

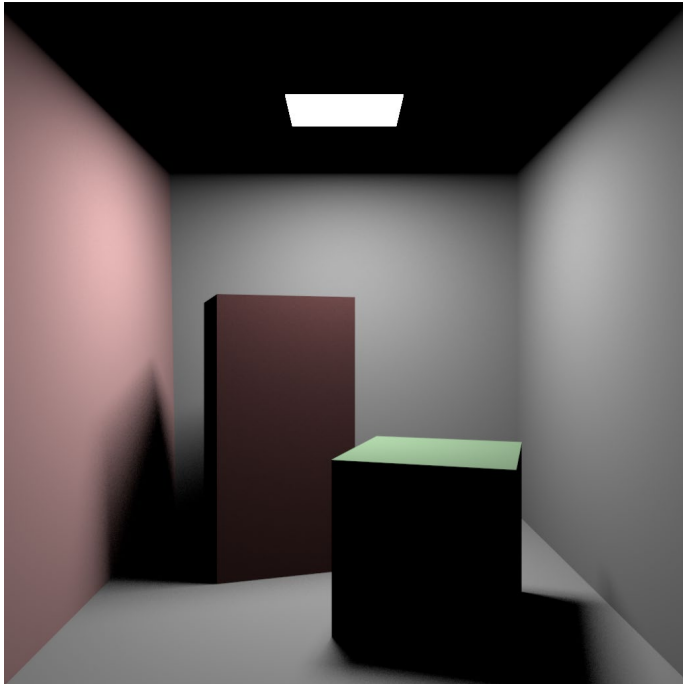
CT5202: Photorealistic Rendering

Global Illumination

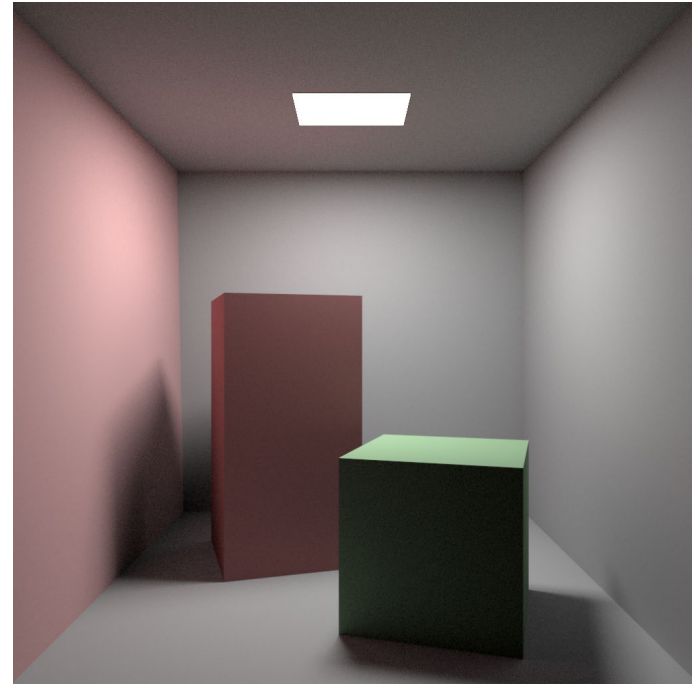
Lecturer: Bochang Moon

Global Illumination

- Global illumination methods consider both direct and indirect lighting



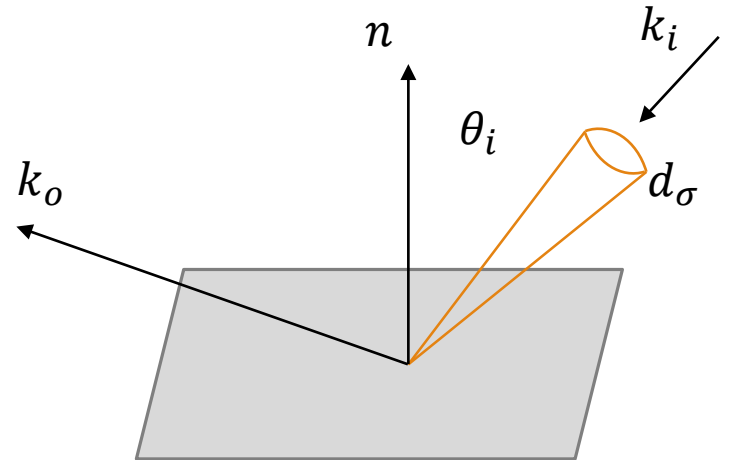
Without indirect lighting



With indirect lighting

Radiosity

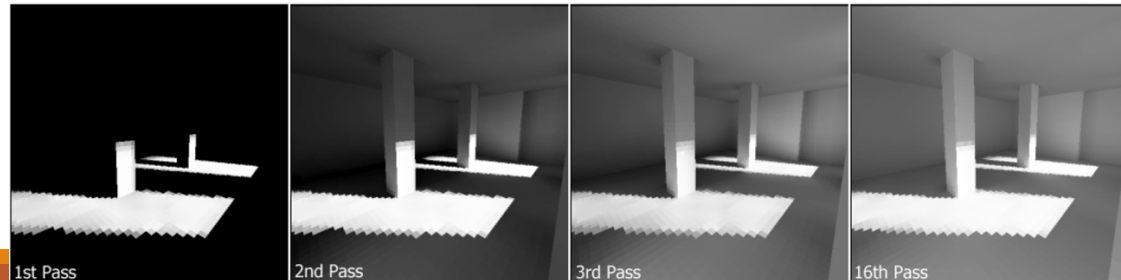
- $L_S(k_o) = \int_{\text{all } k_i} \rho(k_i, k_o) L_f(k_i) \cos\theta_i d\sigma_i$
- Lambertian surfaces (ideal diffuse surface)
 - $\rho(k_i, k_o) = \frac{R}{\pi}$
 - R: diffuse reflectance
- Assumption: all surfaces are Lambertian
- $L_S(k_o) = \int_{\text{all } k_i} \rho(k_i, k_o) L_f(k_i) \cos\theta_i d\sigma_i$
- $= \frac{R}{\pi} \int_{\text{all } k_i} L_f(k_i) \cos\theta_i d\sigma_i$



Radiosity

- $L_s(k_o) = \frac{R}{\pi} \int_{all\ k_i} L_f(k_i) \cos\theta_i d\sigma_i$
- Finite element methods
 - Divide the scene into N small surfaces (patches) with unknown surface radiance L_i , reflectance R_i , and emitted radiance E_i
 - Then, the integral can be approximated with the N linear equations below:
 - $L_i = E_i + \frac{R_i}{\pi} \sum_{j=1}^N k_{ij} L_j$
 - k_{ij} : a constant related to the integral (form factor)
 - Fraction of light leaving a patch i arriving at a patch j
 - This results in N constant-colored polygons
 - Called *radiosity*

from wikipedia



Path Tracing

- $L_s(k_o) = L_e(k_o) + \int_{\text{all } k_i} \rho(k_i, k_o) L_f(k_i) \cos\theta_i d\sigma_i$
- Monte Carlo integration
 - $\int_{x \in S} g(x) d\mu \approx \frac{1}{N} \sum_{i=1}^N \frac{g(x_i)}{p(x_i)}$
 - When $N=1$,
 - $L_s(k_o) \approx L_e(k_o) + \frac{\rho(k_i, k_o) L_f(k_i) \cos\theta_i}{p(k_i)}$
 - Need to do:
 - Select a random direction k_i
 - Evaluate $L_f(k_i)$

