

CT5202: Photorealistic Rendering

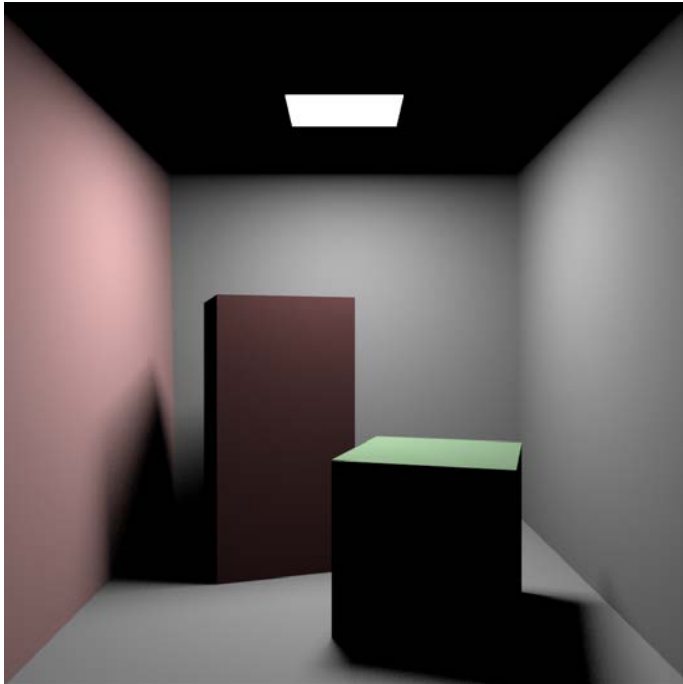
# Global Illumination

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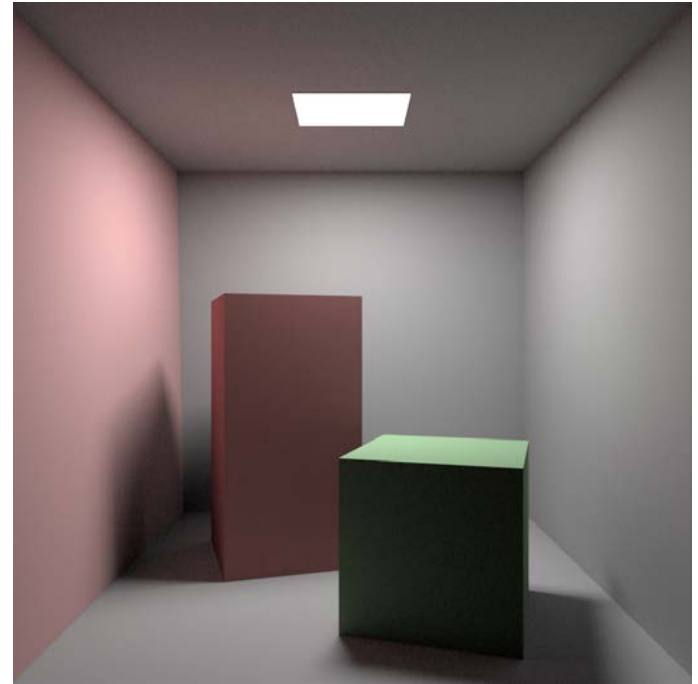
BOCHANG MOON

