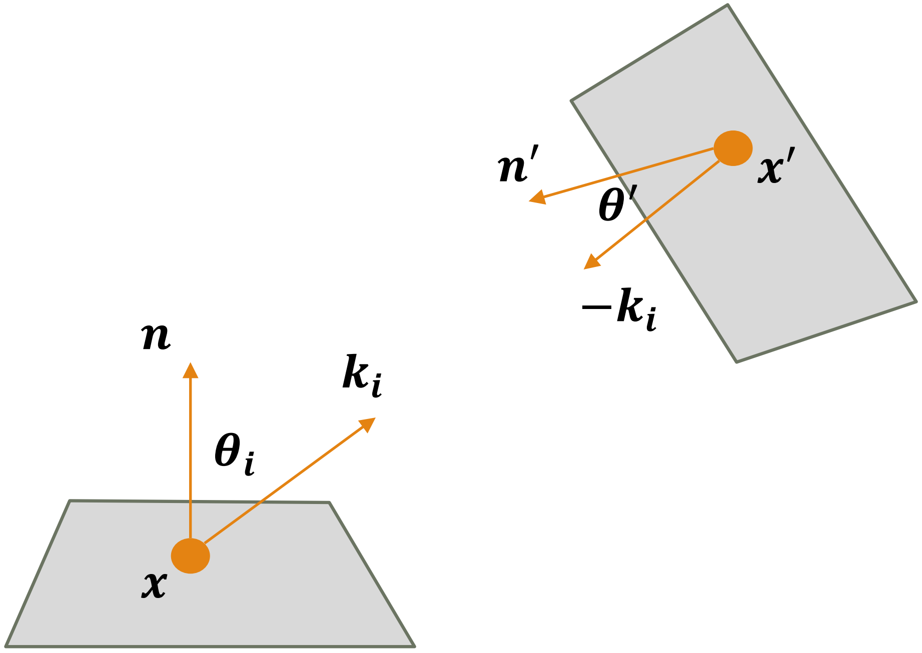
Transport Equation

- $L_s(k_o) = \int_{all\ k_i} \rho(k_i, k_o) L_f(k_i) cos\theta_i d\sigma_i$
- $L_s(-k_i) = L_f(k_i)$
- Solid angle subtended by the point x'
  - Area on a unit sphere
  - $\Delta\sigma_i = \frac{\Delta A' cos\theta'}{||x - x'||^2}$
  - $\Delta A'$ is the area associated with x'
- Differential solid angle
  - $d\sigma_i = \frac{dA' cos\theta'}{||x - x'||^2}$



Transport Equation

- $L_s(k_o) = \int_{all\ x'visible\ to\ x} \frac{\rho(k_i,k_o)L_s(x',x-x\prime)cos\theta_i cos\theta\prime}{||x-x'||^2} dA'$



Transport Equation

- $L_s(k_o) = \int_{all\ x'} \frac{\rho(k_i,k_o)L_s(x',x-x')v(x,x')cos\theta_i cos\theta'}{||x-x'||^2} dA'$
  - Rendering equation [Kajiya, 1986]
  - $v(x, x')$: visibility function
  - 1 if x and x' are mutually visible
  - 0 otherwise

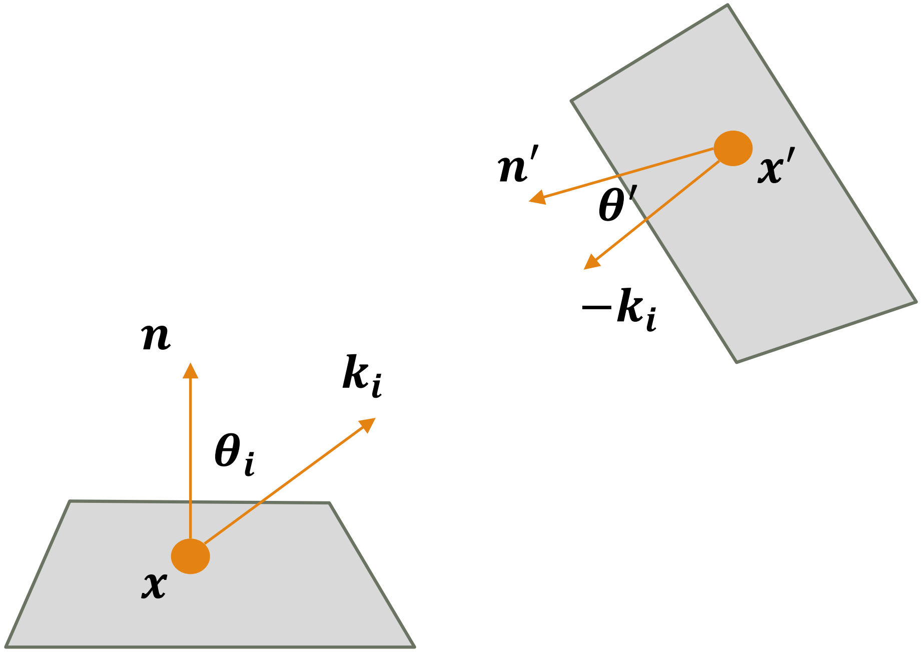


Image Contribution Function

- The value of pixel j:
- $I_j = \int_{\Omega} h_j(x)L(x)d\Omega$
  - Note that we aim to reconstruct a discretized 2D function (i.e., a raster image)
  - $L(x)$: radiance from x
  - h_j: a pixel reconstruction filter of pixel j
    - Typically it depends on the samples' image positions
    - What are the examples?
      - Box filter
      - Gaussian filter
      - Polynomial filters

Image Contribution Function

- The value of pixel j:
- $I_j = \int_{\Omega} h_j(x)L(x)d\Omega$
- Primary sample space formulation:
  - The integration domain Ω can be considered $[0,1)^d$
- e.g.,
  - A typical implementation of path tracing relies on a random number generation that draws uniform samples x
  - Light path vertices are determined by the random samples
  - It always returns a radiance value $L(x)$

Path Space Formulation

- The value of pixel j:
- $I_j = \int_{\mathrm{P}} h_j(\bar{x}) L(\bar{x}) d\mu(\bar{x})$
  - $\bar{x} = \{x_0, \dots, x_n\}$
  - $\bar{x}$ is the path vertices that define a light path from the image plane to light sources
  - x_0: the position on the image plane

  - This is called path integral formulation [Veach and Guibas 1995, 1997]

Conclusion

- The integral of light transport equation can be represented in multiple ways
- Which ones should be used?
  - It depends on the light transport algorithm that you actually relies on.

Paper Presentation & Project

- Each student should select an interesting problem, which is related to this course

  - Present two papers (20 min. presentation + 20 min. Q&A)