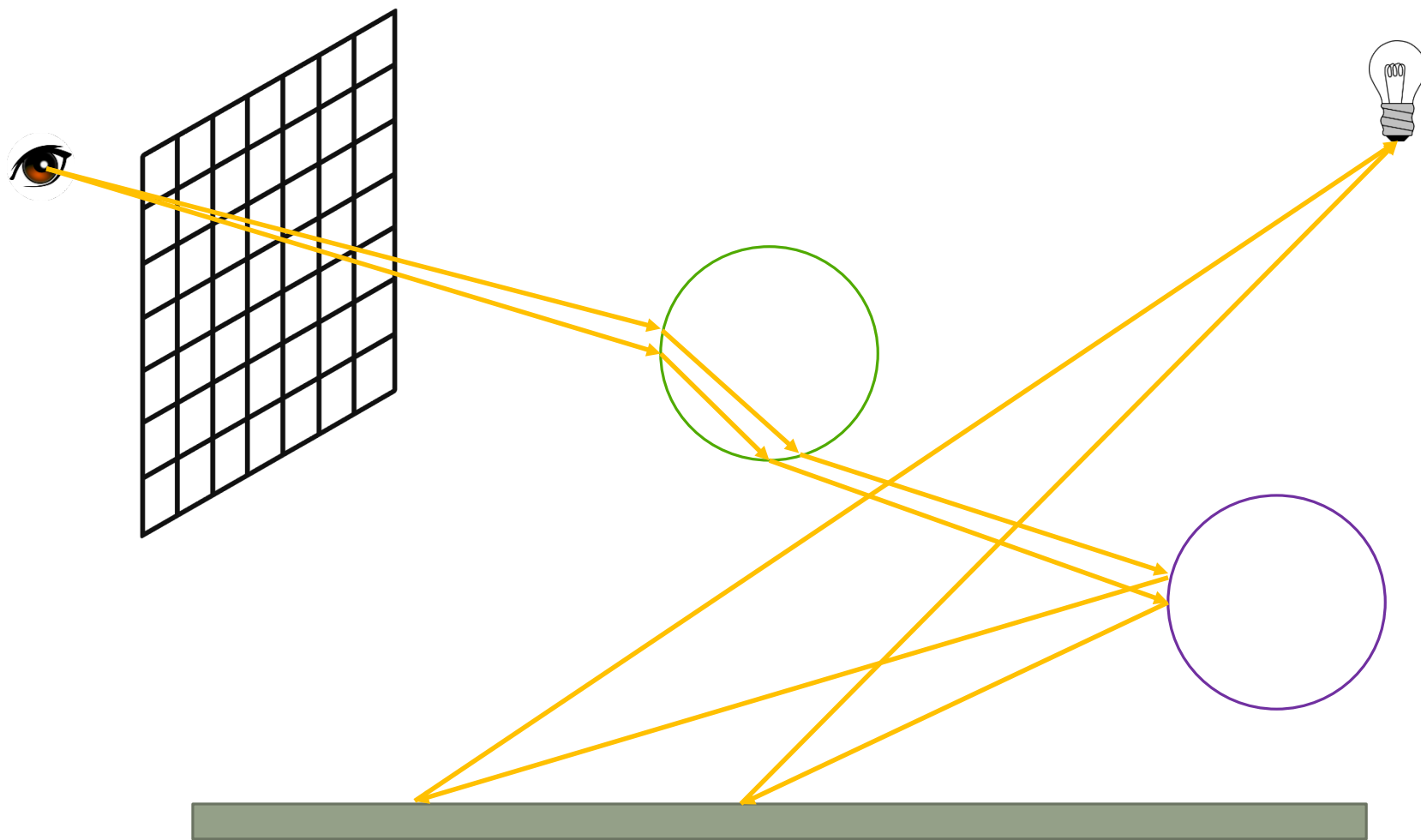
Path Tracing

- $L_S(k_o) \approx L_e(k_o) + \frac{\rho(k_i, k_o) L_f(k_i) \cos\theta_i d\sigma_i}{p(k_i)}$
- In case of the ideal diffuse surface:
 - $\rho = \frac{R}{\pi}$
 - When we choose a density function $p(k_i) = \frac{\cos\theta_i}{\pi}$
 - $L_S(k_o) \approx L_e(k_o) + RL_f(k_i)$
 - Note that we can cancel out the cosign terms