# Global Illumination

- Global illumination methods consider both direct and indirect lighting



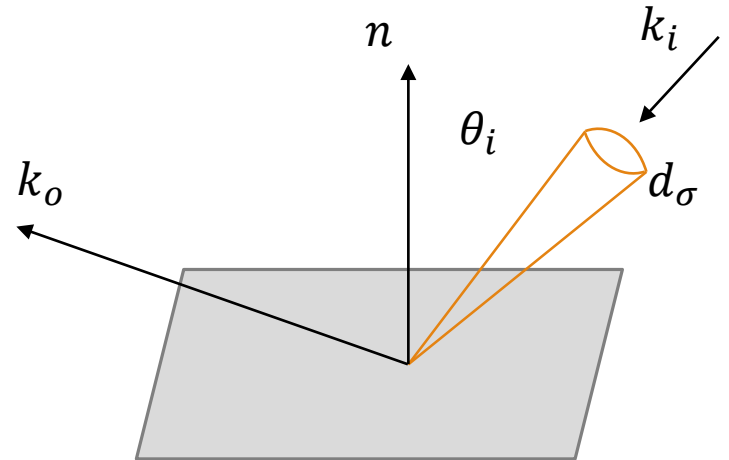
Without indirect lighting



With indirect lighting

# Radiosity

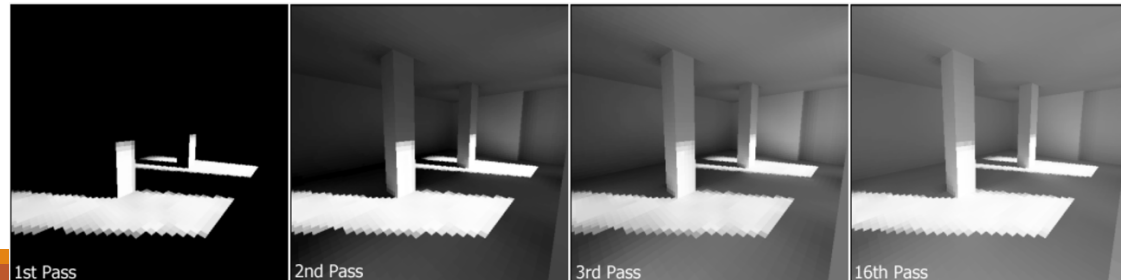
- $L_S(k_o) = \int_{\text{all } k_i} \rho(k_i, k_o) L_f(k_i) \cos\theta_i d\sigma_i$
- Lambertian surfaces (ideal diffuse surface)
  - $\rho(k_i, k_o) = \frac{R}{\pi}$
  - R: diffuse reflectance
- Assumption: all surfaces are Lambertian
- $L_S(k_o) = \int_{\text{all } k_i} \rho(k_i, k_o) L_f(k_i) \cos\theta_i d\sigma_i$
- $= \frac{R}{\pi} \int_{\text{all } k_i} L_f(k_i) \cos\theta_i d\sigma_i$



# Radiosity

- $L_s(k_o) = \frac{R}{\pi} \int_{all\ k_i} L_f(k_i) \cos\theta_i d\sigma_i$
- Finite element methods
  - Divide the scene into N small surfaces (patches) with unknown surface radiance  $L_i$ , reflectance  $R_i$ , and emitted radiance  $E_i$
  - Then, the integral can be approximated with the N linear equations below:
  - $L_i = E_i + \frac{R_i}{\pi} \sum_{j=1}^N k_{ij} L_j$
  - $k_{ij}$ : a constant related to the integral (form factor)
    - Fraction of light leaving a patch i arriving at a patch j
  - This results in N constant-colored polygons
  - Called *radiosity*

from wikipedia



# Path Tracing

- $L_s(k_o) = L_e(k_o) + \int_{\text{all } k_i} \rho(k_i, k_o) L_f(k_i) \cos\theta_i d\sigma_i$
- Monte Carlo integration
  - $\int_{x \in S} g(x) d\mu \approx \frac{1}{N} \sum_{i=1}^N \frac{g(x_i)}{p(x_i)}$
  - When  $N=1$ ,
  - $L_s(k_o) \approx L_e(k_o) + \frac{\rho(k_i, k_o) L_f(k_i) \cos\theta_i}{p(k_i)}$
  - Need to do:
    - Select a random direction  $k_i$
    - Evaluate  $L_f(k_i)$

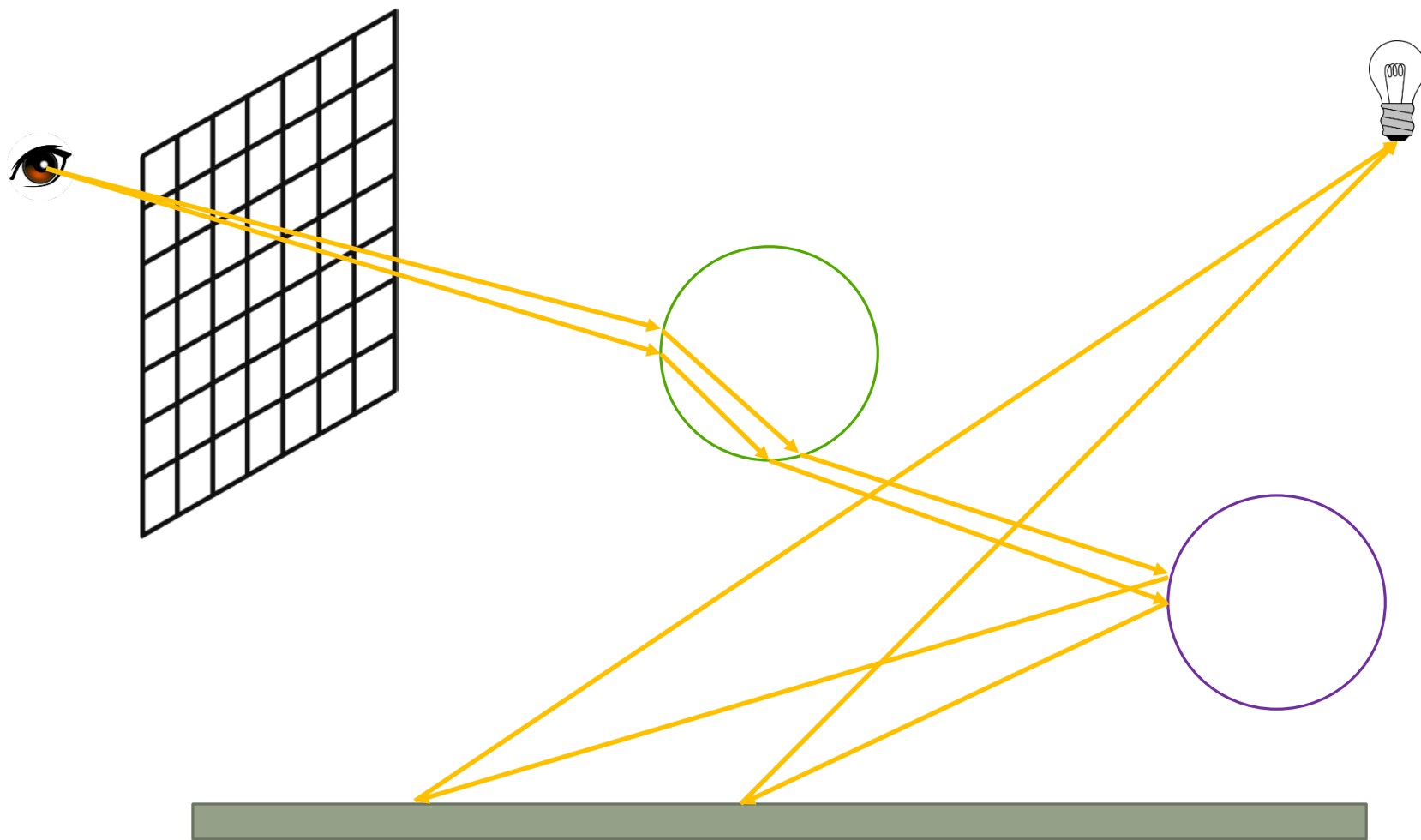
# Path Tracing

- $L_S(k_o) \approx L_e(k_o) + \frac{\rho(k_i, k_o) L_f(k_i) \cos\theta_i d\sigma_i}{p(k_i)}$
- In case of the ideal diffuse surface:
  - $\rho = \frac{R}{\pi}$
  - When we choose a density function  $p(k_i) = \frac{\cos\theta_i}{\pi}$ 
    - $L_S(k_o) \approx L_e(k_o) + RL_f(k_i)$
    - Note that we can cancel out the cosign terms

# Path Tracing

- Procedure
  - *RGB trace(ray a + tb, int depth)*
  - *if (ray hits at a point c)*
    - *RGB col =  $L_e(-b)$*
    - *if (depth < maxdepth)*
      - *compute a random direction d*
      - *return col + R × trace(c + sd, depth + 1)*
  - *else*
    - *return background color*