Path Tracing

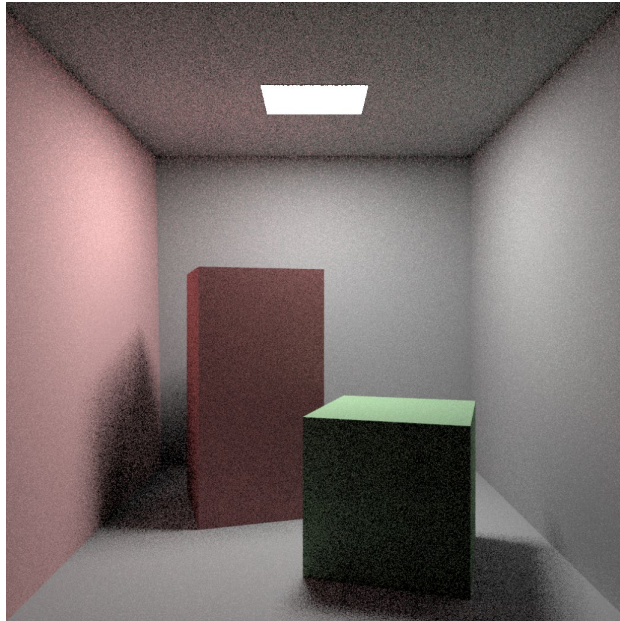
- Procedure
 - *RGB trace(ray $a + tb$, int depth)*
 - *if (ray hits at a point c)*
 - *RGB col = $L_e(-b)$*
 - *if (depth < maxdepth)*
 - *compute a random direction d*
 - *return col + $R \times \text{trace}(c + sd, \text{depth} + 1)$*
 - *else*
 - *return background color*

Path Tracing

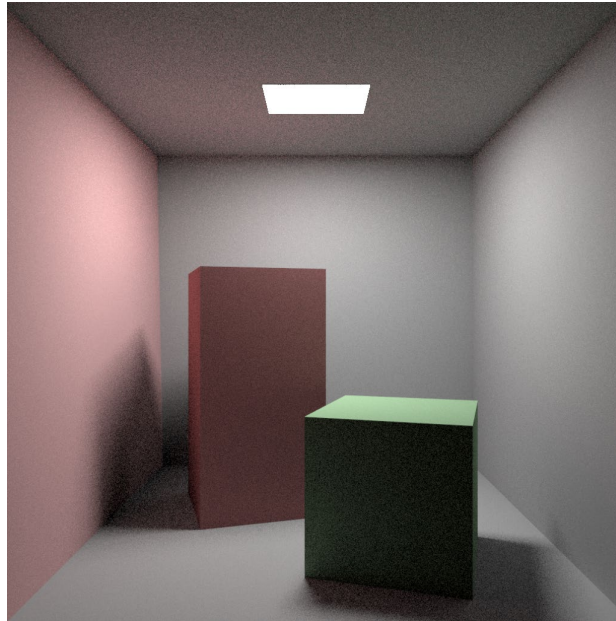


Path Tracing

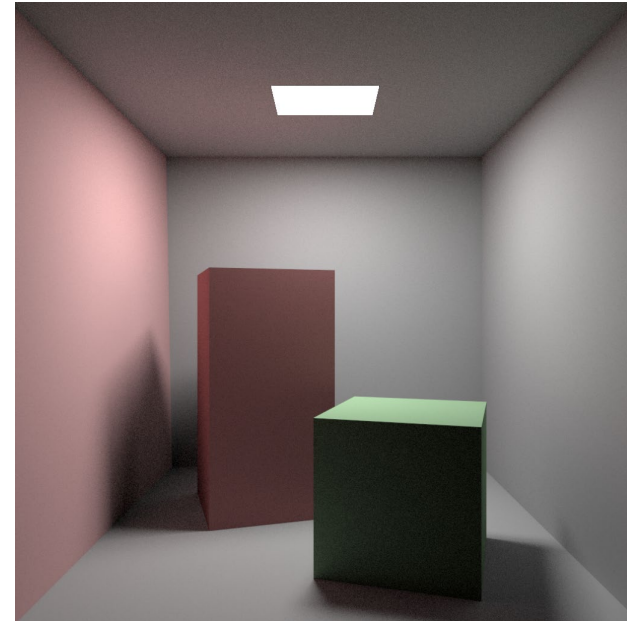
- A general rendering method that solves the full light transport equation (i.e., rendering equation)
- For each pixel color, it makes multiple ray paths, then averages the colors from the ray paths