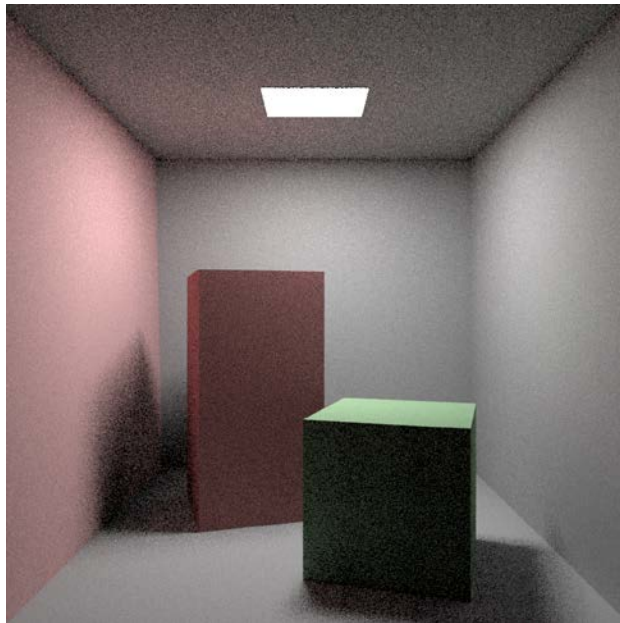
# Path Tracing



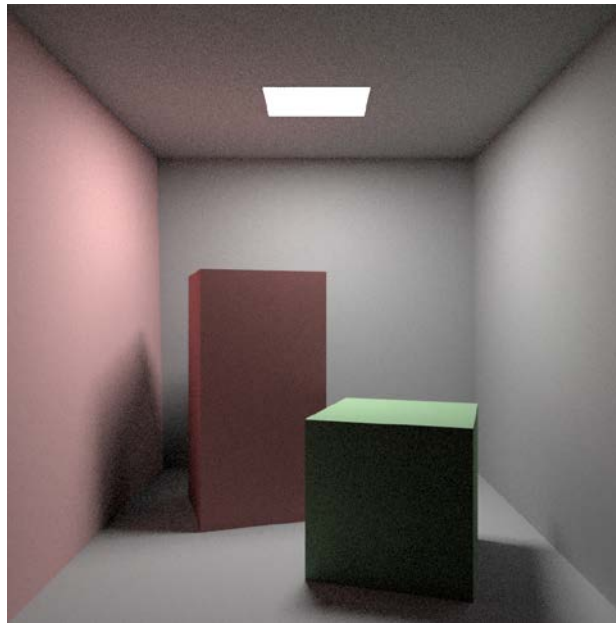


# Path Tracing

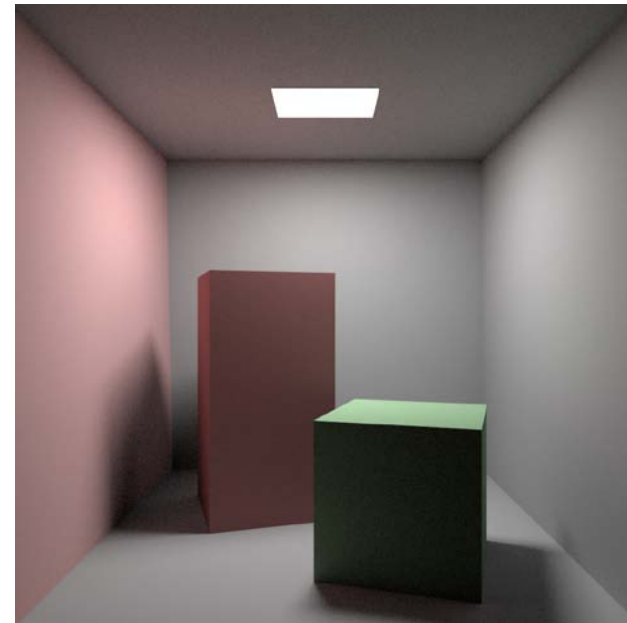
- A general rendering method that solves the full light transport equation (i.e., rendering equation)
- For each pixel color, it makes multiple ray paths, then averages the colors from the ray paths