4 samples / pixel (1.25 secs)



16 samples / pixel (5 secs)



64 samples / pixel (20 secs)

Direct Lighting

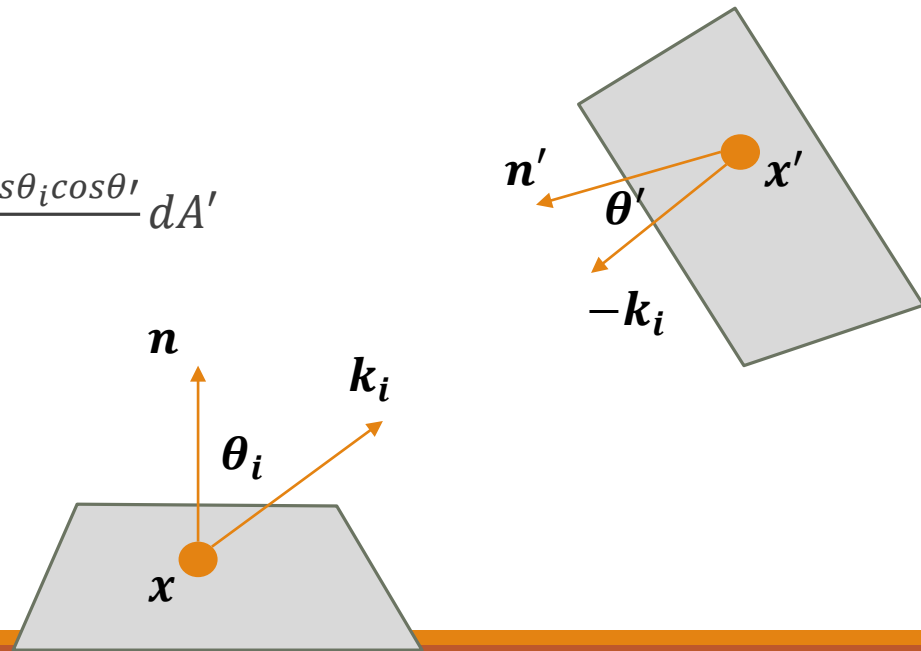
- GI methods include the direct lighting as well as the indirect lighting

- Full light transport

- $$L_S(x, k_o) = \int_{all\ x'} \frac{\rho(k_i, k_o) L_S(x', x-x') v(x, x') \cos\theta_i \cos\theta'}{\|x-x'\|^2} dA'$$

- Light transport for direct lighting

- $$L_S(x, k_o) = \int_{all\ x'} \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta'}{\|x-x'\|^2} dA'$$



Direct Lighting

- $$L_S(x, k_o) = \int_{\text{all } x'} \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta'}{\|x - x'\|^2} dA'$$
- Sample a point x' on a luminaire with density function p ($x' \sim p$)
- $$L_S(x, k_o) \approx \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta'}{p(x') \|x - x'\|^2}$$
- Pick a uniform random point x' from the luminaire
 - $p = \frac{1}{A}$ (A is the area of the luminaire)
 - $$L_S(x, k_o) \approx \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta' A}{\|x - x'\|^2}$$

Direct Lighting

- $$L_S(x, k_o) \approx \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta' A}{\|x - x'\|^2}$$
- Procedure for a planar (e.g., rectangular) light
 - *RGB directLight(x, k_o, n)*
 - *Sample a random point x' with normal vector n' on a light*
 - $d = x' - x$
 - $k_i = \frac{d}{\|d\|}$
 - *if (ray x + td does not hit any objects between the origin and light)*
 - *return $\rho(k_i, k_o) L_e(x', -k_i) \max(0, n \cdot d) \max(0, -n' \cdot d) / \|d\|^4$ (note: $n \cdot d = \|d\| \cos\theta$)*
 - *else*
 - *return 0*