4 samples / pixel (1.25 secs)



16 samples / pixel (5 secs)



64 samples / pixel (20 secs)

# Direct Lighting

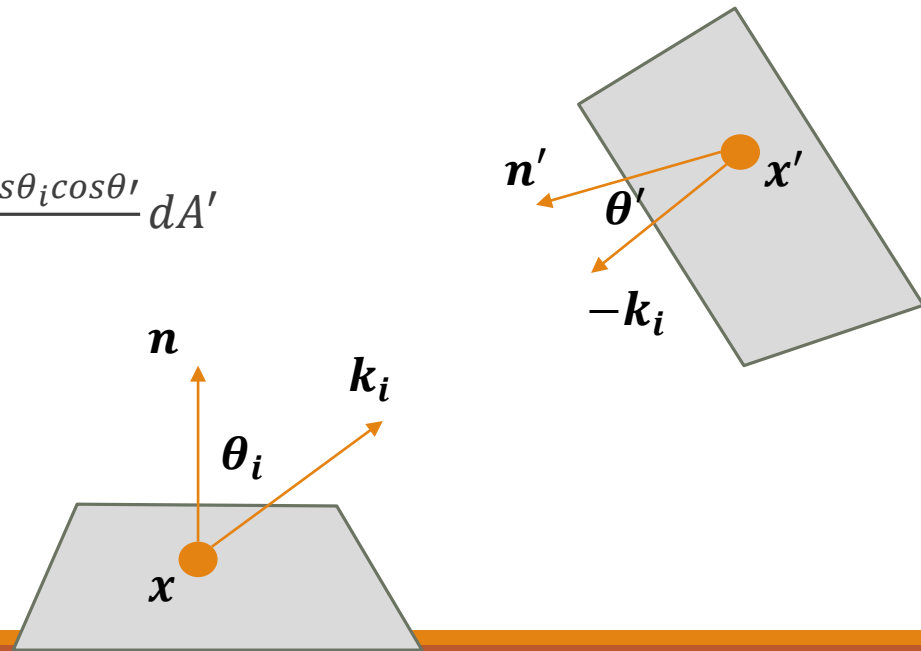
- GI methods include the direct lighting as well as the indirect lighting

- Full light transport

- $$L_S(x, k_o) = \int_{all\ x'} \frac{\rho(k_i, k_o) L_S(x', x-x') v(x, x') \cos\theta_i \cos\theta'}{\|x-x'\|^2} dA'$$

- Light transport for direct lighting

- $$L_S(x, k_o) = \int_{all\ x'} \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta'}{\|x-x'\|^2} dA'$$



# Direct Lighting

- $$L_S(x, k_o) = \int_{all\ x'} \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta'}{\|x - x'\|^2} dA'$$
- Sample a point  $x'$  on a luminaire with density function  $p$  ( $x' \sim p$ )
- $$L_S(x, k_o) \approx \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta'}{p(x') \|x - x'\|^2}$$
- Pick a uniform random point  $x'$  from the luminaire
  - $p = \frac{1}{A}$  (A is the area of the luminaire)
  - $$L_S(x, k_o) \approx \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta' A}{\|x - x'\|^2}$$

# Direct Lighting

- $$L_S(x, k_o) \approx \frac{\rho(k_i, k_o) L_e(x', -k_i) v(x, x') \cos\theta_i \cos\theta' A}{\|x - x'\|^2}$$
- Procedure for a planar (e.g., rectangular) light
  - *RGB directLight(x, k<sub>o</sub>, n)*
  - *Sample a random point x' with normal vector n' on a light*
  - $d = x' - x$
  - $k_i = \frac{d}{\|d\|}$
  - *if (ray x + td does not hit any objects between the origin and light)*
    - *return  $\rho(k_i, k_o) L_e(x', -k_i) \max(0, n \cdot d) \max(0, -n' \cdot d) / \|d\|^4$  (note:  $n \cdot d = \|d\| \cos\theta$ )*
  - *else*
    - *return 0*